A Reliability Approach to Wedge Stability Analysis

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1. INTRODUCTION

Uncertainty and variability in engineering geology are inevitable. Since most engineering geology problems deal with natural materials, most variables governing engineering geology problem are random rather than deterministic. Properties of rocks and soils are inherently heterogeneous, and insufficient information of site conditions and incomplete understanding of a failure mechanism also cause the uncertainty and variability. In rock slope stability analysis, the uncertainty and variability may be in the form of a large scatter in the orientation and the geometry of discontinuities and also test results. Therefore, one of the most difficult and important jobs in rock slope engineering is the selection of the single representative values from widely varied data. Therefore, many engineers and researchers have attempted to limit and quantify the variation and uncertainty in their data and have adopted various methods to indicate the uncertainty and variation in the results of analysis.

As a proposed method to overcome uncertainty and variability, probability theory and statistical techniques have been applied to engineering geology field. Application of probabilistic analysis has provided an objective tool for quantifying and modeling variability and uncertainty.

2. PROBABILISTIC METHOD

A factor of safety is often used to evaluate the stability of a slope system because of its simple calculation and results. The factor of safety is evaluated based on limit equilibrium analysis and it requires fixed single values of discontinuity and strength parameters. However, most input parameters in the factor of safety calculation are not precisely known due to uncertainty and variations in testing, modeling and spatial variation. Thus each of these parameters is a random variable and the analysis with different values for each of these parameters can result in a different factor of safety value. Therefore, the factor of safety itself is also random variable, depending on many input variables.

The probabilistic analysis, as an alternative to the deterministic approach, has been introduced to consider and quantify the uncertainty and variability in parameters and the analytical model. In this analysis, the factor of safety is considered as a random variable and can be replaced by the probability of failure to measure the level of slope stability. The probability of failure is simply defined as the probability of having FS<1 under the probability density function of factor of safety. The physical meaning of probability of failure is interpreted as a measure of relative likelihood of occurrence of failure (Coates, 1981).

In general, probabilistic analysis is performed by two steps: The first step consists of analysis of available geotechnical data to determine the basic statistical parameter(mean and variance) and probability density functions which enable us to represent and predict the random property of geotechnical parameters. In the second step, risk analysis of slope stability is accomplished using the basic statistical parameters and the probability density distribution developed from the previous step.

3. ANALYSIS FOR STOCHASTIC PROPERTIES OF DISCONTINUITY PARAMETERS

3.1 Discontinuity Orientation

Fisher(1953) proposed a distribution on the basis of the assumption that a population of orientation values was distributed about a true value. This assumption is similar to the idea of discontinuity normals being distributed about a true value within a set. He assumed that the probability, $P(\theta)$ that an orientation value selected randomly from the population makes an angle of between θ and $d\theta$ with the true orientation is given by

$$P(\theta) = \eta\, e^{k\cos\theta} d\theta \qquad (1)$$

where k is commonly referred to as Fisher's constant, which is a measure of the degree of clustering within the population and η is a variable expressed as follow:

$$\eta = \frac{k\sin\theta}{e^{k} - e^{-k}} \qquad (2)$$

In view of its simplicity and flexibility, the Fisher distribution provides a valuable model for discontinuity orientation data (Priest, 1993). However, one disadvantage of this distribution is that the distribution provides only an approximation for asymmetric data because this distribution is a symmetric distribution.

3.2 Discontinuity Length

A lognormal distribution has been proposed as a representative distribution model by many different researchers (McMahon, 1971: Bridge, 1976: Barton, 1978: Einstein et al., 1980). However, according to Priest and Hudson (1981), the lognormal distribution is a biased distribution caused by scanline sampling. Their research shows that the negative exponential distribution is an appropriate distribution and it has an advantage that sampling bias caused by scanline method can be canceled out by adopting this distribution.

3.3 Discontinuity Spacing

Priest and Hudson (1976) suggested that the distribution of discontinuity spacing could be modeled by the negative exponential probability density function. This conclusion has been supported by others, such as Wallis and King (1980) and Baecher (1983).

However, according to author's measurements and statistical analysis (Park, 1999), the lognormal distribution is better than the exponential distribution for discontinuity spacing distribution. Approximately 100 spacing values were collected in field by author and then, Chi-square goodness-of-fit tests were performed in order to determine the appropriate distribution of spacing in the study. In this test, the lognormal and negative exponential distributions, which are two possible distribution models for spacing were tested. This is because those theoretical distributions are bounded at zero and are skewed to the right and therefore, those characteristics are similar to the properties of the spacing distribution. According to the results of the goodness-of-fit test, the lognormal distribution is more appropriate for spacing distribution. Some publications such as Rouleau and Gale (1985) and Sen and Kazi (1984), also proposed the lognormal probability distribution for discontinuity spacing.

3.4 Discontinuity Strength Parameters

Compared to other discontinuity parameters, limited research has been accomplished previously regarding statistical evaluation of discontinuity strength parameters. However, although limited work has been accomplished, two different distributions are suggested for shear strength parameters. Mostyn and Li (1993) considered c and ϕ as normally distributed. However, in the paper by Muralha and Trunk(1993), a lognormal distribution is adopted for c and ϕ. Therefore, in the current study, both normal and lognormal distributions are considered as possible distribution models to represent random properties of strength parameters and both distributions are tested for their validity. According to author's test for relative goodness of fit (Park, 1999), both distributions appear to be valid models for internal friction angle, but the normal distribution model is superior to the lognormal model according to the test.

4. PROBABILISTIC ANALYSIS FOR ROCK SLOPE STABILITY

4.1 Conditions for Wedge Failure

4.1.1 Kinematic conditions for wedge failure

The orientation of the line of intersection mainly controls the kinematic analysis of wedge failure. The kinematic instability is based on the following three conditions (Norrish and Wyllie, 1996).

1. The trend of the line of intersection must be similar to the dip direction of the slope face.
2. The plunge of the line of intersection must be less than the dip of the slope face.
3. The plunge of the line of intersection must be greater than the angle of friction of the discontinuity surface.

4.1.2 Kinetic conditions for wedge failure

If kinematic analysis of wedge stability indicates the possibility of a wedge failure, a kinetic analysis must be performed. Analysis of wedge stability requires details of wedge geometry as defined by the location and orientation of the bounding surfaces. Wedge stability can be evaluated using the limit equilibrium concept by resolving the forces acting normal to the discontinuities and in the direction parallel to the line of intersection. The most common procedure used widely is the stereographic projection of Hoek and Bray (1981) (Figure 1).

4.2 Probabilistic Assessment

In previous probabilistic analyses, the concept and procedure of probability of failure were quite confusing. The procedures for evaluating probability of failure are significantly different from one researcher to another because kinematic instability and kinetic instability were not clearly distinguished in most previous approaches. Moreover, the definition of the probability of failure is sometimes quite vague in previous publications. According to Quek and Leung (1995), the probability of failure is expressed as

$$P_f = \frac{N_F}{N_T} \tag{3}$$

where N_F is the number of iterations that the wedges are failed, that is, factor of safety is less than 1, and N_T is the total number of iterations that the wedges analyzed. However, N_T can be interpreted in two different ways: N_T is either the total number of iteration performed or only those iterations that form kinematically unstable wedges. Depending on the definitions, the probability of failure will be different. Therefore, in this paper, for clear definition, the probability of failure is defined as the multiplication of the probability of kinematic instability by the probability of kinetic instability. That is,

$$P_f = \frac{N_m}{N_T} \times \frac{N_f}{N_m} \tag{4}$$

where N_m is the number of iterations that a wedge is kinematically unstable, N_T is the total number of iterations and N_f is the number of iterations that a wedge has factor of safety less than one. The probability of kinematic instability is evaluated as the ratio of the number of iterations (or the number of wedges formed in each iteration) that are determined as kinematically unstable to the number of total iterations, that is, $\frac{N_m}{N_T}$. Since the kinetic analysis is performed only when the wedge is kinematically unstable, the probability of kinetic instability is the ratio of the number of the iterations that the factor of safety is less than one to the number of iterations that the factor of safety is calculated, that is, kinematically unstable ($\frac{N_f}{N_m}$). This multiplication is based on the concept of composite models. Therefore, based on this probabilistic theory, the probability of failure is defined as the ratio of the number of iterations that factor of safety is less than one, which is based on premise that the wedge is kinematically unstable, to the number of total iterations. This method provides a clear definition based on probability theory and simplifies the evaluation of factor of safety without complication.

4.3 Monte Carlo Simulation

The Monte Carlo simulation is frequently used to evaluate the failure probability of a mechanical system, in particular, when direct integration is not practical or when the equation to integrate is difficult to obtain. In this research, the Monte Carlo method is employed because the deterministic model for rock slope failure is not easy to solve by analytical means. This simulation is the most widely used among the probabilistic analysis methods and many others applied it to evaluate slope stability (Kulatilake et al., 1985; Muralha and Trunk. 1993). The Monte Carlo simulation approach is to assume that for a given stability analysis, each variable takes a single value selected randomly from its measured distribution, independent of the other variables. The group of randomly selected parameters is combined with the fixed input data to generate a single random value for the factor of safety. This process is repeated many times to generate a large number of different factors of safety and then the probability of failure is evaluated.

5. APPLICATION OF PROBABILISTIC ANALYSIS

The input parameters used in this analysis are listed in Table 1. The mean orientations and Fisher constants of major discontinuity sets, and the shear strength and geometries for each discontinuity sets were listed in the table. The height of slope cut in this area is approximately 60 m and orientation of the slope are 140°/45°. Based on input values and the simulation procedure discussed previously, probabilistic analyses were performed and the results of analysis for wedge failure are listed in Table 2. In order to compare the probability of slope failure with the deterministic analysis results, the factors of safety for each wedge block were evaluated and then also listed in Table 2.

As observed in Table 2, no discontinuity combinations indicate the unstable condition in the deterministic analysis. However, on the basis of probabilistic analysis results in Table 2, the probabilities of kinematic instability for J1&J2, J1&J3 and J2&J3 combinations are 21.5%, 11.7% and 0.9% respectively. Therefore, there are the possibilities of kinematic instability for all combinations of discontinuity sets on the probabilistic analysis. The probability of kinetic instability are 18.3%, 64.1% and 58.3% for J1&J2, J1&J3, and J2&J3 respectively and therefore, these combinations have quite high probabilities of kinetic instability. The total probabilities of wedge failure for each combination evaluated by the multiplication of the two previous probabilities are 3.9%, 7.5% and 0.5% respectively. Therefore, there is a difference between the results of the deterministic analysis and the probabilistic analysis for J1&J3 combination especially. This is because the deterministic analysis indicates as stable but the probabilistic analysis shows 7.5% probability of failure. Based on 1% of the acceptable failure probability for rock slope suggested by Priest and Brown (1982), J1&J3 combination is interpreted as unstable in a probabilistic analysis but it is analyzed as stable in the deterministic analysis. Consequently, the deterministic analysis based on a fixed representative value of discontinuity parameters fails to indicate the possibility of failure. This is because the deterministic analysis cannot consider the scatter of discontinuity parameters. Therefore, it can be said that there is a possibility that the deterministic analysis based on the single representative value of discontinuity parameters can lead to misinterpretation of rock slope stability.

6. CONCLUSIONS

This study proposed a specific developments that should be considered in a probabilistic analysis. The probability of failure was evaluated based on the multiplication of the probability of kinematic instability by the probability of kinetic instability. In previous probabilistic approaches, the concept of probability of failure was quite confusing. The procedures for evaluating probability of failure are quite different from one researcher to another because these two conditions were not distinguished clearly, and analysis procedures for the two conditions were not clearly divided. Therefore, in this research, analysis procedures for evaluations of kinematic instability and kinetic instability are divided. The probabilities of kinematic instability and kinetic instability are evaluated separately and later are multiplied to determine the probability of failure.

Moreover, as discussed previously, the result of comparison between deterministic analysis and probabilistic analysis in the study area indicates that the analysis result of probabilistic analysis could be quite different from that of the deterministic analysis. The deterministic analysis based on a single value of discontinuity parameters fails to indicate the possibility of slope failure. Consequently, the deterministic analysis is unable to represent the actual condition of rock slope because this analysis does not consider random properties of parameters and therefore, this misinterpretation can cause serious problems. By contrast, the probabilistic analysis is more representative of the actual behavior of parameters and provides analysis results. Therefore, it is recommended that the probabilistic analysis should be used especially in cases when significant scatter in the parameters is observed.

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Table 1 Input values for discontinuity properties

Set I.D.	Mean Orientation	Fisher Const.	Mean Firction Angle (degree)	STD of Fiction Angle	Mean Length (m)	Spacing (m)
J1	·155/22	72	27.03	2.94	2.4	2.4
J2	079/60	29	27.03	2.94	2.4	2.4
J3	273/87	20	27.03	2.94	2.4	2.4

Table 2 Results of wedge failure for deterministic analysis and probabilistic analysis

Set No. 1	Set No. 2	Factor of Safety	Probability of Failure		Total Probability of Failure
			Kinematic	Kinetic	
J1	J2	Stable	0.215	0.183	0.039
J1	J3	Stable	0.117	0.641	0.075
J2	J3	Stable	0.009	0.583	0.005

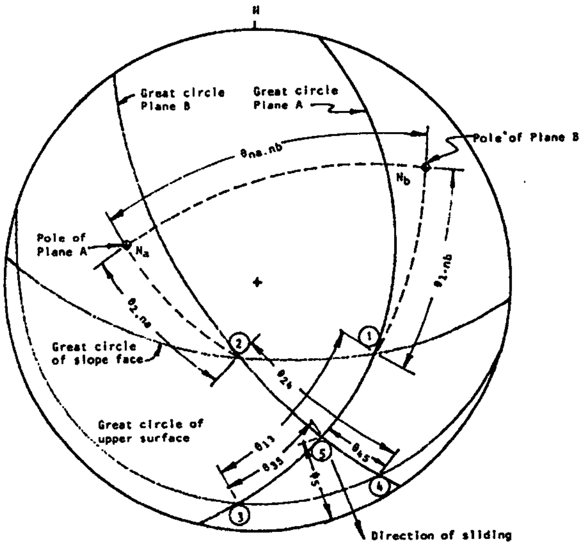

Figure 1 Stereoplot of data required for wedge stability analysis