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능동 소음 및 진동 제어에 사용되는 강인안정한 하이브리드 제어기의 설계

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Design of robust stable hybrid controllers for active noise/vibration control

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Abstracts : Adaptive feedforward control algorithms based largely upon LMS approach have developed in recent two decades, and they have been widely applied to practical sound and vibration control problems in the case of the reference signal is available. Feedforward control can be applied only when reference signals can be measured or regenerated, while feedback controllers are used to reduce sound and vibration when reference signals are not available.

In recent years, hybrid control schemes in which adaptive feedforward controllers are combined with feedback ones have been studied based on simulations and experiments. The results have shown that the hybrid control may have better control performances in convergence speed and steady state error than the single control schemes. Hybrid control has the advantages of improving stability and performance as well as the disturbance rejection property. However, little effort has been made to the analysis or interpretation of hybrid control systems.

In this study, we discussed the feedback controller effects on the stability of feedforward control algorithm in the presence of uncertain error path and a simple example showed that a stable feedback controller could make the feedforward controller unstable. A design criterion of feedback controllers is proposed in order to guarantee the stability of feedforward algorithms in the presence of error paths with uncertainties.

Keywords : active noise control, Filtered-X LMS algorithm, robust control, hybrid control, Kharitonov theorem

1. Introduction

The application of adaptive control approach has been interested extensively in recent decades due to developments in DSP hardware. LMS based algorithm and its derivatives are widely used in practical control applications such as noise control in a car, aircraft and passenger ship. In order to increase the control performance, various feedback control schemes has been added to the adaptive feedforward control algorithms in recent years [1~6]. Figure 1 shows the block diagram of general hybrid control. Typical feedback controllers used together with adaptive algorithms are IMC (Internal model control), LQG and H_∞ control. Better control performance can be achieved by additional feedback loop, due to not only disturbance rejection property of feedback control, but also changing error path from actuators to error sensors in feedforward algorithm. Steady-state mean square errors are reduced by disturbance rejection property and additional damping in closed loop error paths increases convergence speeds of adaptive filters.

Advantages of hybrid controllers are discussed in several simulations and experimental studies [4~6]. However, there have been little efforts exerted to the analysis or interpretation of hybrid control systems.

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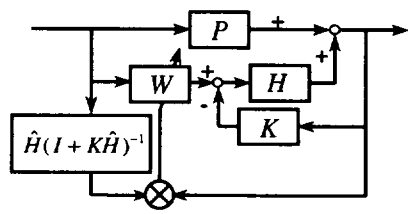

Figure 1. A block diagram of hybrid control

When error path models are exactly same as the actual error paths, Clark [7] verified that separation principle holds in hybrid control design, i.e., feedback controller gives no effects to the stability of feedforward algorithm. Feedback controller changes the performances of feedforward control, but cannot make feedforward controller unstable.

In many cases, however, there are several kinds of uncertainties in error paths, especially in acoustic systems, temperature and humidity changes, moving objects, etc. Therefore feedback controllers are usually designed robustly stable. The robust stable feedback controller is not able to change the stability of the feedforward control in the absence of error path model uncertainties either, but it does not hold any more there are uncertainties in the error paths. Adaptive feedforward algorithms may become unstable regardless of convergence coefficients due to the addition of robust stable feedback controller. In this paper, the influences of feedback controllers upon the stability of feedforward algorithms in the presence of uncertain error paths are studied. A design criterion of feedback controller is proposed in order to guarantee the stability of not only feedback loop but also feedforward algorithm in the presence of error paths with uncertainties.

2. Stability Analysis

2.1 Stability analysis of feedback controller

Consider the feedback loop shown in Figure 2. W_1 and W_2 are input and output weighting functions for uncertainty, W_d and W_e are weighting functions for disturbance and error, respectively. The real plant is modeled as a multiplication of a nominal plant H and an unstructured multiplicative perturbation Δ at the plant output as follows:

$$H(j\omega) = (I + W_1(j\omega)\Delta(j\omega)W_2(j\omega))\hat{H}(j\omega). \quad (1)$$

A H_∞ robust controller stabilizing the feedback loop with multiplicative uncertainty can be designed from the following result [8,9].

$$\|W_2 T_o W_1\|_\infty \le 1, \quad (2)$$

where T_o is the output complimentary sensitivity function of the closed-loop system. The system robust performance is guaranteed if one of the following conditions is satisfied [9]:

$$\bar{\sigma}(W_d)\bar{\sigma}(W_e S_o) + \bar{\sigma}(W_1)\bar{\sigma}(W_2 T_o) \le 1, \quad (3)$$

$$\kappa(W_1^{-1}W_d)\bar{\sigma}(W_e S_o W_d) + \bar{\sigma}(W_2 T_o W_1) \le 1. \quad (4)$$

Where S_o is the output sensitivity function of the closed-loop system, $\bar{\sigma}(\cdot)$ is the maximum singular value and $\kappa(\cdot)$ is the condition number.

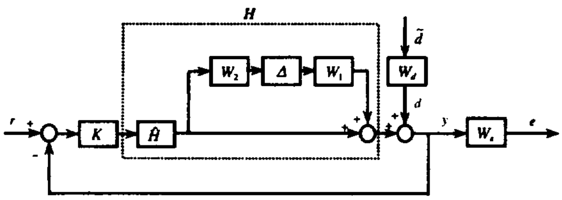

Figure 2. A feedback control system with output multiplicative uncertainty

2.2 Stability analysis of adaptive feedforward controller

Figure 3 shows the block diagram of the conventional LMS-based feedforward control. It was shown that the uncertainty in the error path model has influence upon the stability of Filtered-X LMS algorithm by Snyder [10]. If the phase difference between the error path and its models is greater than 90° in control frequency range, adaptive algorithms cannot be stable for any positive convergence coefficient. The permissible bound of phase difference for the error path models is 90°. The necessary and sufficient condition which stabilize the LMS-based adaptive controller can be expressed as

$$\angle H(j\omega) - \angle\hat{H}(j\omega) < 90^\circ. \quad (5)$$

From the above condition, feedforward stability is not dependent on the control gain but the phase difference only.

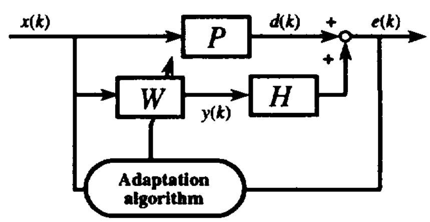

Figure 3. A conventional structure of LMS-based feedforward algorithm

2.3 Stability analysis of hybrid controller

In most of the hybrid control strategies, a fixed feedback controller is designed with the error path models, which were estimated a priori. And, an adaptive controller is adaptively determined to reduce the error. Because the feedback controller is fixed, the feedforward controller cannot affect the feedback stability and performances. So, it just needs to consider the feedforward stability in the hybrid structure after the robust stable controllers are combined. As shown in Figure 1, the feedback controller changes the error path H into $H(I+KH)^{-1}$ and its model $\hat{H}$ into $\hat{H}(I+K\hat{H})^{-1}$ in the hybrid structure, so the stability of the adaptive algorithm is changed accordingly.
The LMS based adaptive filter in the hybrid structure may be stabilized if the following necessary and sufficient condition is satisfied instead of equation (5).

$$\left|\angle\hat{H}(j\omega)\left(I + K(j\omega)\hat{H}(j\omega)\right)^{-1} - \angle H(j\omega)(I + K(j\omega)H(j\omega))^{-1}\right| < 90^\circ. \quad (6)$$

If there is no uncertainty in the error path, (6) always hold independent of the feedback controller and the separation principle can be applied to the design of the hybrid control.

The phase condition of (6) is equivalent to the following magnitude constraint,

$$\left|H(j\omega)(I+K(j\omega)H(j\omega))^{-1}-\hat{H}(j\omega)\left(I+K(j\omega)\hat{H}(j\omega)\right)^{-1}\right|$$
$$<\left|H(j\omega)(I+K(j\omega)H(j\omega))^{-1}+\hat{H}(j\omega)\left(I+K(j\omega)\hat{H}(j\omega)\right)^{-1}\right|. \quad (7)$$

Suppose all the frequency weighting functions are unity and the error path H is single-input single-output system, (7) can be expressed with the uncertainty Δ and $K\hat{H}(j\omega)$ which is already known as follows:

$$|\Delta(j\omega)|<\left|2+\Delta(j\omega)+2(1+\Delta(j\omega))K(j\omega)\hat{H}(j\omega)\right|. \quad (8)$$

Similarly, the necessary and sufficient condition of (5) that stabilize the feedforward controller can be rewritten as

$$|\Delta(j\omega)|<|\Delta(j\omega)+2|. \quad (9)$$

And the sufficient condition that guarantees the feedback stability in Figure 3 is expressed by the small gain theorem as follows:

$$|(1+\Delta(j\omega))|<\left|K(j\omega)\hat{H}(j\omega)\right|^{-1}. \quad (10)$$

The uncertainty satisfying (8), (9) and (10) expressed in terms of $K\hat{H}(j\omega)$ stabilize the hybrid controller, the feedforward controller and the feedback controller, respectively. If any uncertainty which satisfy both (9) and (10) doesn't satisfy the condition (8) for a given $K\hat{H}(j\omega)$ then, the hybrid controller which combined a stable feedforward controller with a stable feedback one may be unstable. Thus, one should re-examine the stability of the adaptive algorithm after combining stable feedback and feedforward controllers.

The uncertainty bounds satisfying (8) and (10) is plotted in Figure 4 (a) and (b), respectively. All the magnitude of the uncertain bounds under fifteen is plotted within $|K\hat{H}|<1$, which guarantees the nominal stability of the feedback controller. Figure 4 (c) is a minimum bound of two plots. Within the region $|K\hat{H}+0.5|<0.5$, we can easily find the uncertain bound of condition (8) is lower than that of (10). It means that the robust stable feedforward algorithm combined with robust stable feedback controller may be unstable for some uncertainty, i.e., the robustly stable feedback controller can make the feedforward controller unstable in the hybrid structure. However, many of the hybrid control were designed assuming the separation principle in practical applications in which uncertainty exists. One cannot say that the

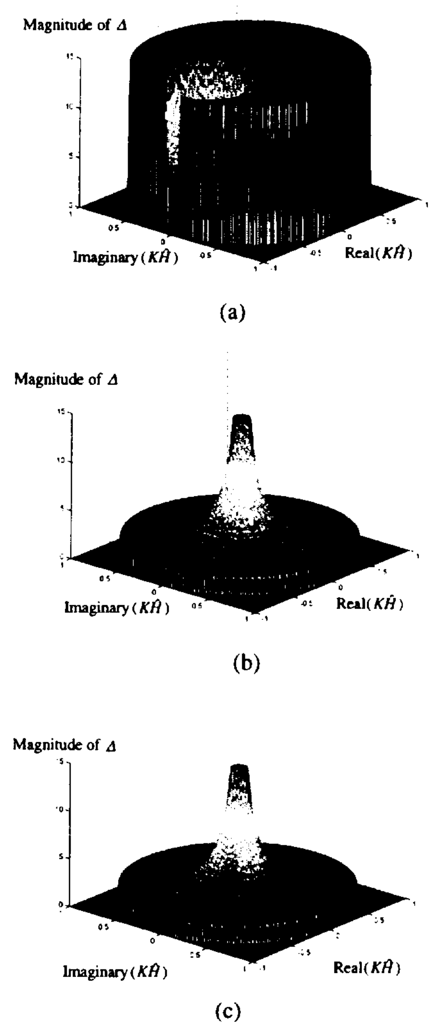

Figure 4. The uncertainty bounds satisfying the stability of (a) the feedforward control in the hybrid structure, (b) the feedback control, and (c) the hybrid control

stability of those hybrid controls is guaranteed for uncertain error paths.

2.4 A counter example

The following problem is a counter example of the separation principle. Consider a single tone noise reduction with a hybrid controller. The plant P and the error path model $\hat{H}$ are FIR filters as follows:

$$P=z^{-7}+0.7z^{-8}-0.8z^{-8}+0.4z^{-9}-0.2z^{-10}, \quad (11)$$

$$\hat{H} = -z^{-1} + 0.7z^{-2} - 0.4z^{-3} + 0.3z^{-4} - 0.2z^{-5} + 0.1z^{-6}, \quad (12)$$

$$H(j\omega) = (1+\varepsilon)\hat{H}(j\omega). \quad (13)$$

At the control frequency 100Hz, the plant and the error path model have a gain of -0.5017+1.2641j and -0.5927+0.1849j respectively. If there is a multiplicative uncertainty ε that has -0.6580+0.9397j at the control frequency in the error path as described in (13), the error path has a gain, -0.3765-0.4937j at 100Hz. Then, the phase difference at the control frequency 100Hz between the error path and its model becomes 70°. So, the Filtered-X LMS algorithm can be stable for the uncertain error path. In order to employ a hybrid control scheme, a robust stable feedback controller was designed which has a gain of 0.6000 - 0.9000j at 100Hz. It makes the feedback loop of Figure 2 stable for the given uncertainty. Time responses before and after feedforward, feedback and hybrid control without uncertainty are plotted in Figures 5 (a), (b), and (c), respectively. All of the controllers are stable. Figures 5 (d), (e), and (f) are the responses before and after feedforward, feedback, and hybrid control in the presence of uncertainty. Control performances are degraded but the stability of the systems is still preserved in single scheme control for the given uncertainty. But the time response diverges in the hybrid control with uncertainty. The error path of the hybrid scheme was changed into $H(I+KH)^{-1}$, and the phase difference between the error path, $H(I+KH)^{-1}$ and its model, $\hat{H}(I+K\hat{H})^{-1}$ at the control frequency, became greater than 90°, exactly 101°. So the hybrid controller cannot be stable for any positive convergence coefficient. From these results, we verified that a robust stable feedforward controller combined with a robust feedback controller is not always stable for given uncertainty, i.e., a robust feedback controller might cause the adaptive feedforward algorithm in hybrid structure unstable in the presence of error path uncertainty.

3. A design criteria for stable hybrid controller

When error path uncertainty Δ exists, the error path model is different from the actual error path. The LMS based feedforward controllers are stable if the phase constraint of (5) holds for a given uncertainty. For SISO systems, the phase constraint of (5) can be expressed in the other inequality [10] as

$$\mathrm{Re}[\hat{H}^*(j\omega)\cdot H(j\omega)] > 0, \text{ for all } \omega, \quad (14)$$

where * is hermitian tramspose. If the number of control outputs is M and the number of error sensors is L, (14) can be rewritten as

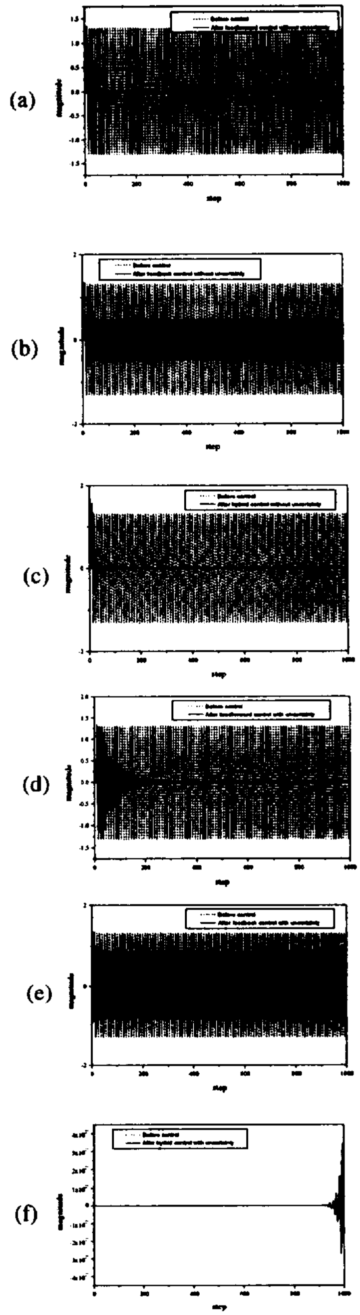

Figure 5. Time responses before and after (a) feedforward control, (b) feedback control, and (c) hybrid control without uncertainty, (d) feedforward control, (e) feedback control, and (f) hybrid control with uncertainty.

$$\mathrm{Re}\left[\lambda_i\left(\hat{\mathbf{H}}^*(j\omega)\cdot\mathbf{H}(j\omega)\right)\right]>0,\ i=1,\cdots,M \quad \text{for all } \omega, \tag{15}$$

where $\lambda_i(\cdot)$ is the ith eigenvalue and

$$\mathbf{H}(j\omega)=\begin{bmatrix} H_{11}(j\omega) & \cdots & H_{1M}(j\omega) \\ \vdots & \ddots & \vdots \\ H_{L1}(j\omega) & \cdots & H_{LM}(j\omega)\end{bmatrix}, \tag{16}$$

$$\hat{\mathbf{H}}(j\omega)=\begin{bmatrix} \hat{H}_{11}(j\omega) & \cdots & \hat{H}_{1M}(j\omega) \\ \vdots & \ddots & \vdots \\ \hat{H}_{L1}(j\omega) & \cdots & \hat{H}_{LM}(j\omega)\end{bmatrix}. \tag{17}$$

(15) means that the real parts of all eigenvalues of $\hat{\mathbf{H}}^*\mathbf{H}$ be positive. Similarly, the feedforward algorithm in hybrid structure is stable if the following condition is satisfied for all ω,

$$\mathrm{Re}\left[\lambda_i\left(\hat{\mathbf{H}}_c^*(j\omega)\cdot\mathbf{H}_c(j\omega)\right)\right]>0,\ i=1,\cdots,M, \tag{18}$$

where

$$\mathbf{H}_c=\mathbf{H}(I+K\mathbf{H})^{-1}. \tag{19}$$

In order to guarantee the stability of hybrid structure, feedback controller should always satisfy not only the feedback loop stability but also the costraint of (18) for any uncertainty. But is is not easy to check whether (18) is satisfied for every uncertainty because we don't know about the exact value but the boundary value of the uncertainty. Thus, in this paper, we tried to estimate whether the feedback controller violates the constraint of (18) for the given uncertainty using Kharitonov theorem [11,12].

We assume that the error path has structured unceratinties that are bounded and their bounds are known. We also assume that the system is single-input single-output case. Then, $\hat{\mathbf{H}}_c^*(j\omega)\cdot\mathbf{H}_c(j\omega)$ can be expressed as the following plant without loss of generality

$$F(s)=\frac{N(s)}{D(s)}, \tag{20}$$

where

$$N(s)=\sum_{i=0}^{m} n_i s^i,\ \mathrm{D}(s)=\sum_{i=0}^{q} d_i s^i, \tag{21}$$

$$\underline{n}_i<n_i<\overline{n}_i,\ \underline{d}_i<d_i<\overline{d}_i. \tag{22}$$

Each coefficient of numerator and denominator polynomials varies independently in a specific interval according to the structured uncertainties. When $s=j\omega$, numerator (denominator) polynomial is located in a rectangular region in complex plane. We introduce four special polynomials that are related to the numerator polynomial

$$\begin{aligned} g_-(s)&=\underline{n}_0+\overline{n}_2s^2+\underline{n}_4s^4+\cdots,\\ g_+(s)&=\overline{n}_0+\underline{n}_2s^2+\overline{n}_4s^4+\cdots,\\ h_-(s)&=\underline{n}_1s+\overline{n}_3s^3+\underline{n}_5s^5+\cdots,\\ h_+(s)&=\overline{n}_1s+\underline{n}_3s^3+\overline{n}_5s^5+\cdots.\end{aligned} \tag{23}$$

Now, we define four important polynomials of the interval numerator polynomial, called Kharitonov polynomials, which are corner polynomials of the rectangular region

$$\begin{aligned} N_{++}(s)&=g_+(s)+h_+(s),\ N_{+-}(s)=g_+(s)+h_-(s),\\ N_{-+}(s)&=g_-(s)+h_+(s),\ N_{--}(s)=g_-(s)+h_-(s).\end{aligned} \tag{24}$$

There exist other four Kharitonov polynomials related to the denominator polynomial and we define them as

$$D_{++}(s), D_{+-}(s), D_{-+}(s), D_{--}(s). \tag{25}$$

We denote by Kharitonov plant that is the ratio of a numerator Kharitonov polynomial to a denominator Kharitonov polynomial. There are sixteen Kharitonov plants. $F_{-+}^{+-}(s)=N_{+-}(s)/D_{-+}(s)$ is one of the sixteen Kharitonov plant.

Now, we prove strict positive realness of the interval plant of (20) with the following theorem

Theorem: An interval family of the proper transfer function is strict positive real if and only if the following eight transfer functions are strict positive real

$$\begin{aligned} &F_{--}^{+-}(s)=N_{+-}(s)/D_{--}(s),\ F_{+-}^{--}(s)=N_{--}(s)/D_{+-}(s),\\ &F_{++}^{+-}(s)=N_{+-}(s)/D_{++}(s),\ F_{+-}^{++}(s)=N_{++}(s)/D_{+-}(s),\\ &F_{-+}^{++}(s)=N_{++}(s)/D_{-+}(s),\ F_{++}^{-+}(s)=N_{-+}(s)/D_{++}(s),\\ &F_{--}^{-+}(s)=N_{-+}(s)/D_{--}(s),\ F_{-+}^{--}(s)=N_{--}(s)/D_{-+}(s).\end{aligned} \tag{26}$$

Detailed proof is explained in [12], so it is omitted in this paper. If all of eight Kharitonov plant are strict positive real, any subset of interval plant family is always strict positive real for any uncertainties. From this result, we can estimate whether the feedback controller violate the constraint which assure the stability of the feedforward algorithm in the hybrid structure. If a robust stable feedback controller doesn't violate (18), the stability of the hybrid controller is guaranteed.

4. Concluding remarks

Stability analysis of the hybrid control (feedback and feedforward) is more complicated than that of single control scheme. In the absence of error path uncertainties, feedback controller has no influence on the stability of feedforward algorithm. But, in the presence of error path uncertainties, it may cause the feedforward algorithm to be unstable regardless of the convergence coefficient. The separation principle does not hold any more in this case. Most of the noise and vibration control systems have these uncertainties, so, design of feedback control in hybrid scheme should be executed carefully. An example showed that robust stable feedback controller could make the hybrid controller unstable.

Finally, a design criterion of robust stabilizing not only feedback but also feedforward loop is proposed. With this method, we can design a feedback controller that guarantees the stability of the hybrid control.

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