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힐버트 변환을 이용한 주기적인 외란 및 잡음제거

PERIODIC DISTURBANCE AND NOISE REJECTION METHOD USING HIRBERT TRANSFORM

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Key Words : Periodic Disturbances and Noise Rejection, Notch Filter, Adaptive Filter, Adaptive Feedforward Controller (AFC), Constrained AFC.

ABSTRACT : In this paper, we propose a novel adaptive feedforward controller for periodic disturbance and noise cancellation, with a frequency tracking capability. It can be added to an existing feedback control system without altering the original closed-loop characteristics, which is based on adaptive algorithm. We introduce novel algorithm "Constrained AFC (adaptive feedforward controller) algorithm" that increase the convergence region regardless of the delay in the closed loop system. In the algorithms, coefficients of the controller are adapted using the residuals of constrained structure which are defined in such a way that the coefficients become time invariant. The proposed algorithm not only estimate the magnitude and phase of the tonal disturbance and noise but also track the frequency of the tone, which changes in quasi-static manner. The frequency tracking algorithm uses the instantaneous frequency approach based on Hilbert transform. A number of computer simulations have been carried out in order to demonstrate the effectiveness of proposed method under various conditions.

1. INTRODUCTION

Periodic disturbance and noise occur in several of control engineering applications, which can be represented as a block diagram in Figure 1. There has been a lot of previous work to design linear controls systems for eliminating periodic disturbances and noise. One of the most widely known methods is based on notch filtering, which adds a notch filter at the synchronous frequency into the loop [1,2]. But removing all signals at the synchronous frequency may leads to closed-loop instability, limiting the usefulness of this approach. The other method is adaptive feedforward controller (AFC) [3,4,5] in which the disturbance/noise is cancelled by adding the negative of its value, $r_\Omega(k)$, as shown in Figure 2. AFC continually estimates the Fourier coefficients describing the sinusoidal disturbance/noise. Simpler methods such as adaptive filter introduced by Glover and Widrow could be used to cancel the tonal disturbance/noise [6]. Since the conventional AFC is designed without considering the underlying structure of the system, it alters the original closed-loop characteristics such as notch filter [7].

In this paper, we propose a novel adaptive feedforward controller (AFC) design method for rejecting periodic disturbance and noise. Contrary to the conventional AFC, it can be added to an existing feedback control system without altering the original closed-loop configuration. Thus, the proposed AFC does not affect loop stability. We derive an adaptive algorithm for the proposed AFC design and introduce novel algorithm "Constrained AFC (adaptive feedforward controller) algorithm". In the algorithm, coefficients are adapted using the residuals of constrained structure which are defined in such a way that the coefficients become time invariant. We also discuss the advantage of the constrained AFC that is proposed to overcome the undesirable effects of AFC algorithm on the convergence.

In many previous works, the frequency of the periodic disturbance/noise is assumed to be known exactly. But it is difficult to measure the precise frequency of disturbance/noise and sometimes the frequency drifts in quasi-static manner in real-world situation. So the frequency tracking capability to provide estimation of the periodic disturbance/noise frequency is necessary in AFC system. We propose an adaptive algorithm with a frequency tracking capability. The proposed algorithm not only estimates the magnitude and phase of the periodic disturbance/noise but also track the frequency of the disturbance/noise. Frequency tracking is based on the instantaneous frequency, which is obtained by Hilbert transform pairs [8].

A number of computer simulations are carried out to demonstrate the salient features of the proposed AFC compared to the conventional one.

2. THE CONCEPT OF PERIODIC DISTURBANCE AND NOISE REJECTION BASED ON ADAPTIVE FEEDFORWARD CONTROL

Let us consider the set-up in Figure 1, which shows a linear time-invariant plant *P(s)* perturbed by a periodic measurement disturbance and noise, $d_\Omega(t)/n_\Omega(t)$ represented as follows:

$$n_\Omega(t) = N\sin(\omega_0 t + \varphi_n) = w_{n0}\sin(\omega_0 t) + w_{n1}\cos(\omega_0 t)$$
$$d_\Omega(t) = D\sin(\omega_0 t + \varphi_d) = w_{d0}\sin(\omega_0 t) + w_{d1}\cos(\omega_0 t) \tag{1}$$

where $N = \sqrt{w_{n0}^2 + w_{n1}^2}$, $D = \sqrt{w_{d0}^2 + w_{d1}^2}$, $\varphi_n = \tan^{-1}\left(\frac{w_{n1}}{w_{n0}}\right)$, $\varphi_d = \tan^{-1}\left(\frac{w_{d1}}{w_{d0}}\right)$ and w_0, w_1 are the Fourier coefficients of the synchronous disturbance/noise at the frequency ω_0. The sensor signal $y_\Omega(s)$ contains a synchronous measurement disturbance/noise, $d_\Omega(s)/n_\Omega(s)$. Compensator $C(s)$ is assumed to be designed to stabilize the system.

Let us introduce the adaptive feedforward control method and the drawbacks to design linear control systems for eliminating periodic disturbances/noise. The adaptive feedforward cancellation (AFC) is shown in Figure 2. The disturbance/noise is cancelled by adding the canceling signal $r_\Omega(k)$ that has identical magnitude but opposite phase of disturbance/noise [3,4,5]. The canceling signal has the form

$$\begin{aligned} r_\Omega(k) &= w_{n0}(k)\sin(k\omega_0 T) + w_{n1}(k)\cos(k\omega_0 T) \\ &\quad + w_{d0}(k)\sin(k\omega_0 T) + w_{d1}(k)\cos(k\omega_0 T) \\ &= \{w_{n0}(k) + w_{d0}(k)\}\sin(k\omega_0 T) \\ &\quad + \{w_{n1}(k) + w_{d1}(k)\}\cos(k\omega_0 T) \\ &= w_0(k)\sin(k\omega_0 T) + w_1(k)\cos(k\omega_0 T) \end{aligned} \quad (2)$$

where T is a sampling time and $w_0(k) = w_{n0}(k) + w_{d0}(k)$, $w_1(k) = w_{n1}(k) + w_{d1}(k)$, k = 0, 1, 2, ... are discrete time-varying Fourier coefficients updated on-line by adaptive algorithm for magnitude and phase estimation.

The conventional adaptive law governing $w_0(k)$ and $w_1(k)$ is based on LMS algorithm as follows [9]:

$$\begin{aligned} w_0(k+1) &= w_0(k) + 2\eta e(k)\sin(k\omega_0 T) \\ w_1(k+1) &= w_1(k) + 2\eta e(k)\cos(k\omega_0 T) \end{aligned} \quad (3)$$

where η is the step size that regulates the speed and stability of the adaptation. But it is well known that the conventional AFC design and adaptive law alters the original closed-loop characteristics such as the notch filter[7]. Making a notch within or near system bandwidth can reduce the stability margin of the system at frequencies near ω_0.

A novel adaptive feedforward controller (AFC) design method for disturbance rejection that does not alter the original closed-loop characteristics, was proposed in previous work [7]. In this paper, we add to noise to closed loop with disturbance and derive the cost function for eliminating the periodic disturbance/noise by same manner.

Periodic disturbance/noise can be cancelled by adding a secondary synchronous signal $r_\Omega(k)$ to the reference input. The discrete system output is then decomposed as:

$$y(k) = T(z^{-1})r(k) + T(z^{-1})(n_\Omega(k) - r_\Omega(k)) - Q(z^{-1})d_\Omega(k) \quad (4)$$

where $T(z^{-1}) = \frac{P(z^{-1})C(z^{-1})}{1+P(z^{-1})C(z^{-1})}$, $Q(z^{-1}) = \frac{1}{1+P(z^{-1})C(z^{-1})}$

and z^{-1} is a delay operator. Note that $T(z^{-1})$, $Q(z^{-1})$ are the closed-loop and the sensitivity transfer function respectively. $r_\Omega(k)$ is designed to cancel the component of the system output due to $d_\Omega(k)$ by making the factor $T(z^{-1})(n_\Omega(k) - r_\Omega(k)) - Q(z^{-1})d_\Omega(k) = 0$ in equation (4) without changing the frequency response of $T(z^{-1})$. This approach is desirable because the compensation $C(z^{-1})$ can now be designed without considering disturbance/noise.

To make the factor $T(z^{-1})(n_\Omega(k) - r_\Omega(k)) - Q(z^{-1})d_\Omega(k) = 0$ in equation (4) without affecting the loop stability or the system bandwidth, the cost function J is defined as

$$\begin{aligned} J = \varepsilon^2(k) &= \{\hat{y}(k) - \hat{T}(z^{-1})r(k)\}^2 \\ &= \{\hat{T}(z^{-1})(n_\Omega(k) - r_\Omega(k)) - \hat{Q}(z^{-1})d_\Omega(k)\}^2 \end{aligned} \quad (5)$$

where $\hat{T}(z^{-1}) = \frac{\hat{P}(z^{-1})C(z^{-1})}{1+\hat{P}(z^{-1})C(z^{-1})}$, $\hat{Q}(z^{-1}) = \frac{1}{1+\hat{P}(z^{-1})C(z^{-1})}$.

Note that $\hat{T}(z^{-1}), \hat{Q}(z^{-1})$ is the nominal transfer function. Since the sensor cannot measure the output signal y(k), signal $\hat{y}(k)$ should be estimated by $\hat{y}(k) = \hat{P}(z^{-1})u(k)$. The estimate of the closed loop and sensitivity transfer function is assumed to be perfect at disturbance/noise frequency ω_0, i.e., $\hat{T}(e^{-j\omega_0 T}) = T(e^{-j\omega_0 T})$, $\hat{Q}(e^{-j\omega_0 T}) = Q(e^{-j\omega_0 T})$. The cost function in equation (5) can be simplified by the sinusoidal transfer function $T(e^{-j\omega_0 T})$ under steady state. The modulus and phase of its transfer function are given by $Ae^{-j\Phi}$. The filtered signal of $r_\Omega(k)$, i.e., $T(z^{-1})\ r_\Omega(k)$ is represented as follows:

$$\begin{aligned} &T(z^{-1})r_\Omega(k) \\ &= Aw_0(k-n)\sin(k\omega_0 T - \Phi) + Aw_1(k-n)\cos(k\omega_0 T - \Phi) \end{aligned} \quad (6)$$

where Φ is a system delay and n is integer number of $\frac{\Phi}{\omega_0}$, i.e., $Int(\frac{\Phi}{\omega_0})$. Substituting equation (6) into equation (5) yields the following cost function

$$\begin{aligned} \varepsilon(k) &= T(z^{-1})n_\Omega(k) - Q(z^{-1})d_\Omega(k) \\ &\quad - \{Aw_0(k-n)\sin(k\omega_0 T - \Phi) + Aw_1(k-n)\cos(k\omega_0 T - \Phi)\} \end{aligned} \quad (7)$$

3. AFC (Adaptive Feedforward Controller) AND CONSTRAINED AFC ALGORITHM

The optimum set of Fourier coefficients, which minimizes J, can be obtained using a steepest gradient descent method. The update equation, adaptive law is as follows:

$$w_i(k+1) = w_i(k) - \eta\frac{\partial \varepsilon^2(k)}{\partial w_i(k)} \quad (i = 0, 1) \quad (8)$$

where η is a step size, which determines the speed of adaptation. Substitution of equation (7) into equation (8) yields the adaptation rule. But the partial derivative in equation (8) cannot be given in an explicit manner. So a modified form of adaptive algorithm is derived by the assumption that each w_i is time invariant, i.e., $w_i(k-n) = w_i(k)$ [9,10]:

$$\begin{aligned} w_0(k+1) &= w_0(k) + 2\eta\varepsilon(k)A\sin(k\omega_0 T - \Phi) \\ w_1(k+1) &= w_1(k) + 2\eta\varepsilon(k)A\cos(k\omega_0 T - \Phi) \end{aligned} \quad (9)$$

Equation (9) is titled "AFC (Adaptive Feedforward Controller) algorithm" and proposed algorithm tracks the amplitude and phase of the disturbance/noise by driving

$w_0(k) \to w_0$ and $w_1(k) \to w_1$. So the system output response due to the synchronous disturbance/noise is cancelled at steady state. Figure 3 is a block diagram of the proposed AFC scheme.

The adaptive algorithm, shown in equation (9), is identical to the delayed-x LMS algorithm, which is a special form of the filtered-x LMS algorithm. In a case of delayed-x LMS algorithm, Snyder [11], Morgan [12] and Long [13] derived the stable region of the algorithm for n-step delay as follows:

$$0 < \eta < \frac{1}{A^2\lambda}\sin\{\frac{\pi}{4n+2}\} \tag{10}$$

where n is $Int(\frac{\Phi}{\omega_0})$ and λ is an eigenvalue of the AFC input autocorrelation matrix, $Expectation\begin{bmatrix} \sin^2(k\omega_0 T) & \sin(k\omega_0 T)\cos(k\omega_0 T) \\ \cos(k\omega_0 T)\sin(k\omega_0 T) & \cos^2(k\omega_0 T) \end{bmatrix}$. It should be noted that the delay, n reduces the convergence region of η by a factor of $\sin(\frac{\pi}{4n+2})$.

Let us consider another set of error $\varepsilon_c(k)$ defined by

$$\varepsilon_c(k) = T(z^{-1})n_\Omega(k) - Q(z^{-1})d_\Omega(k)$$
$$-\{Aw_0(k)\sin(k\omega_0 T - \Phi) + Aw_1(k)\cos(k\omega_0 T - \Phi)\} \tag{11}$$

which is obtained by imposing the constraint $w_i(k-n) = w_i(k)$ on the original error $\varepsilon(k)$ [14,15,16]. The novel adaptation rule is obtained using a steepest gradient descent method in the same manner:

$$w_0(k+1) = w_0(k) + 2\eta\varepsilon_c(k)A\sin(k\omega_0 T + \Phi)$$
$$w_1(k+1) = w_1(k) + 2\eta\varepsilon_c(k)A\cos(k\omega_0 T + \Phi) \tag{12}$$

This is so called the "Constrained AFC algorithm". In this approach, coefficients of the controller are adapted using the residuals of constrained structure which are defined in such a way that they should become time invariant.

Modified Error $\varepsilon_c(k)$ can be represented in terms of the original error from equations (7) and (11) as follows:

$$\varepsilon_c(k) \tag{13}$$
$$= \varepsilon(k) - \{Aw_0(k)\sin(k\omega_0 T + \Phi) + Aw_1(k)\cos(k\omega_0 T + \Phi)\}$$
$$+ \{Aw_0(k-n)\sin(k\omega_0 T - \Phi) + Aw_1(k-n)\cos(k\omega_0 T - \Phi)\}$$

The convergence analysis of CAFC algorithm is identical to that of LMS algorithm owing to the residual error $\varepsilon_c(k)$ having a constrained structure $w_i(k-n) = w_i(k)$. Therefore the stable range of η can be derived [9,14,15] as follows:

$$0 < \eta < \frac{1}{A^2\lambda} \tag{14}$$

It should be noted that the convergence region of η becomes independent of the delay. So the constrained AFC algorithm increase the convergence region of stable step size compare with the AFC algorithm.

4. COMPUTER SIMULATIONS

Let us consider the simplified 2nd order integral system $p(s) = \frac{1}{ms^2}$ for computer simulations such as previous work [7]. In order to increase the system stability, the PD control is added to feedback system. Thus, the discrete closed-loop transfer function is obtained as follow [17]:

$$T(z^{-1}) = \frac{0.198z^{-1} + 0.198z^{-2}}{1 - 1.044z^{-1} + 0.442z^{-1}}$$

where the sampling time is 1 sec. We perform a simulation for non-zero reference tracking problem. The frequencies of harmonic reference signal and periodic disturbance are 0.01 Hz and 0.1 Hz, respectively as follows:

$$r(k) = 1.0\sin[2\pi * 0.01 * k]$$
$$n_\Omega(k) = 0.3\sin[2\pi * 0.1 * k] + 0.5\cos[2\pi * 0.1 * k]$$
$$d_\Omega(k) = 0.7\sin[2\pi * 0.1 * k] + 1.5\cos[2\pi * 0.1 * k]$$

A number of computer simulations have been conducted for $d_\Omega(k) = 0$, $n_\Omega(k) = 0$, $d_\Omega(k), n_\Omega(k) \neq 0$ in order to demonstrate the effectiveness of the proposed AFC method compare with the conventional AFC. The initial values of the Fourier coefficients were set to zero and the convergence factor η was fixed at 0.07. In case of periodic disturbance rejection ($n_\Omega(k) = 0$), Figure 4 and 5 show the resulting time history plots of the Fourier coefficients $w_0(k), w_1(k)$ and output signal y(k). AFC compensations are enabled at the 100th step. Before AFC is enabled (t < 100 step), a synchronous component due to $d_\Omega(k)$ can be seen clearly in the signal y(k). Within 20 steps after the compensation is on, the synchronous component in y(k) is removed in both cases. In the conventional AFC case, the Fourier coefficients do not converge to constants as shown in Figure 4. Instead, they fluctuate around constant levels. By increasing the convergence factor, their fluctuation is increased. In the proposed AFC case, Fourier coefficients converge exactly to the steady state values after about 20 steps as shown in Figure 5. This simple example illustrates better performance of the proposed AFC compared with the conventional AFC. Simulations for rejection of periodic noise ($d_\Omega(k) = 0$) have already been conducted in previous paper [7]. We also calculated the frequency response of the closed-loop system with the proposed and the conventional AFC compensations. The conventional AFC made a notch within the system bandwidth at the vicinity of the sinusoidal frequency such as a notch filter. In the proposed AFC, the frequency response was not altered and therefore the loop stability or the system bandwidth is not changed at all [7]. Figure 6 and 7 show the simulation results under the periodic disturbance and noise ($d_\Omega(k), n_\Omega(k) \neq 0$). The proposed AFC has a good performance rather than the conventional AFC such as above simulation.

The step size η provides a balance between speed and stability of convergence. Figure 8 shows a typical convergence pattern of the output signal and coefficients for the proposed AFC and CAFC algorithm respectively. It is clear that large value of η, e.g. η=0.3 make the convergence of AFC algorithm become unstable. On the contrary, It makes the convergence of CAFC algorithm fast. It is noted the stable convergence region of the constrained AFC algorithm is enlarged compare with the AFC algorithm due to independent of the delay.

5. FREQUENCY TRACKING BASED ON INSTANTANEOUS FREQUENCY

In previous works, the frequency of the periodic disturbance is assumed to be known exactly. But it is difficult to measure the precise frequency of disturbance/noise in real-world situation. So the frequency tracking algorithm for frequency estimation is necessary in AFC system.

In the proposed AFC system, we perform a simulation with the desired and the measured frequencies at 0.1 Hz ($f_d = \frac{\omega_d}{2\pi}$), 0.102 Hz ($f_e = \frac{\omega_e}{2\pi}$) respectively. Figure 9 shows the resulting time history plots of the filter coefficients $w_0(k), w_1(k)$ and output signal y(k). As expected, the coefficients do not converge to constants and oscillate. Figure 10 shows the spectrum of time history of the filter coefficients. The time varying filter coefficients oscillate with the fundamental frequency, $f_d - f_e$ (0.002 Hz). The other component having two times of the true frequency f_d (0.1x2 Hz), i.e., carrier frequency is added to the fundamental. Therefore time varying filter coefficients has useful information which can be utilized for frequency error estimation.

In the study, we use instantaneous frequency approach for frequency tracking. The time history of the filter coefficients $w_0(k), w_1(k)$ is sine and cosine curve respectively, which is Hilbert pairs signal as shown in Figure 9 (b). The fundamental instantaneous frequency, i.e., the difference of the frequency can be obtained by differentiating the phase of analytic signal, which can be calculated from the Hilbert transform pairs of the time varying coefficients $w_0(k)$ and $w_1(k)$. Let the analytic signal z(k) has the coefficient $w_0(k)$ and its Hilbert pair, $w_1(k)$ [8].

$$z(k) = w_0(k) + jw_1(k) \tag{15}$$

The envelope and phase of z(k) can be written as

$$|z(k)| = \sqrt{w_0^2(k) + w_1^2(k)} \tag{16}$$

$$\varphi(k) = \tan^{-1}\left(\frac{w_1(k)}{w_0(k)}\right) \tag{17}$$

Therefore, the radial frequency ω can be obtained from the derivative of equation (17). The instantaneous frequency *f(k)* i.e. the estimation frequency error $f_d - f_e$ is

$$f(k) = \frac{1}{2\pi}\frac{d\varphi(k)}{dk} = \frac{1}{2\pi}\frac{d}{dk}\tan^{-1}\left(\frac{w_1(k)}{w_0(k)}\right) = \frac{\frac{dw_1(k)}{dk}w_0(k) + \frac{dw_0(k)}{dk}w_1(k)}{2\pi|z^2(k)|} \tag{18}$$

where $\frac{dw_0(k)}{dk}$ is the time derivative of $w_0(k)$. So we can calculate instantaneous frequency from the $w_0(k)$, $w_1(k)$ and their derivatives.

In case of the 0.02 Hz frequency error, we perform a simulation with the same condition of the previous works. AFC compensation with frequency tracking is enabled. Due to the frequency tracking, the coefficients converge to constant steady state values as shown in Figure 11. After the frequency tracking is on, the undesirable effect of measurement frequency error is removed within 300th step. It is also interesting to investigate the frequency tracking capability in case of slowly changing frequency. Figure 12 show the stable tracking capability of the proposed AFC system. The drift of coefficients in Figure 12 (b) enables not only the estimation of the magnitude and phase of the periodic disturbance/noise but also track the slowly changing frequency of the tone.

6. CONCLUSION

The periodic disturbance and noise rejection method for feedback control system is proposed. A novel adaptive feedforward controller (AFC) design method can be applied to an existing feedback control system without altering the original closed-loop configuration. For cancellation of the periodic disturbance/noise, the adaptive algorithm is developed including the constrained AFC algorithm. The convergence region of constrained AFC algorithm increase and becomes independent of the delay compare with the AFC algorithm. For the real-word situation, AFC system with frequency tracking capability is proposed. We use the instantaneous frequency approach for frequency tracking, which the Hilbert pairs signal is used. A number of computer simulations were carried out for simplified model in order to compare the conventional AFC and the proposed AFC.

The proposed method can be useful for the control engineers to design an adaptive feedforward controller for periodic disturbance/noise rejection.

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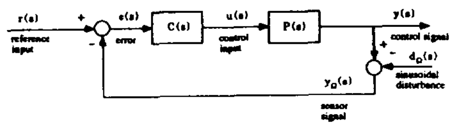


Fig. 1 Block diagram of feedback control system with periodic disturbance

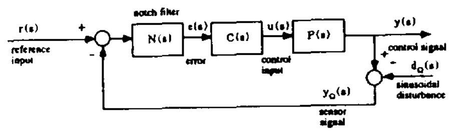


Fig. 2 Block diagram of feedback control using a notch filter for rejecting periodic disturbance

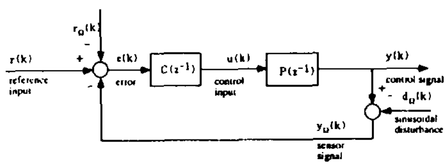


Fig. 3 Block diagram of feedforward control for rejecting periodic disturbance

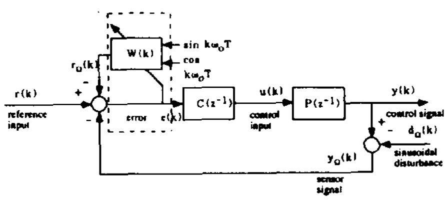


Fig. 4 Block diagram of feedforward control using an adptive notch filter for rejecting periodic disturbance

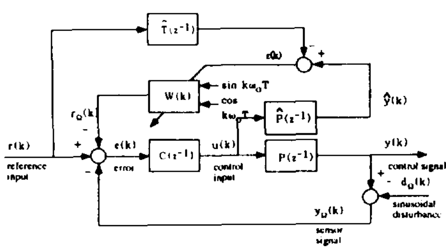
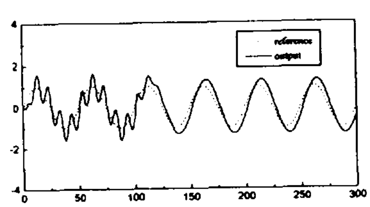
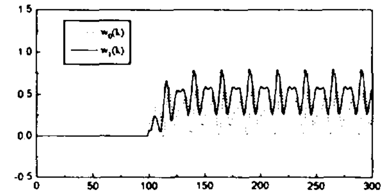


Fig. 5 Block diagram of feedforward control without altering the original closed-loop configuration

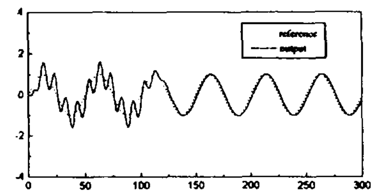


(a) output response

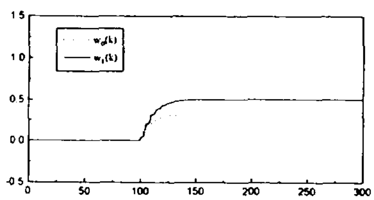


(b) time history of coefficient update

Figure 6. Response of the conventional adaptive feedforward controller

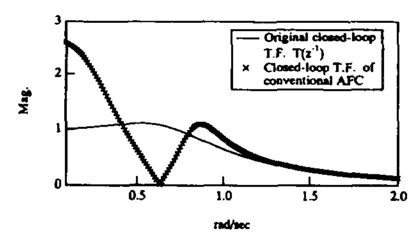


(a) output response

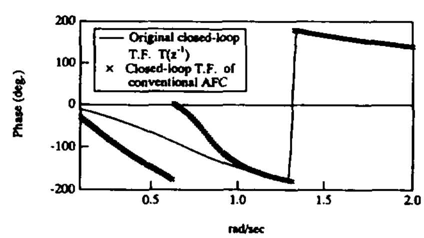


(b) time history of coefficient update

Figure 7. Response of the proposed adaptive feedforward controller

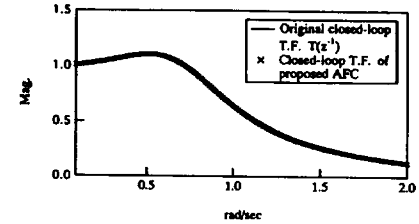


(a) Magnitude plot

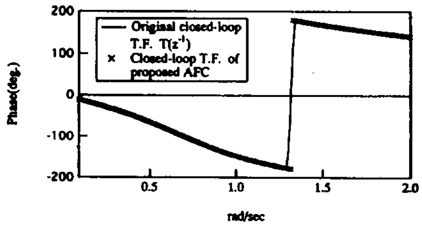


(b) Phase plot

Figure 8. Transfer function of the conventional adaptive feedforward controller(η=0.2)

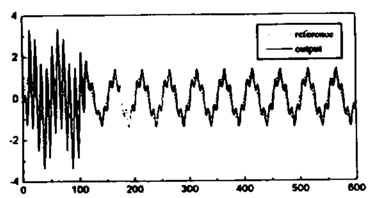


(a) Magnitude plot

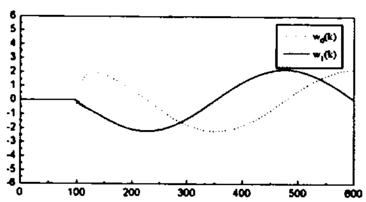


(b) Phase plot

Figure 9. Transfer function of the proposed adaptive feedforward controller(η=0.2)



(a) output response



(b) time history of coefficient update

Figure 10. The resulting time history plots of the filter coefficients $w_0(k)$, $w_1(k)$ and output response for estimated frequency error in case of the proposed AFC

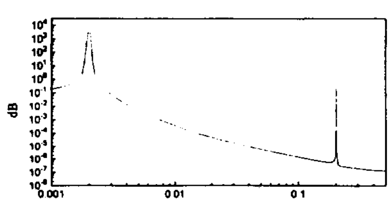
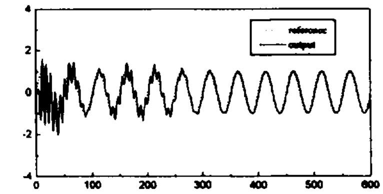
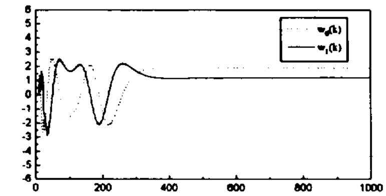


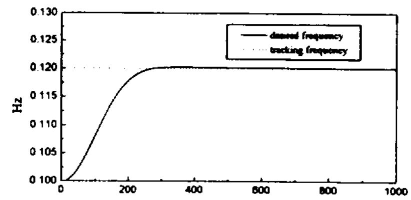
Figure 11. Spectrum of time varying coefficient after proposed AFC control



(a) output response

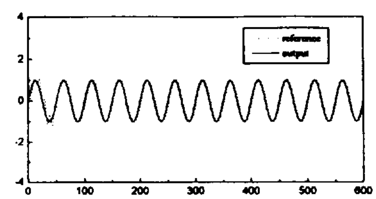


(b) time history of coefficient update

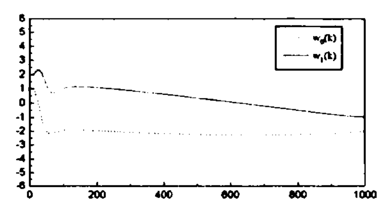


(c) tracking capability

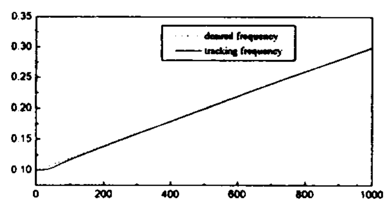
Figure 12. The resulting time history plots of the filter coefficients $w_0(k)$, $w_1(k)$ and output response in case of the proposed AFC with frequency tracking capability



(a) output response



(b) time history of coefficient update



(c) tracking capability

Figure 13. The resulting time history plots of the filter coefficients $w_0(k)$, $w_1(k)$ and output response for slowly changing frequency in case of the proposed AFC with frequency tracking capability