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Modeling of the electron kinetics in proportional counters

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INTRODUCTION

Spherical or cylindrical proportional counters are widely utilized to measure various radiation quantities, such as radiation energy, radiation quality, and dose equivalent using tissue equivalent (TE) gases [1,2]. To design the appropriate counters for each application, it is essential to know the gas gain for gas filled proportional counters as a function of gas composition, pressure, voltage applied and counter geometry. The aim of this article is to show a self-consistent methodology using the Boltzmann equation, which enables us to describe accurately the electron multiplication processes in cylindrical or spherical field geometries and to gain a physical perspective in simulating the dynamic properties of the counters.

FORMULATION USING THE BOLTZMANN EQUATION

To describe the behavior of electrons in non-uniform electric fields in proportional counters, we start with the general form of the Boltzmann equation which gives the spatio-temporal variation of the electron distribution function f(**r**,**v**,t),

$$\left\{\frac{\partial}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}+\frac{e\mathbf{E}}{m}\cdot\frac{\partial}{\partial \mathbf{v}}+J\right\}f(\mathbf{r},\mathbf{v},t)=0\,, \qquad (1)$$

where, **r** is the position of electron, **v** is the velocity, e and m are the charge and mass of electron, and **E** is the electric field. J represents the collision operator that includes the electron molecule collision processes [3]. Performing coordinate transformation into the cylindrical and spherical configurations, the second term in Eq.(1) can be written as,

$$\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}=v_r\frac{\partial f}{\partial r}+v_\varphi\frac{1}{r}\frac{\partial f}{\partial \varphi}+v_z\frac{\partial f}{\partial z} \qquad (2\text{-a})$$

and

$$\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}}=v_r\frac{\partial f}{\partial r}+v_\theta\frac{1}{r}\frac{\partial f}{\partial \theta}+v_\varphi\frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi} \qquad (2\text{-b})$$

for the cylindrical and spherical cases, respectively. Here, (v_r,v_φ,v_z) and (v_r,v_θ,v_φ) are the elements of the velocity vector **v** in their coordinates. Thus, integrating Eq.(1) over all spatial variables except r, we can derive a partial differential equation for f(r,v,t) in one-dimension in space. Letting the electric field **E** depend only on position r (i.e., $E=(E_r,0,0)=E(r)$) and using that the spatial segment d**r** is given by $d\mathbf{r}=dxdydz=rd\varphi\, dzdr=r^2\sin\theta\, d\theta\, d\varphi\, dr$, the general form of the one-dimensional equation arising from Eq.(1) is represented by

$$\left\{\frac{\partial}{\partial t}+v_r\frac{\partial}{\partial r}+\frac{eE(r)}{m}\frac{\partial}{\partial v_r}+J\right\}F(r,\mathbf{v},t)=0\,, \qquad (3)$$

where $F(r,v,t) = r^s f(r,v,t)$ (s=1 for cylindrical, s=2 for spherical). The geometry is shown schematically in Fig.1. It should be noted that the case with s=0 corresponds to the parallel geometry and the formalism for solving F(r,v,t) ($= r^s f(r,v,t)$) is equivalent independent of the field geometry.

ARRIVAL-TIME SPECTRA METHOD

Next, we will relate the gas gain of the counters to the solution of Eq.(3). Usually, the gas gain G of the proportional counters is expressed as [4],

$$\ln G = \int_A^B \alpha(r)dr, \tag{4}$$

where α is the ionization coefficient, and A and B are boundary of a line between which describes the gas amplification region which follows the electric field lines. Typically, the parameter α(r) in Eq.(4) is calculated empirically or estimated using simple theory that may be inaccurate for complex gases. To represent the space-time evolution of the electrons in an amplification region under the uniform electric fields, Kondo and Tagashira [5] proposed a general theory of electron swarms including a method for describing the arrival-time spectra (ATS) of electrons. By "swarms" we mean the stage of an electron avalanche in either space or time that is characterized by a charge density that is low enough not to perturb the background electric field structure. In the present study, the ATS method is applied to the non-uniform field conditions. In this method, an equation describing the evolution of the swarm is introduced. The evolution equation is given by

$$\begin{aligned} \frac{\partial}{\partial r}N(r,t) &= \sum_j \alpha^{(j)}(r)\left(-\frac{\partial}{\partial t}\right)^j N(r,t) \\ &= \alpha^{(0)}(r)N(r,t) - \alpha^{(1)}(r)\frac{\partial N(r,t)}{\partial t} + \alpha^{(2)}(r)\frac{\partial^2 N(r,t)}{\partial t^2} - + \ldots \end{aligned} \tag{5}$$

Here, $N(r,t) = \int F(r,v,t)dv = \int r^s f(r,v,t)dv$, and the coefficients $\alpha^{(j)}(r)$ (j=0,1,2...) are given by taking successive moments with $T(r) \equiv t - \langle t \rangle_{(r)}$ on the arrival-time spectra of electrons, N(r,t), as follows:

$$\alpha^{(0)}(r) = \frac{d}{dr}(\ln N_a(r)) \tag{6-a}$$

$$\alpha^{(1)}(r) = \frac{d}{dr}\langle t \rangle_{(r)} \tag{6-b}$$

$$\alpha^{(2)}(r) = \frac{1}{2!}\frac{d}{dr}\langle T^2 \rangle_{(r)}, \tag{6-c}$$

where $N_a(r) \equiv N(r) = \int N(r,t)dt$.

It should be noted that Eq.(5) is a kind of continuity equation of electrons describing arrival-time distributions at fixed position r, which is deduced by integration of eq.(3) over **v** after the division by v_r. The solution of the distribution function F(r,v,t) in eq.(3) can be represented as:

$$F(r,v,t) = \sum_K g^{(K)}(r,v) \cdot \left(-\frac{\partial}{\partial t}\right)^K N(r,t). \tag{7}$$

Substituting eqs.(5) and (7) into Eq.(3) gives the following series of equations for $g^{(K)}(v)$

(K=0,1,2...) associated with the $(\partial/\partial t)^K$,

$$\left(v_r\frac{\partial}{\partial r}+\frac{eE(r)}{m}\frac{\partial}{\partial v_r}+J\right)g^{(0)}(r,v)=-\alpha^{(0)}(r)v_r g^{(0)}(r,v)$$

$$\left(v_r\frac{\partial}{\partial r}+\frac{eE(r)}{m}\frac{\partial}{\partial v_r}+J\right)g^{(K)}(r,v) \qquad (8)$$

$$=g^{(K-1)}(r,v)-\sum_{j=0}^{K}\alpha^{(j)}(r)v_r g^{(K-j)}(r,v), \qquad (K=1,2...).$$

DISCUSSION

As is shown in Eq.(6-a), $\alpha^{(0)}(r)$ (the lowest order parameter in Eq.(5)) exactly corresponds to the ionization coefficient in Eq.(4), while $N_a(r)$ represents the total number of electrons passing through a position r. If r is chosen to the anode radius, $N_a(r)$ ends up to be the gas multiplication factor in the proportional counters. Therefore, one can obtain the multiplication factor (or gain) by counting the arriving electrons at the anode surface. Positive ions remaining in tracks of the electrons in the multiplication volume is also N_a in total number, and the flux of these ions contributes to the slow signal of the counters.

Conventionally, the ionization coefficient in Eq.(4) has been estimated from the data under the uniform electric field and the steady-state current conditions. The equations derived above illuminate a direct and natural way to analyze the electron multiplication process in the cylindrical or spherical counters. Although it may be difficult to solve Eq.(8) actually as a function of position r, this equation can be a general formula to lead to an accurate procedure for calculating gas gain in the counters.

CONCLUSION

In this study, we have obtained the general expression of the Boltzmann equation for electrons under the non-uniform field geometry in the proportional counters, and have shown the formalism that gives the exact gas gain of the counters in a direct and self-consistent way taking advantage of the arrival-time spectra (ATS) method.

References

[1] P. Colautti, V. Conte, G. Talpo, G. Tornielli and M. Bouchefer, Nucl. Instr. and Meth. A321 (1992) 392.

[2] P. Kliauga, A. J. Waker and J. Barthe, Radiat. Protect. Dos. 61 (1995) 309.

[3] H. Date, K. Kondo and H. Tagashira, J. Phys. D: Appl. Phys. 23 (1990) 1384.

[4] e.g., T. Aoyama, Nucl. Instr. and Meth. A234 (1985) 125.

[5] K. Kondo and H. Tagashira, J. Phys. D: Appl. Phys. 23 (1990) 1175.

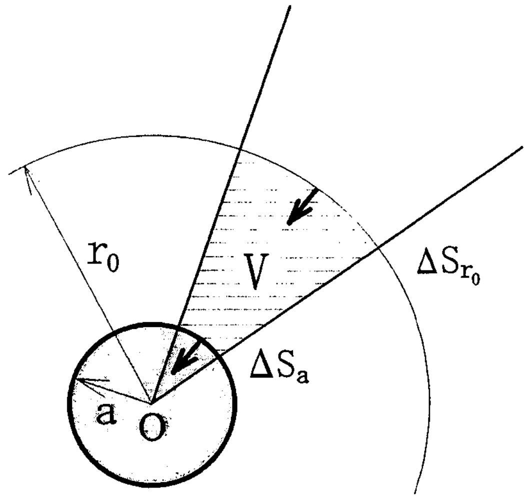

Fig.1. Schematic geometry of the proportinal counter. Here, a is the anode radius, and V represents the gas amplification region. ΔS_a and ΔS_{r0} denote segments of surface area for r=a and a=r_0, respectively. $\Delta S_a : \Delta S_{r0} = a^s : r_0^s$ (s=1 for cylindrical, s=2 for spherical).