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Iterative Least-Squares Image Reconstruction Methods for Emission Tomography

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Introduction

Iterative methods for nonlinearly unconstrained and constrained optimization problems are applied to image reconstruction for emission tomography. Non-negativity of pixel values are considered as the constraints, as in most cases. The iterative methods are evaluated from the following three points: closeness of the original image and the reconstruction, statistical noise characteristics and convergence properties. The noise in the reconstruction originated from statistical fluctuations in the projection data detected, and it grows with increasing iteration. Approaches to this problem are tried in the present work.

Method

Let $\vec{x} = (x_1, x_2, \ldots, x_n)^T$ be the column vector representation of images where T is the transpose symbol, and $f(\vec{x})$ be the least-squares objective function with the penalty function $P(\vec{x})$. Then, $f(\vec{x})$ can be expressed as follows:

$$f(\vec{x}) = \sum_{i=1}^{m} \frac{1}{\sigma_i^2}(y_i - z_i)^2 + \zeta P(\vec{x}) = \sum_{i=1}^{m} \frac{1}{\sigma_i^2}(y_i - \sum_{j=1}^{n} a_{ij}x_j)^2 + \zeta P(\vec{x}) \qquad (1)$$

where y_i $(i = 1, 2, \ldots, m)$ is the detected projection with an uncertainty of σ_i^2, z_i is the reprojection calculated as $z_i = \sum_{j=1}^{n} a_{ij}x_j$ from a_{ij} whose values are predetermined, and ζ is a positive constant. It is required to find an optimal solution, $\vec{x} = \vec{x}^*$, that minimizes $f(\vec{x})$. The method of searching for this solution depends on whether or not the variables $\vec{x}$ are constrained. It will be explained below for the unconstrained case.

A sequence $\vec{x}^0, \vec{x}^1, \ldots$ is generated in such a manner that it should satisfy the relation $f(\vec{x}^{k+1}) < f(\vec{x}^k)$. From this relation, we can obtain that the sequence converges to $\vec{x}^*$. The $\vec{x}^k (k = 0, 1, \ldots)$ is created using unconstrained optimization methods as follows:

$$\vec{x}^{k+1} = \vec{x}^k + \alpha^k \vec{d}^k \tag{2}$$

where $\vec{d}^k = (d_1^k, d_2^k, \ldots, d_n^k)^T$ is the vector of search direction and α^k is a positive scalar called the step size. Here, $\vec{d}^k$ is calculated by the conjugate gradient(CG) method, an unconstrained optimization method, and α^k is obtained using an iterative step-size algorithm.

For the constrained case, the constraints $x_j \geq 0, j = 1, 2, \ldots, n$ are considered, and the augmented Lagrangian function(ALF) method is used for this optimization. In this case, an augmented Lagrangian function is minimized instead of the original function $f(\vec{x})$.

Results and Discussion

SPECT images for numerical phantoms and patients were obtained by the CG and ALF methods for the objective function $f(\vec{x})$ which took $\sigma_i^2 = 1$ and $\zeta = 0$ in Eq.(1)[1,2]. SPECT images reconstructed with the two methods showed a close resemblance to the original images. In addition, the objective function $f(\vec{x})$ decreased with increasing iteration and almost reached a convergence beyond a certain number of iterations, irrespective of initial values such as $\vec{x}^0$. Thus, the two methods showed excellent convergence properties.

Since statistical fluctuations in the reconstruction $\vec{x}^{k+1}$ were due to those in the search direction $\vec{d}^k$ in Eq.(2), a weighted smoothing was performed for $\vec{d}^k$, and it resulted in a marked suppression of statistical noise in the reconstructed image. The ALF method showed a greater improvement in noise characteristics than the CG method, since the non-negativity constraint in the ALF method made a distributed interval of pixel values narrower and it yielded smaller fluctuations in the pixel value.

It is expected that an introduction of penalty functions largely reduces statistical noise in the reconstruction. A further study on this is in progress.

Summary

Nonlinear optimization algorithms, the CG and ALF algorithms, were used for SPECT image reconstruction. They produced reconstructions sufficiently close to the original images, and showed excellent convergence properties. A major problem to be solved is to suppress a rise in statistical noise in the reconstruction with increasing iteration, and the attempts are made.

References

1) Suzuki S, Wakabayashi M, Okuyama K *et al*: Photon Attenuation Correction Technique in SPECT Based on Nonlinear Optimization. Med Imag Tech 16 : 237-251, 1998

2) Suzuki S, Wakabayashi M, Kuwamura S *et al*: Application of Constrained Optimization using Augmented Lagrange Functions to Photon Attenuation Correction in SPECT. Med Imag Tech 17 : 245-260, 1999