

# 풍하중을 받는 벤치마크 구조물의 진동제어를 위한 외란 예측기가 포함된 슬라이딩 모드 퍼지 제어

◦김상범\*\*, 윤정방\*, 구자인\*\*\*

## Application of Sliding Mode Fuzzy Control with Disturbance Estimator to Benchmark Problem for Wind Excited Building

◦Saang Bum Kim\*\*, Chung Bang Yun\*, and Ja In Gu\*\*\*

### ABSTRACT

A distinctive feature in vibration control of a large civil infrastructure is the existence of large disturbances, such as wind, earthquake, and sea wave forces. Those disturbances govern the behavior of the structure, however, they cannot be precisely measured, especially for the case of wind-induced vibration control. The sliding mode fuzzy control (SMFC), which is of interest in this study, may use not only the structural response measurement but also the wind force measurement. Hence, an adaptive disturbance estimation filter is introduced to generate a wind force vector at each time instance based on the measured structural response and the stochastic information of the wind force. The structure of the filter is constructed based on an auto-regressive with auxiliary input model. A numerical simulation is carried out on a benchmark problem of a wind-excited building. The results indicate that the overall performance of the proposed SMFC is as good as the other methods and that most of the performance indices improve as the adaptive disturbance estimation filter is introduced.

### 1. INTRODUCTION

Vibration control of a large civil infrastructure generally involves large external disturbances, such as wind, earthquake, and sea wave forces. The structural responses are mainly influenced by the external load, and the control force only works as a small modifier. However many control algorithms can not directly utilize the characteristics of the environmental loading, and the algorithms take the loading as unknown disturbance only. Sliding mode fuzzy control (SMFC) is one of the nonlinear and intelligent control methods, which may consider the external load data directly. For the case of wind vibration control, however, it is very difficult to precisely measure the wind forces on a large-scale structure. To overcome the above difficulty, the disturbance estimation filter is introduced to the SMFC. This filter generates a wind force vector based on the measured signal, and the controller produces a control signal using the measured signal and the estimated wind force. The proposed SMFC with adaptive disturbance

estimation filter is applied to the benchmark problem for response control of a wind-excited tall building, which was formulated by Yang *et al.* (1998). The results from the numerical simulation study indicate that the present control algorithm is very efficient for reducing the wind-induced vibration.

### 2. MODEL OF CONTROL SYSTEM

#### 2.1 Modeling of Structure

Referring to a building structure for a benchmark problem of wind vibration control as in Figure 1, the dynamic behavior of the structure with a control device can be modeled as

$$\mathbf{M}_s \ddot{\mathbf{y}}_s(t) + \mathbf{C}_s \dot{\mathbf{y}}_s(t) + \mathbf{K}_s \mathbf{y}_s(t) = \mathbf{f}_s(t) + \mathbf{B}_s \mathbf{u}_s(t) \quad (1)$$

where  $\mathbf{y}_s(t)$ ,  $\mathbf{f}_s(t)$ , and  $\mathbf{u}_s(t)$  = displacement, wind

\* 한국과학기술원 토목공학과 교수

\*\* 한국과학기술원 토목공학과 박사과정

\*\*\* 한국과학기술원 토목공학과 석사과정

force, and control force vectors;  $M_s$ ,  $C_s$ , and  $K_s$  = mass, damping, and stiffness matrices; and  $B_s$  = boolean matrix representing the effects of the control force. By converting into a state space form and selecting appropriate control and measurement variables, the following state and measurement equations can be obtained

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}_u\mathbf{u}_s(t) + \mathbf{B}_f\mathbf{f}_s(t) + \mathbf{B}_w\mathbf{w}(t) \\ \mathbf{y}_c(t) &= \mathbf{C}_c\mathbf{x}(t) + \mathbf{D}_{cu}\mathbf{u}_s(t) + \mathbf{D}_{cf}\mathbf{f}_s(t) + \mathbf{D}_{cw}\mathbf{w}(t) \quad (2) \\ \mathbf{y}_m(t) &= \mathbf{C}_m\mathbf{x}(t) + \mathbf{D}_{mu}\mathbf{u}_s(t) + \mathbf{D}_{mf}\mathbf{f}_s(t) + \mathbf{D}_{mw}\mathbf{w}(t) + \mathbf{v}(t)\end{aligned}$$

where  $\mathbf{x}(t)$ ,  $\mathbf{y}_c(t)$ ,  $\mathbf{y}_m(t)$ ,  $\mathbf{w}(t)$ , and  $\mathbf{v}(t)$  = state, control signal, measured signal, un-modeled error, and measurement noise vectors; and  $\mathbf{A}$ ,  $\mathbf{B}_u$ ,  $\mathbf{B}_f$ ,  $\mathbf{B}_w$ ,  $\mathbf{C}_c$ ,  $\mathbf{D}_{cu}$ ,  $\mathbf{D}_{cf}$ ,  $\mathbf{D}_{cw}$ ,  $\mathbf{C}_m$ ,  $\mathbf{D}_{mu}$ ,  $\mathbf{D}_{mf}$ , and  $\mathbf{D}_{mw}$  = system matrices.

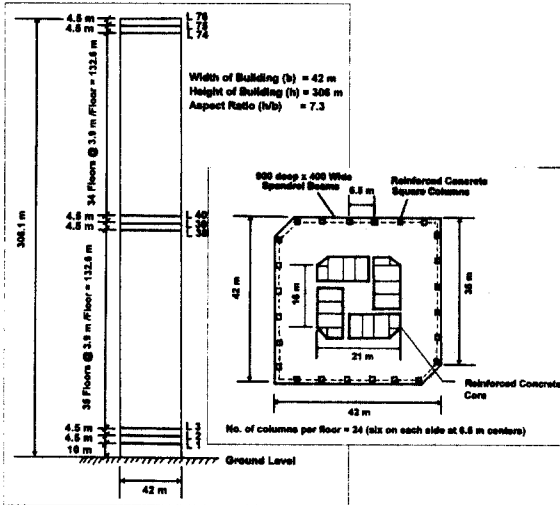


Figure 1. Structure for Benchmark Problem of Wind Vibration Control

## 2.2 Model Reduction and State Observer

Before designing the controller, two stages of pre-design were carried out. The first stage is for model reduction. For the computational efficiency, the control law is designed based on a reduced model obtained using the balanced truncation method. The state space system is transformed into a balanced system, of which the controllability and observability Gramians are diagonal and identical. Then taking the largest Hankel singular values, a reduced-order system is obtained. The second stage is for design of observer. The Kalman-Bucy filter is used to estimate the state from the measured signal as (Goodwin & Sin 1984)

$$\begin{aligned}\dot{\hat{\mathbf{x}}}_r &= \mathbf{A}_r\hat{\mathbf{x}}_r(t) + \mathbf{B}_{ru}\mathbf{u}_s(t) \\ &+ \mathbf{L}_{obs}(\mathbf{y}_m(t) - \mathbf{C}_{mr}\hat{\mathbf{x}}_r(t) - \mathbf{D}_{mru}\mathbf{u}_s(t))\end{aligned} \quad (3)$$

where  $\hat{\mathbf{x}}_r$  = estimated state vector; and  $\mathbf{L}_{obs} = (\mathbf{P}_{obs}\mathbf{C}_{mr}^T + \mathbf{B}_{rw}\mathbf{S}_{obs})\mathbf{R}_{obs}^{-1}$  = observer gain matrix. The observer gain matrix can be obtained by solving the following algebraic Riccati equation for  $\mathbf{P}_{obs}$

$$\begin{aligned}\bar{\mathbf{A}}_r\mathbf{P}_{obs} + \mathbf{P}_{obs}\bar{\mathbf{A}}_r^T - \mathbf{P}_{obs}\mathbf{C}_{mr}^T\mathbf{R}_{obs}^{-1}\mathbf{C}_{mr}\mathbf{P}_{obs} \\ + \mathbf{B}_{rw}\mathbf{Q}_{obs}\mathbf{B}_{rw}^T - \mathbf{B}_{rw}\mathbf{S}_{obs}\mathbf{R}_{obs}^{-1}\mathbf{S}_{obs}^T\mathbf{B}_{rw}^T = \mathbf{0}\end{aligned} \quad (4)$$

where

$$\begin{aligned}\bar{\mathbf{A}}_r &= \mathbf{A}_r - \mathbf{C}_{mr}^T\mathbf{R}_{obs}^{-1}\mathbf{S}_{obs}^T\mathbf{B}_{rw}^T \\ \begin{bmatrix} \mathbf{Q}_{obs} & \mathbf{S}_{obs} \\ \mathbf{S}_{obs}^T & \mathbf{R}_{obs} \end{bmatrix} \delta(\tau) &= \mathbf{E} \begin{bmatrix} \mathbf{w}_r(t) \\ \mathbf{v}_r(t) \end{bmatrix} \begin{bmatrix} \mathbf{w}_r(t) \\ \mathbf{v}_r(t) \end{bmatrix}^T\end{aligned}$$

## 3. ADAPTIVE DISTURBANCE ESTIMATION FILTER FOR WIND FORCE

### 3.1 Stochastic Model for Wind Force

The stochastic wind force  $\mathbf{f}_s(s)$  can be modeled by its spectral density function as (Jin *et al.* 1998)

$$\mathbf{f}_s(s) = \mathbf{H}_{fw}(s)\mathbf{w}_f(s) \quad (5)$$

where  $\mathbf{S}_{f_s}(\omega) = \bar{\mathbf{H}}_{fw}(j\omega)\mathbf{H}_{fw}(j\omega)^T$ ;  $\mathbf{S}_{f_s}(\omega)$  = spectral density matrix of wind force;  $\mathbf{w}_f$  = white noise source with unit intensity; and  $\mathbf{H}_{fw}(s)$  = transfer function. Figure 2 shows power spectral density functions of the wind force fluctuations at various nodes along the building. The transfer function model can be converted into a following discrete auto-regressive with auxiliary input (ARX) model as (Ljung 1987)

$$\mathbf{f}_s[k+1] = \mathbf{A}_{fD}\mathbf{f}_s[k] + \mathbf{B}_{fD}\mathbf{w}_f[k] \quad (6)$$

where  $\mathbf{A}_{fD}$  and  $\mathbf{B}_{fD}$  = ARX coefficient matrices ( $\mathbf{H}_{fw}(z) \approx (z\mathbf{I} - \mathbf{A}_{fD})^{-1}\mathbf{B}_{fD}$ ). The AR matrix  $\mathbf{A}_{fD}$  is obtained by minimizing the mean squares of estimation error, while  $\mathbf{B}_{fD}$  is taken as an identity matrix.

### 3.2 Adaptive Disturbance Estimation Filter

To generate a wind force vector in on-line mode, measurement error equation can be derived as

$$\begin{aligned}e_f[k] &= \mathbf{y}_{mr}[k] - \mathbf{D}_{mrf}\mathbf{A}_{fD}\hat{\mathbf{f}}_s[k-1|k-1] \\ &- \mathbf{C}_{mr}\hat{\mathbf{x}}_r[k] - \mathbf{D}_{mru}\mathbf{u}_s[k]\end{aligned} \quad (7)$$

By minimizing the mean square of the measurement error, following adaptive filter equation can be obtained as (Kim

$$\hat{\mathbf{f}}_s[k|k] = \mathbf{A}_{fD} \hat{\mathbf{f}}_s[k-1|k-1] + \mathbf{M}_{LMS}[k] e_f[k] \quad (8)$$

where

$$\mathbf{M}_{LMS}[k] = \mathbf{P}_{LMS}[k|k-1] \mathbf{D}_{mf}^T \cdot (\mathbf{R}_{LMS} + \mathbf{D}_{mf} \mathbf{P}_{LMS}[k|k-1] \mathbf{D}_{mf}^T)^{-1}$$

$$\mathbf{P}_{LMS}[k|k] = (\mathbf{I} - \mathbf{M}_{LMS}[k] \mathbf{D}_{mf}) \mathbf{P}_{LMS}[k|k-1]$$

$$\mathbf{P}_{LMS}[k+1|k] = \mathbf{A}_{fD} \mathbf{P}_{LMS}[k|k] \mathbf{A}_{fD}^T + \mathbf{B}_{fD} \mathbf{Q}_{LMS} \mathbf{B}_{fD}^T$$

and  $\mathbf{M}_{LMS}[k]$  = filter gain matrix;  $\mathbf{P}_{LMS}[k|k]$  and  $\mathbf{P}_{LMS}[k+1|k]$  = prediction error covariance matrices;  $\mathbf{Q}_{LMS}$  and  $\mathbf{R}_{LMS}$  = covariance matrices of  $\mathbf{w}_f$  and  $\mathbf{v}$ . Figure 3 shows the time histories of true and estimated wind forces and estimation error for the 9th component of the reduced wind force vector  $\mathbf{B}_{mf} \mathbf{f}_s$ .

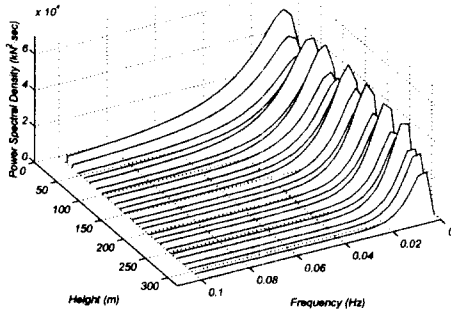


Figure 2. Power Spectral Densities of Wind Force Fluctuations at Various Nodes

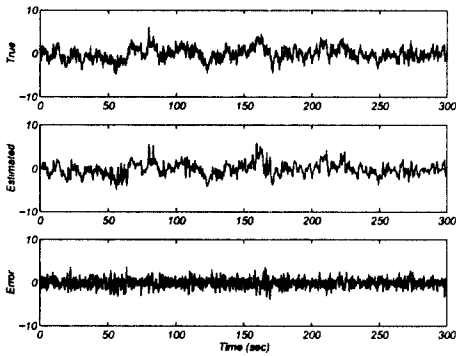


Figure 3. True and Estimated Disturbance and Error : 9<sup>th</sup> component of  $\mathbf{B}_{mf} \mathbf{f}_s$

## 4. SLIDING MODE FUZZY CONTROLLER

### 4.1 Structure of SMFC

As in the sliding mode control (SMC), the basic strategy of the SMFC is forcing the state of the system to stay on the sliding surface, where the response of the system can be reduced rapidly. The structure of the SMC is constructed using the Lyapunov's direct method. Then the controller is composed of a conventional feed-back part and a feed-forward part as (Khalil 1996)

$$\mathbf{u}_s(t) = -(\mathbf{P}\mathbf{B}_{ru})^{-1} (\mathbf{P}\mathbf{A}_r + [\cdot \cdot \cdot \eta \cdot \cdot] \mathbf{P}) \hat{\mathbf{x}}_r(t) - (\mathbf{P}\mathbf{B}_{ru})^{-1} \mathbf{P}\mathbf{B}_f \hat{\mathbf{f}}_s(t) \quad (9)$$

where  $\hat{\mathbf{f}}_s$  = a disturbance vector generated using an adaptive filter;  $\mathbf{P} = [P_1 \dots P_n]^T$ ; and  $P_i$  = direction vector of the sliding surface for the  $i$ -th control force. Converting the above sliding mode controller into a fuzzy form, a sliding mode fuzzy controller can be obtained as (Kim & Yun 1999)

$$\mathbf{u}_s(t) = \mathbf{K}_{Fuzzy}(t, \hat{\mathbf{x}}_r(t), \hat{\mathbf{f}}_s(t)) \quad (10)$$

where  $\mathbf{K}_{Fuzzy}$  is the fuzzy controller. Figure 4 shows the overall structure of the SMFC with an adaptive disturbance estimation filter.

### 4.2 Design of Fuzzy Control

The present fuzzy controller consists of 5 modules : (1) Normalization, (2) Fuzzification, (3) Inference Engine, (4) De-Fuzzification, and (5) De-Normalization (Driankov *et al.* 1993). In the first module named *Normalization*, the observed signal are normalized as

$$\begin{aligned} s_n(t) &= \alpha_n \mathbf{K}_n \mathbf{x}_{rE}(t) \\ s_t(t) &= \alpha_t \mathbf{K}_t \mathbf{x}_{rE}(t) \\ s_f(t) &= \alpha_f \mathbf{K}_f \hat{\mathbf{f}}(t) \end{aligned} \quad (11)$$

where  $\mathbf{K}_n = (\mathbf{P}\mathbf{B}_{ru})^{-1} (\mathbf{P}\mathbf{A}_r + [\cdot \cdot \cdot \eta \cdot \cdot] \mathbf{P})$ ;  $\mathbf{K}_t = \|\mathbf{B}_{ru}\|^{-1} \|\mathbf{K}_n\| \mathbf{B}_{ru}^T - (\|\mathbf{K}_n\| \|\mathbf{B}_{ru}\|)^{-1} \mathbf{K}_n \mathbf{B}_{ru} \mathbf{K}_n$ ;  $\mathbf{K}_f = -(\mathbf{P}\mathbf{B}_{ru})^{-1} \mathbf{P}\mathbf{B}_f$ ;  $s_n(t)$  and  $s_t(t)$  = normal and tangential components of the state vector with respect to the transformed sliding surface;  $s_f(t)$  = auxiliary state representing the feed-forward control force; and  $\alpha_n$ ,  $\alpha_t$  and  $\alpha_f$  are scale factors.

*Fuzzification* module converts the scaled crisp values to fuzzy numbers with singleton membership functions. *Fuzzy Rule Base* is constructed based on the SMC as shown in Table 1. In this study, the implication of the fuzzy relation from *If-then* rule is realized by Mamdani's

method, and triangular membership functions are used for various fuzzy numbers as in Figure 5. Aggregation for several fuzzy relations is carried out by disjunction. In the module of *Inference Engine*, approximate reasoning is conducted using the generalized modus ponens. In the module of *De-Fuzzification*, the fuzzy control force is transformed into a crisp number by the center of gravity method.

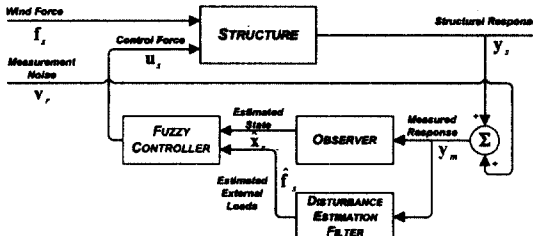


Figure 4. Schematic Diagram of SMFC with Adaptive Disturbance Estimation Filter

Table 1. Rule Table for  $\tilde{u}_{FB}$

$\tilde{s}_n$	NB	NM	NS	Z	PS	PM	PB
PB	Z		NS		NM		NB
PM		Z		NS		NM	
PS	PS		Z		NS		NM
Z		PS		Z		NS	
NS	PM		PS		Z		NS
NM		PM		PS		Z	
NB	PB		PM		PS		Z

Note : The first character in the fuzzy number denotes negative or positive, and the second character denotes big, medium, or small. Z means zero.

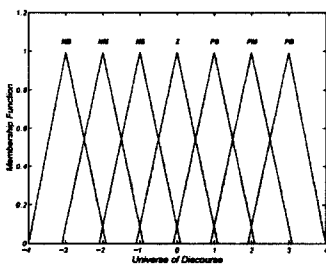


Figure 5. Triangular Membership Functions

## 5. NUMERICAL SIMULATION STUDY FOR BENCHMARK PROBLEM

A numerical simulation study is carried out on the benchmark structure proposed by Yang, *et al.* (1998). The structure is a 76-story concrete office tower with an active tuned mass damper (ATMD) installed on the top floor. Detailed data related to this problem are given at the web site of the problem organizer (Agrawal, 2000). It is modeled as a system with 24 dof's for structural analysis. Then it is reduced to a system with 12 dof's using the balanced truncation method for the computational efficiency in the control process. 12 performance indices are defined to compare the control performance by the problem organizer. J1 and J2 are for rms acceleration performance, J3 and J4 are for rms displacement performance, and J5 and J6 are performance related to rms displacement and velocity of the ATMD. On the other hand, J7 and J8 are for peak acceleration performance, J9 and J10 are for peak displacement performance, and J11 and J12 are performance related to peak displacement and velocity of the ATMD. Control performance of the present SMFC is compared with those obtained by other methods in Table 2. Figure 6 shows time histories of the wind forces at 260 and 173m above ground, the responses of the top floor for uncontrolled and controlled cases and the control forces.

The results indicate that the overall performance of the proposed SMFC is as good as the other methods with the adaptive disturbance estimator. However, the SMFC requires slightly larger displacement and velocity of the ATMD as the performance indices J5, J6, J11 and J12 indicate. The results also show that most of the performance indices improve as the adaptive disturbance estimation filter is introduced, which indicates that the wind force estimation using the disturbance estimation filter makes the control algorithm more effective.

## 6. CONCLUDING REMARKS

Adaptive disturbance estimation filter is presented to estimate the un-measured wind force vector, and it is employed in sliding mode fuzzy controller (SMFC) for wind vibration control. The present method is applied to a benchmark problem of a wind-excited building. The results from the numerical simulation study show the effectiveness of the present algorithm.

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Table 2. Comparison of Control Performance Criteria

	LQG*	FWLQG* (Filter 1)	$H_{\infty, \mu}$ *	Quadratic Stability	SMCS*	SMFC w/o Filter	SMFC w/ Filter
J1	0.369	0.351	0.357	0.418	0.367	<b>0.383</b>	<b>0.373</b>
J2	0.606	0.614	0.608	0.584	0.608	<b>0.589</b>	<b>0.575</b>
J3	0.509	0.501	0.490	0.515	0.507	<b>0.520</b>	<b>0.536</b>
J4	0.491	0.499	0.509	0.484	0.493	<b>0.480</b>	<b>0.464</b>
J5	1.870	1.817	1.879	1.481	1.858	<b>2.492</b>	<b>2.250</b>
J6	1.891	1.840	2.046	1.580	1.889	<b>2.327</b>	<b>2.309</b>
J7	0.481		0.476	0.542	0.477	<b>0.500</b>	<b>0.483</b>
J8	0.460		0.438	0.423	0.464	<b>0.412</b>	<b>0.394</b>
J9	0.569		0.569	0.568	0.568	<b>0.559</b>	<b>0.569</b>
J10	0.423		0.423	0.423	0.424	<b>0.434</b>	<b>0.424</b>
J11	2.401		2.551	2.027	2.369	<b>2.562</b>	<b>2.401</b>
J12	2.618		2.898	2.278	2.617	<b>2.581</b>	<b>2.619</b>

Note : \* The results are from the references (Yang *et al.* 1998, Jin *et al.* 1998, Watanabe *et al.* 1998, Srivastava *et al.* 1998, and Wu *et al.* 1998)

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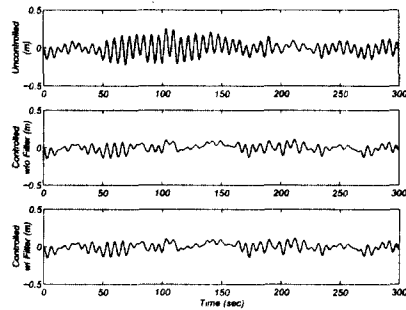
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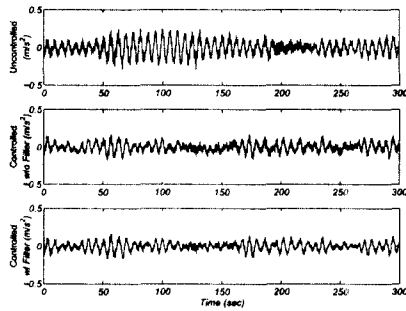
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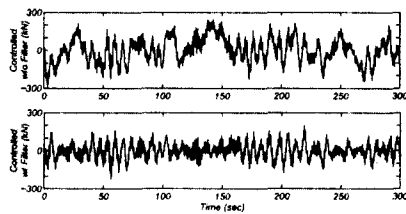
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(a) Top Floor Displacements



(b) Top Floor Accelerations



(c) Control Forces

Figure 6. Time Histories of Wind Forces, Uncontrolled and Controlled Responses and Control Forces