# Wavenumber correlation analysis of satellite magnetometer observations

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## ABSTRACT

Identifying anomaly correlations between data sets is the basis for rationalizing geopotential interpretation and theory. A procedure is presented that constitutes an effective process for identifying correlative features between the two or more geopotential data sets. Anomaly features that show direct, inverse, or no correlations between the data may be separated by applying filters in the frequency domains of the data sets. The correlation filter passes or rejects wavenumbers between co-registered data sets based on the correlation coefficient between common wavenumbers as given by the cosine of their phase difference. This study includes an example of Magsat magnetic anomaly profile that illustrates the usefulness of the procedure for extracting correlative features between the data sets.

## 1. WAVENUMBER CORRELATION ANALYSIS

To resolve anomaly feature correlations between co-registered data sets, a procedure is required to estimate the wavenumber correlation coefficient $CC_k$ for each wavenumber $k$. Such a procedure is evident if we consider the transforms at any given wavenumber as vectors in the complex plane. These wavevectors can be represented in polar coordinates as follows

$$\overline{X}(k) = |\overline{X}(k)|\, e^{-j\theta \overline{X}(k)}, \qquad (1)$$
$$\overline{Y}(k) = |\overline{Y}(k)|\, e^{-j\theta \overline{Y}(k)},$$

where for the transforms corresponding to wavenumber $k$, $|\overline{X}(k)|$ and $|\overline{Y}(k)|$ are the amplitudes; and $\theta \overline{X}(k)$ and $\theta \overline{Y}(k)$ are the phase angles; so that $\triangle\, \theta_k = (\theta\, \overline{Y}(k) - \theta\, \overline{x}(k))$ is the phase difference; and $j = \sqrt{-1}$ . The CC between two vectors is simply their normalized dot product so that the correlation spectrum is given by

$$CC_k = \cos(\triangle\, \theta_k) = \frac{\overline{X}(k) \cdot \overline{Y}(k)}{|\overline{X}(k)||\overline{Y}(k)|}\ . \qquad (2)$$

In other words, the correlation coefficient between k-th wavenumber components of $X$ and $Y$ is given simply by the cosine of the shift or difference in the phase of these components.

This result has been widely used to extract static lithospheric components of the satellite magnetometer observations (Kim, 1996; Arkani-Hamed, 1988; Alsdorf et al., 1994). Jones (1988) extended the use of equation (2) in the equivalent form given by Arkani-Hamed and Strangway (1986).

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$$CC_k = \frac{Re[\overline{X}(k)\ \overline{Y}^*(k)]}{[\overline{X}(k)\ \overline{X}^*(k)][\overline{Y}(k)\ \overline{Y}^*(k)]}, \tag{3}$$

To implement the WCF, the correlation spectrum between the two signals $X$ and $Y$ are determined from either equations (2) or (3). Based on the correlation spectrum, notch filters are applied so that only those wavenumber components of $X$ and $Y$ are inverse transformed which correspond to the feature correlations desired. As with any spectral filtering application, the filtered output must be compared against the input signals to judge the reasonableness of the results and to establish the most effective values of the CC to use in any investigation.

## 2. EXAMPLE OF APPLICATION

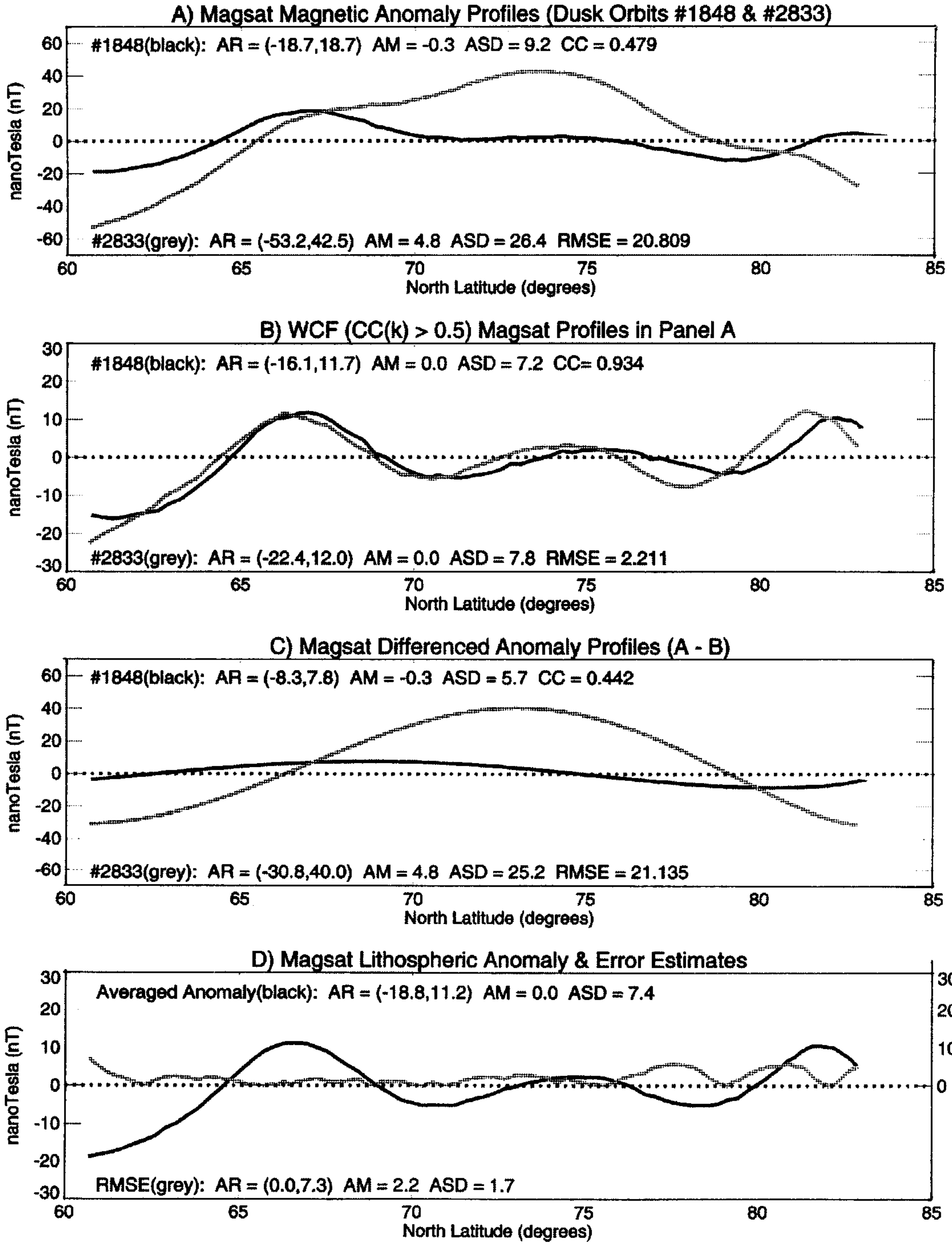
In the example, which is given by Fig. 1, we consider the problem of extracting polar lithospheric anomalies from orbital satellite magnetic data contaminated by highly dynamic external fields from auroral electrojets, field-aligned currents, large-scale ring currents, and other effects.

Analysis of the correlation spectrum between the two track signals shows that all the wavenumbers except the second one (i.e., $k=2$) are relatively well correlated. In fact, the CC of wavenumber 2 ( $CC_{k=2}$) is 0.442 while the $CC_s$ of the other wavenumbers are greater than 0.91. Accordingly, a cutoff value ($CC_k \geq 0.5$) is chosen to estimate the lithospheric anomaly components from the dusk orbits as shown in Fig. 1.B. The anomaly components corresponding to the second wavenumber that are rejected by this application of WCF are given in Fig. 1.C. These rejected components are partly coherent and long wavelength trends that appear to be related more readily to external field effects, induced currents in the mantle, and errors in the core field reduction than to magnetic variations of the underlying lithosphere.

In summary, this analysis suggests that the satellite magnetometer observations of the example in Fig. 1.A are essentially made up of high coherent components in Fig. 1.B that presumably are caused by magnetic sources of the lithosphere, and partly coherent components in Fig. 1.D that probably are related to non-lithospheric effects. Accordingly, a least-squares estimate of the lithospheric anomalies in the satellite magnetic observations can be obtained by averaging point-by-point the coherent signals of Fig. 1.B as shown in 1.D. The differences between the coherent signals in Fig. 1.B are also presented as point-by-point RMSEs in Fig. 1.D to constrain interpretations of the averaged lithospheric anomaly estimates.

## 3. CONCLUSIONS

A procedures is presented that constitutes an effective process for extracting correlative features between the two or more geopotential data sets. Feature correlations between data sets may be isolated by the application of correlation filters in the wavenumbers between co-registered data sets based on the correlation coefficient between common wavenumbers as given by the cosine of their phase difference. The presented wavenumber correlation filtering procedure can be implemented to obtain improved estimates of the lithospheric anomaly components from satellite magnetic observations.


A) Magsat Magnetic Anomaly Profiles (Dusk Orbits #1848 & #2833)

B) WCF (CC(k) > 0.5) Magsat Profiles in Panel A

C) Magsat Differenced Anomaly Profiles (A - B)

D) Magsat Lithospheric Anomaly & Error Estimates

Figure 1. Wavenumber correlation analysis for spatially adjacent dusk orbits #1848 and #2833 at about 330 km altitude across the Arctic from northern Greenland to southern Finland. Note the changes in amplitude scale between panels.