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°김태우, 김지환*

Nonlinear vibration analysis of viscoelastic laminated plates undergoing large deflection

Tae-Woo Kim and Ji-Hwan Kim

ABSTRACT

Dynamic behavior of laminated composite plates undergoing moderately large deflection is investigated taking into account the viscoelastic behavior of material properties. Based on von Karman's non-linear deformation theory and Boltzmann's superposition principle, non-linear and hereditary type governing equations are derived. Finite element analysis and the method of multiple scales is applied to examine the effect of large amplitude on the dissipative nature of viscoelastic laminated plates.

INTRODUCTION

When polymeric matrix based composites, such as graphite-epoxy or glass-epoxy, are used for structural components, viscoelastic behavior is expected due to the time dependent properties of the matrix. In a certain environment of high temperature and/or high moisture, the viscoelastic motion of structures made up of polymeric composite comes to be prominent and cannot be neglected. For more accurate prediction of the structural behavior, many researchers incorporated the time dependent behavior of polymeric composite into their field of studies.

In this study, geometrically non-linear analysis of a laminated composite plate undergoing moderately large deflection is carried in consideration of the time dependent behavior of polymeric fiber reinforced composites. A lot of literatures are available on large deflection of elastic system and most of them treat frequency ratio at given deflection order as a main topic. For viscoelastic system, it seems necessary to examine non-linear characteristics of dissipation provided by viscoelastic material properties as well as frequency ratio.

There are some studies dedicated to geometrically non-linear analysis of a structure where viscoelastic material property is considered. Vinogradov [1] investigated creep of a viscoelastic column, and showed that there is no infinite increase in deflection after creep buckling when geometric non-linearity is taken into account. Aboudi [2] analyzed the postbuckling behavior of viscoelastic laminated plates. The time dependent postbuckling behavior was presented and results based on the different plate theories were compared with one another. Fung et al. [3] studied the dynamic stability of a viscoelastic beam subjected to harmonic and parametric excitations simultaneously, and showed variation of stability boundaries due to the non-linear deformation caused by some parameters.

In this work, governing equation is derived from Hamilton's principle within von Karman's non-linear plate theory and Boltzmann's superposition principle. To

* 서울대학교 기계항공공학부

treat the non-linear and hereditary type governing equations, finite element method and the method of multiple scales is employed. For the hereditary characteristics of the governing equations make it difficult to decouple the flexure motion, all coupled equations are attempted to solve simultaneously at the cost of efficient computation. The exponential decay ratio is used as a parameter for measuring viscoelastic dissipation and numerical results are presented for rectangular plates in large amplitude vibration.

FORMULATION

Fig. 1 shows geometric configuration of a rectangular plate undergoing moderately large deflection.

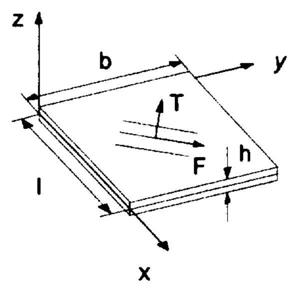

Fig. 1 Geometric configuration of a laminated composite plate

In the theory of first order shear deformation, the displacement fields are assumed as

$$\begin{aligned} u_1(x,y,z,t) &= u(x,y,t) + z\psi_x(x,y,t) \\ u_2(x,y,z,t) &= v(x,y,t) + z\psi_y(x,y,t) \\ u_3(x,y,z,t) &= w(x,y,t) \end{aligned} \qquad (1)$$

where u_1,u_2,u_3 are components of the three-dimensional displacement vector in the x, y and z directions respectively while u,v,w denote the displacements at the mid-plane and ψ_x and ψ_y are the rotations of the normals to the mid-plane about the y and x axis.

The strain-displacement relations based on von Karman's large deformation theory of plate are written in the following form

$$\begin{aligned} \varepsilon_1 &= u,_x + z\psi_x,_x + w,_x^2/2 \\ \varepsilon_2 &= v,_y + z\psi_y,_y + w,_y^2/2 \\ \varepsilon_3 &= u,_y + z\psi_x,_y + v,_x + z\psi_y,_x + w,_x\, w,_y \\ \varepsilon_4 &= \psi_y + w,_y \\ \varepsilon_5 &= \psi_x + w,_x \end{aligned} \qquad (2)$$

where contracted notations are used for engineering strains and comma denotes derivative.

For linear viscoelastic constitutive equations, Boltzmann's superposition principle is employed, which is given in a convolution form as follows

$$\sigma_i(t) = \int_{0^-}^{t} \sum_{j=1}^{5} \overline{Q}_{ij}(t-\tau)\dot{\varepsilon}_j(\tau)d\tau, \qquad i = 1,2,\ldots,5 \quad (3)$$

where $\overline{Q}_{ij}$ are the relaxation functions referred to x-y coordinate, which is obtained from the axis transformation of the relaxation moduli Q_{ij} referred to principal material axes.

The equations of motion is derived from extended Hamilton's principle for non-conservative system:

$$\int_{t_1}^{t_2} (\delta T - \delta U)dt = 0 \qquad (4)$$

where δT, δU are variation of the kinetic energy and the virtual work by the internal forces respectively.

Now, one can obtain following discretized governing equation by interpolating displacement and rotation fields in terms of nodal values and substituting them into eqn (4) considering eqn(1) to (3)

$$\mathbf{M}\ddot{\mathbf{x}} + \int_{0^-}^{t} \sum_{i=1}^{6} Q_i(t-\tau)\mathbf{K}_i\dot{\mathbf{x}}d\tau = 0 \qquad (5)$$

where $\mathbf{x}$ is the global nodal vector, $\mathbf{M}$ is the mass matrix and the contracted indices are used for relaxation moduli referred to principal material axes as $Q_1=Q_{11}$, $Q_2=Q_{12}$, $Q_3=Q_{22}$, $Q_4=Q_{33}$, $Q_5=Q_{44}$, $Q_6=Q_{55}$.

METHOD OF ANALYSIS

The relaxation moduli Q_{ij} referred to the principal material direction can be represented in terms of exponential series, which is practically one of the widely used model for the approximation of viscoelastic behavior of material. Hence, without taking into account the variation of temperature and moisture, any relaxation modulus is assumed in the following form

$$Q_i(t) = Q_i^\infty + \sum_{j=1}^{N_i} Q_i^j \exp(-d_i^j t) = Q_i(0) f_i(t) \quad i=1,2,\ldots,6 \quad (6)$$

where N_i is the number of exponential terms required for approximation, Q_i^∞ is final stiffness of $Q_i(t)$, Q_i^j is a constant coefficient, d_i^j is a relaxation parameter and $f_i(t)$ is a time function that characterizes relaxation phenomenon.

After substituting eqn(6) into eqn(5), one gets a nondimensional equation by introducing parameters such as u/b, v/l, w/h :

$$\overline{\mathbf{M}}\ddot{\overline{\mathbf{x}}} + \int_{0^-}^{T} \frac{1}{Q_1(0)} \sum_{i=1}^{6} Q_i(T-\tau)\overline{\mathbf{K}}_i \dot{\overline{\mathbf{x}}} d\tau = 0 \qquad (7)$$

where bars denote nondimensional values and $T=\omega_L t$ with ω_L being linear frequency. As the relaxation develops slowly, eqn(7) is expanded in series of nondimensional relaxation parameters η_i^j [5][6]:

$$\overline{\mathbf{M}}\ddot{\overline{\mathbf{x}}} + \sum_{i=1}^{6} \{Q_i(0)\int_{0^-}^{T} \overline{\mathbf{K}}_i \dot{\overline{\mathbf{x}}} d\tau - \sum_{j=1}^{N} \eta_i^j Q_i^j \int_0^T \overline{\mathbf{K}}_i \overline{\mathbf{x}} d\tau\} / Q_1(0) + O(\eta^2) = 0 \qquad (8)$$

where $\eta_i^j = d_i^j / \omega_L$.

For demonstration purposes, relaxation moduli are assumed that $Q_{11}(t)$ is independent of time and the other moduli have the same time function $f(t)$ [7]. Furthermore, the standard solid model, which is simplest and has a single exponential term in eqn (6), is used as the time function [8]. The non-linear mode vector is assumed to be the same as the linear mode and single-mode analysis is carried out. Let

$$\overline{\mathbf{x}} = \boldsymbol{\phi} q(T) \qquad (9)$$

where $\boldsymbol{\phi}$ is the normalized linear mode vector which has maximum transverse displacement as unity. Substituting eqn(9) into eqn(8) and multiplying eqn (8) by a vector that normalizes the coefficient of second derivative term as unity, one gets a following equation after performing integration by parts

$$\ddot{q} + q + \alpha_2 q^2 + \alpha_3 q^3 - \eta \int_0^T (\beta_1 q + \beta_2 q^2 + \beta_3 q^3) d\tau + \eta^2 \int_0^T 2(T-\tau)(\beta_1 q + \beta_2 q^2 + \beta_3 q^3) d\tau + O(\eta^3) = 0 \qquad (10)$$

In case of pure elastic structures, first four terms of eqn(10) compose of the well-known geometrically nonlinear equation which governs vibrational motion with moderately large amplitude and it can be analyzed by various method. However, the integral term, induced by dissipative nature, in eqn(10) makes it difficult to apply any available method to the equation. In this study, the solution of eqn (10) is sought by means of method of multiple scales [9].

Now, one can write a solution to eqn (17) as follows

$$q(T_0, T_1, T_2, \ldots) = \varepsilon q_1 + \varepsilon^2 q_2 + \varepsilon \eta q_3 + \varepsilon^3 q_4 + \varepsilon^2 \eta q_5 + \varepsilon \eta^2 q_6 + \ldots \qquad (11)$$

where ε denotes a small parameter that is a measure of the amplitude of oscillation and different time scales are defined by

$$\begin{aligned} &\text{Order } 0: T_0 = T, \\ &\text{Order } 1: T_1 = \varepsilon T,\ T_2 = \eta T \\ &\text{Order } 2: T_3 = \varepsilon^2 T,\ T_4 = \varepsilon\eta T,\ T_5 = \eta^2 T \end{aligned} \qquad (12)$$

Substituting eqn (11) into eqn (10) and collecting terms of like powers of ε and η, this leads to a system of equations and one can solve them sequentially for q_1,

$q_2,\ldots,q_6$. The first order solution of the system of equations can be expressed in the following form

$$q_1 = A(T_1,\ldots,T_6)\exp(iT_0) + \overline{A}(T_1,\ldots,T_6)\exp(-iT_0) \quad (13)$$

where $\overline{A}$ is complex conjugate of A. The detailed expression of coefficient A and $\overline{A}$ on each time scale are determined from conditions that $q_2,\ldots,q_6$ have no secular terms which increase with time infinitely. The conditions for uniform expansion are obtained in course of solving the equation set sequentially.

Initial conditions are imposed as velocity zero and amplitude-to-thickness ratio w_0. By incorporating of conditions for uniform expansion and initial conditions into eqn(13), the lowest order solution up to second order time scale comes to

$$q(T) = w_0 \exp\{m(T)T\}\cos\{n(T)T\} \quad (14)$$

where

$$m(T) = -\eta\beta_1/2 - h(T)\sin(\eta^2\beta_1^2 T/4) \quad (15.1)$$

$$n(T) = 1 + \eta^2\beta_1(3\beta_1/8 - 1) + h(T)\cos(\eta^2\beta_1^2 T/4) \quad (15.2)$$

with

$$h(T) = w_0^2(3\alpha_3/8 - 5\alpha_2^2/12)\exp(-\beta_1\eta T) \quad (16)$$

Eqn(15.1), normalized exponential decay ratio, measures dissipation and eqn(15.2) means ratio of nonlinear damped frequency to linear undamped frequency. As one can see, the last terms of eqn(15.1) and eqn(15.2) reflect geometric nonlinearity. The ratio of nonlinear to linear frequency for pure elastic case can be easily induced from eqn(15.2) by making η go to zero, which means available data of literatures on frequency ratio for elastic vibration analysis with large amplitude can be used directly to investigate the nonlinear effect on dissipation parameter, for the ratio is also a factor in eqn(24.1) as well, though limited in small range of deflection order due to breakdown of the perturbation method. From eqn(15.1) and eqn(15.2) one can also expect that nonlinear effect on dissipation is not as apparent as on frequency, for magnitude of nonlinear term in those each are subject to harmonic functions with long period and opposite phase, this to cosine function of value around one and that to sine function of value around zero.

NUMERICAL EXAMPLES AND DISCUSSION

For finite element analysis, sixteen-node Lagrangian rectangular element is introduced and a 5×5 mesh over the whole plate is used after a convergence study test which is omitted in this paper for brevity. In the subsequent section, following geometric, material properties and boundary conditions are used, not otherwise stated, to obtain the numerical results

l/t=100, b/l=1, $E_1(0)/E_2(0)$=40,
$G_{12}(0)/E_2(0)$=$G_{13}(0)/E_2(0)$=0.5, $G_{23}(0)/E_2(0)$=0.2,
$\nu_{12}(0)$=0.25, k=5/6, $f(T)$=0.4+0.6exp(-0.5T),
B.C.: u=v=w=0 along x=0,b and u=v=w=0 along y=0,l.

To determine range of the small parameter, namely initial maximum amplitude-to-thickness ratio, within which results from the present formulation is reasonable, results of present work for pure elastic cases, η=0, are compared with those of a available literature. Fig. 2 shows the frequency ratio of nonlinear to linear fundamental frequency versus amplitude-to-thickness ratio for a simply supported laminated composite plate. It can be concluded that in the order of amplitude-to-thickness ratio smaller than 0.4, the present approach gives reasonable results in both cases. Discrepancy found in the area of larger amplitude for thicker plates is ascribed to difference in theory and method of analysis employed by Ref. 11, high order shear deformation theory and direct integration method, from present approach.

Noting that m_L means linear dissipation parameter, $-\eta\beta_1/2$, and n_L linear damped frequency, $1+\eta^2\beta_1(3\beta_1/8-1)$, the effect of large deflection on dissipation and frequency is shown in Fig. 3. One can see that large deflection has far less effect on dissipation than on frequency, as mentioned above, by observing that nonlinear effect leads at maximum to only 4% increase in dissipation parameter compared with that of linear analysis, while over 20% increase in frequency for initial amplitude-to-thickness ratio w_0=0.6. The amount of increase caused by nonlinear effect is proportional to square of w_0 as can be easily understood from eqn(15.1) and eqn(15.2). Fig.4 show the effect of relaxation parameter on the nonlinear behavior. It is presented that more viscoelastic material induces more rapid development of damped behavior without affecting peak value of the dissipation parameter and initial frequency variation. The time taken for dissipation parameter to reach the peak value is plotted against relaxation parameter in Fig.5, noting T_p is peak time. It is in inverse proportional to η, which is derived easily through differentiating eqn(15.1) with respect to time and seeking the time for the derivative of eqn(15.1) to vanish. Finally, time history of transverse displacement at the maximum deflection point is plotted in Fig. 6 using eqn(14) and compared with that from linear analysis. One can see that variation of amplitude at maximum deflection point shows little difference between linear and nonlinear analysis in contrast to apparent difference in frequency for a case which has about 4% and 20% difference in dissipation parameter and frequency respectively as nonlinear to linear ratio. From these figures, nonlinear effect appears to be a more important factor to be considered in frequency analysis rather than magnitude analysis.

For more parametric studies on the effect of aspect ratio, slenderness ratio, boundary condition, staking sequence and so forth, frequency ratio from numerous previous literatures on elastic analysis of nonlinear vibration can be used in a limit where the perturbation method makes sense. It is to be noted that in this paper two small parameters, ε and η, are assumed to be in same order and there should be rearrangement among terms to be considered in eqn(11) and eqn(12) according to order of magnitude if the two parameters have different orders.

CONCLUSION

Geometric nonlinear behavior of laminated composite plates undergoing free vibration with large amplitude has been analyzed taking into account the time dependent material properties and results were compared with those of linear analysis. Within a limit on magnitude of a small parameter where perturbation method is applicable, it is shown that large deflection increases dissipative nature of viscoelastic laminated plates and nonlinear effect is in proportion to square of initial deflection for both dissipation parameter and frequency. For the nonlinear effect on the former is not as much as on the latter, nonlinear effect is more to be considered for frequency analysis rather than for magnitude analysis.

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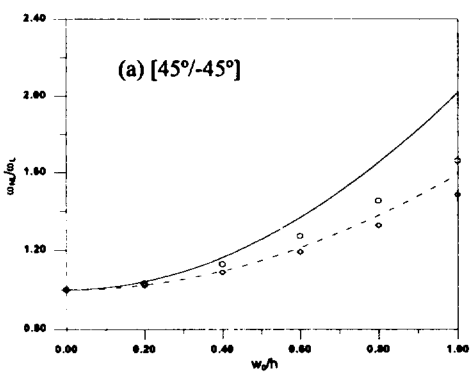


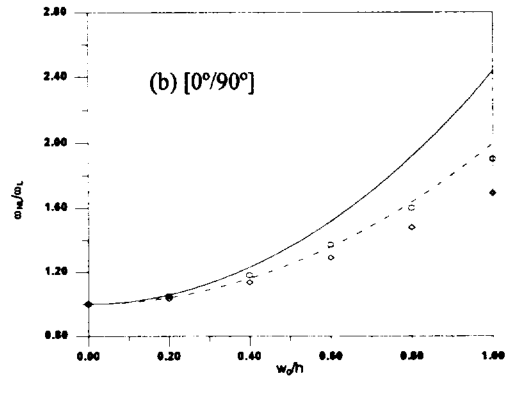


Fig. 2 Frequency ratio of the 1st frequency versus amplitude-to-thickness ratio: pure elastic case (η=0): present — l/t=10 ---- l/t=100, Ref.11 O l/t=10 ◇ l/t=100

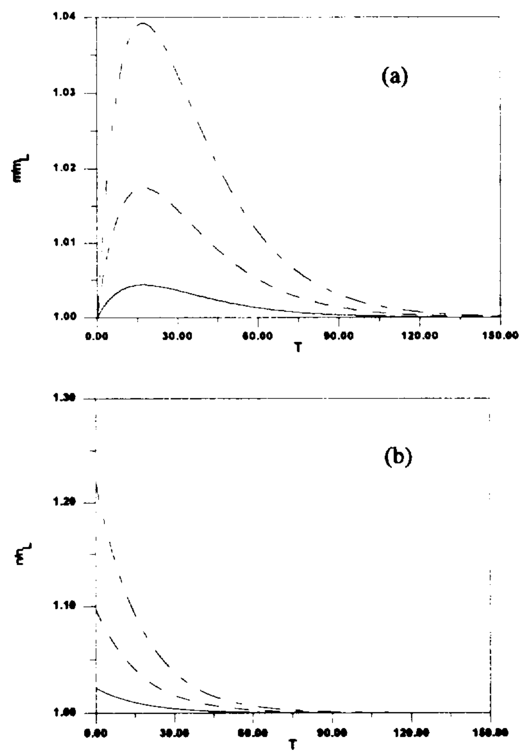
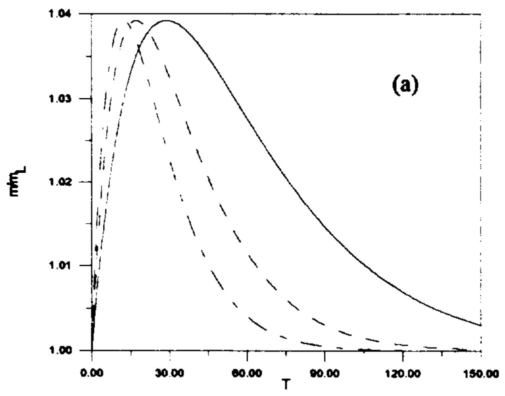


Fig. 3 Effect of large amplitude on the dissipation parameter and frequency: [45°/-45°], w_0 — 0.2 --- 0.4 -- 0.6



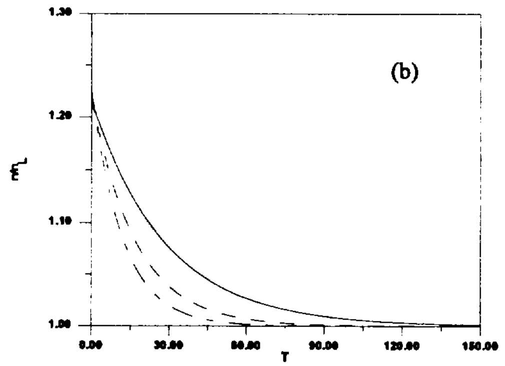


Fig. 4 Effect of relaxation parameter on the dissipation parameter and frequency: [45°/-45°], η — 0.3 --- 0.5 -·- 0.7

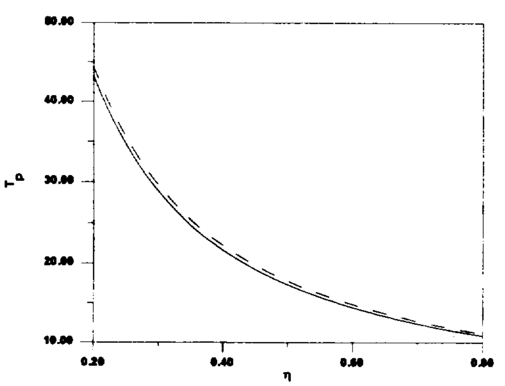


Fig. 5 Peak time of m/m_L versus relaxation parameter: — [45°/-45°], --- [0°/90°]

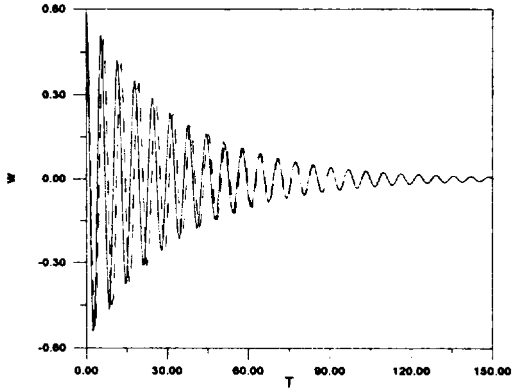


Fig. 6 Time history of maximum deflection point: [45°/-45°], w_0=0.6, --- linear — nonlinear