Optimal Burn-In under Warranty

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Abstract

This paper discusses an optimal burn-in procedure to minimize total costs based on the assumption that the failure rate pattern follows a bimodal mixed Weibull distribution. The procedure will consider warranty period as a factor of the total expected burn-in cost. A cost model is formulated to find the optimal burn-in time that minimizes the expected burn-in cost. Conditional reliability for warranty period will be discussed. An illustrative example is included to show how to use the cost model in practice.

Keywords: Bimodal Mixed Weibull Distribution; Burn-In; Conditional Reliability; Warranty Cost

1. Introduction

Burn-in is described by the *AT&T* Reliability Manual (1990) as one effective method of screening using two types of stress, temperature and electrical stresses. For each product/component, there is an optimum burn-in time period. If we burn-in for shorter than optimum, the extra field repair/replace costs are greater than the savings in burn-in costs. If we burn-in for a period longer than optimum, the extra burn-in costs exceed the savings from field repair/replace costs. Therefore, finding optimal burn-in time is very important to save costs.

In the past, mixture distribution models have been suggested to describe the failure patterns of electrical and micro-electrical components. Kao (1959) introduced a bimodal mixed Weibull distribution to describe the failure time of electronic tubes. Subsequently, many researchers (see, Stitch (1975), Reynolds & Stevens (1978), Boardman & Colvert (1978)) have found that failure rate patterns of many components/devices follow a mixed Weibull distribution law. Recently, Block, Mi and Savits (1993) investigated a general mixture model to determine the optimal burn-in time. Spizzichino (1995) discussed another model for mixtures in heterogeneous that was related to burn-in. Block and Savits (1997) gave an excellent review of burn-in for mixture models. Kim (1998) presented an optimal burn-in model for mixed populations with multiobjectives. In this paper, we present an optimal burn-in procedure under warranty using the bimodal mixed Weibull distribution.

2. Bimodal Mixed Weibull Distribution

A bimodal mixed distribution is composed of two cumulative density functions (CDF). Let $F_1(t)$ be the CDF of weak population, $F_2(t)$ be the CDF of strong population and $F(t)$ be the total CDF for the entire population. Then, $F(t)$ is constructed by taking a weighted average of the CDFs for the weak and strong populations. The weights are the proportions of each type of subpopulation. Thus, if the weak population has p proportion and the strong population has $(1-p)$ proportion, then

$$F(t) = pF_1(t) + (1-p)F_2(t) \tag{1}$$

Typically, $F_1(t)$ has a high probability of failing early while $F_2(t)$ has a low early failure rate that either stays constant or increases very late in life.

Assume that $f_i(t)$ is the probability density function (pdf) for $F_i(t)$ where $i = 1, 2$. Then, the failure rate of the two-mixed distribution is expressed as

$$h(t) = \frac{pf_1 + (1-p)f_2(t)}{1-\{pF_1(t) + (1-p)F_2(t)\}} \tag{2}$$

where $h(t)$ is interpreted as the instantaneous failure rate at time t.

Now, let us consider the two-mixed Weibull distribution with two parameters for each population. From the equation (1), the CDF of the two-mixed Weibull distribution is

$$F(t) = 1 - p(\exp[-(t/\eta_1)^{\beta_1}]) - (1-p)(\exp[-(t/\eta_2)^{\beta_2}]) \tag{3}$$

where

β_1 : shape parameter of the weak population,
β_2 : shape parameter of the main population

η_1 : scale parameter of the weak population,
η_2 : scale parameter of the main population
p : proportion of the weak population

The two-mixed Weibull distribution will be used to formulate the cost model.

2.1. Time Dependent Conditional Reliability

The explicit treatment of early failures or wear effects requires time dependent conditional reliability modeling. In this situation, the reliability of a component or system becomes a strong function of its age. In a conditional reliability model, we consider the effect of the accumulated operating time, t, on the probability that a system can survive for an additional time, x.

Suppose that we define $R(x \mid t)$ as the reliability of a system that has previously been operated for a time t. The conditional reliability under the given condition of t is

$$R(x \mid t) = P(X > x + t \mid X > t) = \frac{P(X > x + t)}{P(X > t)} \tag{4}$$

where

$X = x + t$ is the time elapsed at failure since the system was new and
$P(X > x + t) = R(x + t)$ is the probability that a device or system will operate for $x + t$ time without a failure.

When the times are exponentially distributed, conditional reliability is just a simple exponential calculation, as follows:

$$\frac{\exp[-\lambda(x + t)]}{\exp[-\lambda t]} = \exp[-\lambda t] \tag{5}$$

Since the reliability of a new system is just $R(x) \equiv R(x \mid t = 0) = P(X > t)$, we obtain $R(x \mid t) = R(x + t) / R(t)$. Finally, the conditional reliability function can be expressed in terms of the hazard function as $R(x \mid t) = \exp(-\int^{x+t} h(u)du)$.

We may interpret the above t as the duration of past usage (which may include physical "wear"), and x as a future period of interest (regarding a reliability prediction). In other cases, this "time" variable may be replaced by the load condition which the device or system has experienced. The reliability of such a device depends on the result of the cumulative load applied over the total previous history of the device.

3. Cost model under warranty

The proposal for an optimal burn-in procedure assumes that the failure pattern of components follows the bimodal mixed Weibull distribution. It is further assumed that the probability of the occurring failures in any components during the warranty period is negligible. Using the formulated cost model, we determine the optimum burn-in time such that the total cost for burn-in under warranty is minimized.

Notation

p : probability that a component is initially bad,

p_G : probability that an initially strong (good) component survives burn-in.

p_B : probability that an initially bad (weak) component survives burn-in. This event can occur when the burn-in time is too short or the accelerated conditions are not severe enough.

C_B : burn-in cost for a component

C_{GF} : cost when a strong component fails burn. The cost includes replacement cost and loosing the value of a strong component. This cost occurs when accelerated conditions are too severe and/or the burn-in time is too long under accelerated conditions.

C_{BP} : cost when a bad component passes burn-in. The cost is a repair/replace cost possibly further downstream assembly.

C_{BF} : cost when a bad component fails burn-in. This is the replacement cost.

C_{AF} : cost per component that survived the burn-in but failed during warranty period W_t. The cost is field failure cost or repair/replace cost in field.

W_t : warranty period.

T : burn-in time

$R(W_t \mid T)$: conditional reliability of a burned-in component during warranty period

$R(T)$: reliability of a component during burn-in

N : number of components to be placed on burn-in.

N_W : number of components that survives the burn-in. $N_W = N \times R(T)$.

In practice, components on test during burn-in will either fail or survive. Some will be from the bad components while others will be from the good components. Table 1 shows the associated probabilities.

Table 1: Probabilities during Burn-In and warranty period

	Good components	Bad components
Survive in burn-in	$(1-p)p_G$	pp_B
Fail in burn-in	$(1-p)(1-p_G)$	$p(1-p_B)$
Survive in warranty	$R(W_t \mid T)$	
Fail in warranty	$1-R(W_t \mid T)$	

In Table 1, pp_B indicates the probability that a component survives the burn-in and was from the weak portion of the components. Probability that components survived the burn-in but failed during warranty period, W_t, is $1-R(W_t \mid T)$.

Table 2 includes the cost function for the situations presented in Table 1. For a component that survived the burn-in test and was from the good portion of the components, the total cost is the cost for performing the test C_B.

Table 2: Cost function during Burn-In and warranty period

	Good components	Bad components
Survive in burn-in	C_B	$C_B + C_{BP}$
Fail in burn-in	$C_B + C_{GF}$	$C_B + C_{BF}$
Fail in warranty	C_{AF}	C_{AF}

Then, the total expected cost for burn-in under warranty is given by

$$C_T = N\{(1-p)p_G C_B + pp_B(C_B + C_{BP}) + (1-p)(1-p_G)(C_B + C_{GF}) + p(1-p_B)(C_B + C_{BF})\} + N_W[(1-R(W_t \mid T)]C_{AF}) \quad (6)$$

where $p_G = \exp[-(T/\eta_2)^{\beta_2}]$, $p_B = \exp[-(T/\eta_1)^{\beta_1}]$, and $R(W_t \mid T) = \dfrac{R(W_t + T)}{R(T)}$.

In equation (6), $N_W = NR(T)$. Therefore, the expected total cost in the equation (6) can be expressed as

$$\begin{aligned} C_T = N(&p\{\exp[-(T/\eta_1)^{\beta_1}]\}(C_{BP} + C_{AF} - C_{BF}) \\ &+ (1-p)\{\exp[-(T/\eta_2)^{\beta_2}]\}(C_{AF} - C_{GF}) \\ &- (p\{\exp[-(W_t + T/\eta_1)^{\beta_1}]\} + (1-p)\{\exp[-(W_t + T/\eta_2)^{\beta_2}]\})C_{AF} \\ &+ p(C_{BF} - C_{GF}) + C_{GF} + C_B) \end{aligned} \tag{7}$$

The optimal burn-in time is determined to minimize the total cost defined in equation (7). In the next section, optimal burn-in procedure will be presented.

4.Optimizing expected total cost

The optimal burn-in time, T^*, is determined by minimizing the expected total cost. Three possible cases are considered.

- If replacement and repair costs for components during burn-in are higher than field or downstream replacing and repair costs, $\partial C_T / \partial T > 0$ and $\partial^2 C_T / \partial T^2 < 0$. Therefore, C_T is an increasing function of T and concave. In this case, not to burn-in is the optimum.

- If replacement and repair costs for components during burn-in are much lower than field or downstream replacing and repair costs, the minimum total cost is obtained at the time that begins useful failure rate region in the bathtub curve.

- The nontrivial optimal burn-in time with the minimum expected burn-in costs could be achieved by taking the first partial derivative of C_T as

$$\frac{\partial C_T}{\partial T} = N(\frac{\beta_1}{\eta_1}\left(\frac{T}{\eta_1}\right)^{\beta_1 - 1} \exp\left[-\left(\frac{T}{\eta_1}\right)^{\beta_1}\right][p(C_{BP} + C_{AF} - C_{BF})]$$

$$
\begin{aligned}
&+\frac{\beta_2}{\eta_2}\left(\frac{T}{\eta_2}\right)^{\beta_2-1}\exp\left[-\left(\frac{T}{\eta_2}\right)^{\beta_2}\right][(1-p)(C_{AF}-C_{GF})] \\
&-\frac{\beta_1}{\eta_1}\left(\frac{W_t+T}{\eta_1}\right)^{\beta_1-1}\exp\left[-\left(\frac{W_t+T}{\eta_1}\right)^{\beta_1}\right][pC_{AF}] \\
&-\frac{\beta_2}{\eta_2}\left(\frac{W_t+T}{\eta_2}\right)^{\beta_2-1}\exp\left[-\left(\frac{W_t+T}{\eta_2}\right)^{\beta_2}\right][(1-p)C_{AF}]\,) \qquad (8)
\end{aligned}
$$

For the bimodal mixed Exponential distribution, equation (8) becomes

$$
\exp\left[-\left(\frac{T}{\eta_1}\right)\right][p(C_{BP}+C_{AF}-C_{BF})]+\exp\left[-\left(\frac{T}{\eta_2}\right)\right][(1-p)(C_{AF}-C_{GF})]
$$

$$
=\exp\left[-\left(\frac{W_t+T}{\eta_1}\right)\right][pC_{AF}]+\exp\left[-\left(\frac{W_t+T}{\eta_2}\right)\right][(1-p)C_{AF}] \qquad (9)
$$

Then, the optimal burn-in time is defined using quasi newton method. We can also find the optimal burn-in time from the cost model (equation (7)) directly.

5. An Illustrative example

A numerical example from Jensen and Petersen (1982) is considered to discuss the optimal burn-in procedure under warranty. A manufacturer receives a bath of 1000 CMOS transistors. From the previous experience, they expect to find the proportion of the weak transistors in the bath is $p = 0.05$. Assume that the time-to-failure distribution of weak population is exponential with an mean time to failure equals to 200 hours at a test temperature of $125^0 C$. At the same temperature, the mean time to failure of strong population is known to be around 100,000 hours. Past investigation suggests that the time-to-failure distribution of the strong population is also exponential.

Now, let us investigate this example further. Assume that we defined the following values of the constraints and parameter values of the mixed population for the transistors are:

$$\hat{p} = 0.05,\ \hat{\beta}_1 = \hat{\beta}_2 = 1,\ \hat{\eta}_1 = 200 \text{ hours},\ \hat{\eta}_2 = 100{,}000 \text{ hours}.$$

and

$$C_B = \$1.00, C_{BF} = \$2.00, C_{GF} = \$4.00, C_{BP} = \$200.00, C_{AF} = \$300.00, N = 1000$$

Then, from equation (7)

$$C_T = 1000(0.05\{\exp[-(T/200)]\}(498) + (0.95)\{\exp[-(T/100000)]\}(296) \\ - (0.05\{\exp[-(W_t + T/200)]\} + (0.95)\{\exp[-(W_t + T/100000)]\})(300) + 4.9)$$

If $W_t = 50{,}000$ hours, the expected total cost is

$$C_T = 1000\left(24.9\left\{\exp\left[-\left(T/200\right)\right]\right\} + 281.2\left\{\exp\left[-\left(T/100000\right)\right]\right\}\right) \\ - 1000\left(15\left\{\exp\left[-\left(T + 50000/200\right)\right]\right\} + 285\left\{\exp\left[-\left(T + 50000/100000\right)\right]\right\} + 4.9\right)$$

Table 3 shows the relation among burn-in times, total cost, and conditional reliability at warranty period = 50,000 hours.

5. Conclusion

In practice, we often encounter the reliability situation with weak and strong subpopulations for many components/systems. In this paper, an expected burn-in cost model was formulated to determine the optimal burn-in time, T^*, when components have warranty period. We also formulated burn-in probability and cost models.

The optimal burn-in time is determined to minimize the total expected burn-in. The proposed formulas will perform well as long as moderately accurate estimated parameter values are used. The finding of this study can be used by a reliability engineer or analyst as a guide for planning an optimal burn-in procedure for a process with mixed population for components/systems.

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