Verifying Little’s Law on Discrete Event Simulation

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Abstract

MQL (Mean Queue Length) is one of the key factors to evaluate a system. MQL can be easily calculated by a well-known formula, Little’s Law. Little’s Law is a fundamental theorem of the queuing theory. We will give an easy example for a given statistics collection period T. Let \( L(t) \) be the number of parts present in a buffer. The Mean Queue Length is defined by

\[
L = \frac{1}{T} \cdot \int_{0}^{T} L(t) \, dt
\]

If \( t_i \) denotes the time of the \( j \)th part spent in the buffer, then it is easily seen that

\[
L = \frac{1}{T} \cdot \sum_{i=1}^{N} t_i,
\]

where \( N \) is the number of entrances of parts into the buffer under consideration. In other words, this sum is taken for all movable elements. Note that \( t_i \) is not equal to the processing time of a part, since there is possibly a waiting time of the part. The following figure shows the validity of the above formula in case of 2 movable elements:

![Diagram showing the number of parts in the buffer over time](image)
If the processing time is equal to the time $t_j$ spent the part on the buffer, then Mean Queue Length can be determined by the utilization, which is defined by

$$utilization = \frac{\sum t_j}{capacity \times T}$$

Therefore, $L = buffer\cdot capacity \times buffer\cdot utilization$. This equation holds only if the parts are never blocked (waiting) on the buffer. In the model Little's Law this is ensured by vanishing of the processing time of the drain.

We had built a model to analyze EMSs (Electric Monorail Systems) in a car-body shop of “D” car manufacturer using SIMPLE++. The arrival rate $\lambda = \frac{N}{T}$ can easily modelled by the interarrival time (attribute interval) of the basic building block “source”. The mean time spent a part in the buffer is

$$W = \frac{1}{N} \cdot \sum_{j=1}^{N} t_j$$

In case of finite many parts (customers) Little’s Law $L = \lambda \cdot W$ can easily verify.

The model MeanQueueLength shows how the values $L$ can be determined in the model. Waiting parts on/before the buffer are allowed.