$L-R$ 형 퍼지수의 집합-이론적 연산자들에 관한 연구

On the set-theoretic operations of $L-R$ type fuzzy numbers

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<Abstract>

In this paper, we define a concept of some set-theoretical operations of $L-R$ type fuzzy numbers and discuss some properties of these concepts. Using these results, we discuss a concept of cardinality of type-two fuzzy sets.

1. Introduction

Let $X=\{x_1, x_2, \cdots, x_n\}$ be a finite set. A fuzzy set M in X is defined by

$$M=\{(x, \mu_M(x)) \mid x \in X\}$$

where $\mu_M : X \to [0,1]$ is the membership function of M. The complement $\overline{M}$ of a fuzzy set M has the membership function $\mu_{\overline{M}} = 1-\mu_M$. Zadeh[15, 16], A. Ralescu[9], D. Ralescu[10], Wygralak[12], Dubois and Prade[1,2,3], and Yager[14] investigated concepts of cardinality of a fuzzy set and obtained some properties of these concepts.

In this paper, we define the set-theoretic operations; max*, min*, and complement of $L-R$ type fuzzy numbers on $[0,1]$, and investigate some properties of these operations. Using these properties , we define a new concept of cardinality of type-two fuzzy sets. Furthermore, we obtain several properties results of this concept.

2. Preliminaries and definitions.

In this section, we introduce fuzzy numbers , L-R fuzzy numbers, and some operations of $L-R$ type fuzzy numbers.

Definition 2.1 [2] A fuzzy number is a fuzzy set M of the interval $[0,1]$ if
(1) M is normal, i.e. $\exists_1\ x_0 \in [0,1]$ such that $\mu_M(x_0)=1$;
(2) M is convex, i.e. $\mu_M(\lambda x+(1-\lambda)y) \geq \min(\mu_M(x), \mu_M(y))$ for all $x, y, \lambda \in [0,1]$;
(3) μ_M is piecewise continuous.

Let L and R be strictly decreasing continuous functions from $[0,1]$ to $[0,1]$ such that $L(0)=R(0)=1$ and $L(1)=R(1)=0$. Then, L and R is called the left and the right shape function, respectively(see [7]).

Definition 2.2 [2,7] A fuzzy number M is said to be an $L-R$ type fuzzy number it its membership function is defined by

$$\mu_M(x)=\begin{cases} L\left(\frac{m-x}{\alpha}\right) & \text{for } m-\alpha \leq x \leq m \leq 1, m \geq \alpha > 0 \\ R\left(\frac{x-m}{\beta}\right) & \text{for } m+\beta \geq x \geq m \geq 0, 1-m \geq \beta > 0 \\ 0 & \text{otherwise} \end{cases}$$

Symbolically, we write $M=(m,\alpha,\beta)_{LR}$ and denote that A_{LR} is the class of all such $L-R$ type fuzzy numbers.

Definition 2.3 Let $M=(m,\alpha,\beta)_{LR}$ and $N=(n,\gamma,\delta)_{LR}$ belong to A_{LR}. Then $\max^*(M,N)$ and $\min^*(M,N)$ are defined by

$$\max{}^*\{M,N\}=(m \vee n, \alpha \wedge \beta, \beta \vee \delta)_{LR} ;$$

$$\min{}^*\{M,N\}=(m \wedge n, \alpha \vee \beta, \beta \wedge \delta)_{LR}.$$

Using the next proposition, we can discuss the order of $L-R$ type fuzzy numbers.

Proposition 2.4 Let $M=(m,\alpha,\beta)_{LR}$ and $N=(n,\gamma,\delta)_{LR}$ be elements of A_{LR}. Then, we have

$$\max{}^*(M,N)=M, \quad \min{}^*(M,N)=N \quad \text{if and only if} \quad m \leq n, \ \alpha \geq \gamma, \text{ and } \beta \leq \delta$$

Using the definition([2,3]) of substraction $M \ominus N$ of fuzzy numbers M, N and the Eq.(4.2.1) of [3], we can obtain the following proposition.

Proposition 2.5 [3] (1) If $M=(m,\alpha,\beta)_{LR}$ and $N=(n,\gamma,\delta)_{RL}$ is a element of A_{LR} and A_{RL}, respectively, then $M \ominus N=(m-n, \alpha+\gamma, \beta+\delta)_{LR}$.
(2) If $M=(m,\alpha,\beta)_{LR}$ and $\tilde{1}=(1,0,0)_{RL}$ is a element of A_{LR} and A_{RL}, respectively, then $\tilde{1} \ominus M=(1-m,\beta,\alpha)_{RL}$.

Using the proposition 2.5(2), we can the following complement of $L-R$ type fuzzy numbers.

Definition 2.6 Let $M=(m,\alpha,\beta)_{LR}$ and $\tilde{1}=(1,0,0)_{RL}$ be a element of A_{LR} and A_{RL}, respectively. The complement $\overline{M}^*$ of M is defined by

$$\overline{M}^* \equiv \tilde{1} \ominus M = (1-m, \beta, \alpha)_{RL}$$

3. Type-two fuzzy cardinality.

Let $X=\{x_1, x_2, \cdots, x_n\}$ be a finite set. Using the set-theoretic operations of $L-L$ type fuzzy numbers on $[0,1]$, we define the following new concept of type-two fuzzy cardinality.

Definition 3.1 Let $F: X \to A_{LL}$ be a type-two fuzzy set and $F(x_k)=M_k=(m_k, \alpha_k, \beta_k)_{LL}$ for $k=1,2,\cdots,n$. Then, a type-two fuzzy cardinality of F is a function f_2 -card F: $\{0,1,\cdots,n\} \to A_{LL}$ defined by

$$f_2\text{ -card } F(k) = \min{}^*\{M_{(k)}, \ \overline{M}^*_{(k+1)}\} \quad \text{for } k=1,2,\cdots,n$$

where $M_{(1)}, M_{(2)}, \cdots, M_{(n)}$ are $L-L$ type fuzzy numbers of $M_1, M_2, \cdots, M_n$ arranged in decreasing order of magnitude of the normal points m_k, for $k=1,2,\cdots,n$, and $M_{(0)}=(1,0,1)_{LL}$, $M_{(n+1)}=(0,1,0)_{LL}$.

From this definition, we can obtain the following proposition.

Proposition 3.2 Let $F: X \to A_{LL}$ be as in the definition 3.1. Then we have that f_2 -card $F(k)=(m_{(k)} \wedge (1-m_{(k+1)}), \alpha_{(k)} \vee \beta_{(k+1)}, \beta_{(k)} \wedge \alpha_{(k+1)})_{LL}$ for $k=0,1,\cdots,n$, where $m_{(0)}=1$, $m_{(n+1)}=0$.

Proposition 3.3 Let $F: X \to A_{LL}$ be a type-two fuzzy set. The

$$f_2-\text{card}F(k)=\begin{cases} \tilde{1} & \text{if } k=r \\ \tilde{0} & \text{if } k \neq r \end{cases}$$

if and only if F is a nonfuzzy set with r elements.

In order to discuss properties of f_2 -card F of type-two fuzzy set F , we define the order($\leq$) of $L-L$ type fuzzy numbers and the convexity of f_2 -card F .

Definition 3.4 Let $M=(m, \alpha, \beta)_{LL}$ and $N=(n, \gamma, \delta)_{LL}$ be elements of A_{LL} . Then, we define the order $\leq$ of M and N :

$$M \leq N \text{ if and only if } m \leq n, \ \alpha \geq \gamma, \text{ and } \beta \leq \delta.$$

Definition 3.5 Let $F: X \to A_{LL}$ be a type-two fuzzy set. Then, f_2 -card F is convex if

$$f_2\text{ -card } F(l) \geq \min{}^*\{f_2 - \text{card}F(k), \ f_2 - \text{card}F(r)\} \quad \text{whenever } k \leq l \leq r .$$

Proposition 3.6 Let $F: X \to A_{LL}$ be a type-two fuzzy set and

$$F(x_k)=M_k=(m_k, \alpha_k, \beta_k)_{LL} \text{ for } k=1,2,\cdots,n.$$

Assume that $M_{(0)} \geq M_{(1)} \geq M_{(2)} \geq \cdots \geq M_{(n)} \geq M_{(n+1)}$. Then, f_2 -card F is convex.

Proposition 3.7 Let $F: X \to A_{LL}$ be a type-two fuzzy set and

$$F(x_k) = M_k = (m_k, \alpha_k, \beta_k)_{LL} \text{ for } k=1,2,\cdots,n.$$

Assume that $M_{(0)} \geq M_{(1)} \geq M_{(2)} \geq \cdots \geq M_{(n)} \geq M_{(n+1)}$. Then, we have

$$f_2\text{-card } \overline{F}^*(k) = f_2 \text{-card } F(n-k) \text{ for } k=0,1,\cdots,n$$

where $\overline{F}^*(x_k) = \overline{M}^*_k = (1-m_k, \beta_k, \alpha_k)_{LL}$ for $k=1,2,\cdots,n$.

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