An Mingling Algorithm for Intersecting Planar Parametric Curves Using Curve Charaterization and Approximation

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Abstract

Intersection problem occurs in various engineering application areas, such as CAD/CAM, GIS, computer graphics, etc. Most of all, intersection algorithms are fundamental to CAD/CAM.

Parametric curves have been frequently used in CAGD and thus intersection algorithm between parametric curves been studied intensively in several respects such as the speed, the robustness and the efficiency. Although many intersection algorithms have been published, there exists no algorithm that is satisfactory in all the above three aspects.

The intersection techniques that appear in the literature can be classified into three categories; Newton-Raphson iteration method, subdivision method and implicitization method. Newton-Raphson iteration-wise method shows a good convergence rate in case that a good initial seed is given. Otherwise, it provides a wrong solution or diverges. Bezier clipping algorithm copes with intersection problem like an intelligent Newton method. Though it is faster than Implicitization algorithm and Interval subdivision for curve of degree more than and equal to 5, it is slower than Implicitization algorithm for curve of degree less than 5[]. Intersection algorithm based on subdivision method divides the original intersection problem into the easier ones and then conquers the each divided problem. Bézier subdivision and interval subdivision algorithm is included in this category. Implicitization method transforms intersection problem to the problem of finding a single polynomial root by substituting a parametric form curve into the implicitized curve. This approach is known to be fastest in computing the intersections between curves of degrees less than quintics.

In this paper, an algorithm for intersecting Bezier curves is provided and is extended to an algorithm for intersections between NURBS curves. The algorithm characterizes both curves to be intersected and approximates them in lower degree curves. Implicitization technique is applied to the intersections between approximated low degree curves for locating initial solution. Then a good initial solution is obtained and Newton-Raphson iteration converge to a true intersection quickly and robustly. Tangential case and overlapping case are not considered in the pro+posed algorithm

1. Introduction

The intersection techniques that appear in the literature can be classified into three categories; Newton-Raphson iteration method, subdivision method and implicitization method. Newton-Raphson iteration-wise method shows a good convergence rate in case that a good initial seed is given[1]. Otherwise, it provides a wrong solution or diverges. Bezier clipping algorithm copes with intersection problem like an intelligent Newton method. Though it is faster than Implicitization algorithm and Interval subdivision for curve of degree more than and equal to 5, it is slower than Implicitization algorithm for curve of degree less than 5[2]. Intersection algorithm based on subdivision method divides the original intersection problem into the easier ones and then conquers the each divided problem. Bézier subdivision and interval subdivision algorithm is included in this category. Implicitization method transforms intersection problem to the problem of finding a single polynomial root by substituting a parametric form curve into the implicitized curve. This approach is known to be fastest in computing the intersections between curves of degrees less than quintics[3].

Sederberg and Parry compare three planar curve/curve intersection algorithms, that is Bezier subdivision algorithm, Interval subdivision algorithm and Implicitization algorithm[3]. From their conclusion, Implicitization algorithm excels the other two algorithms in speed for curves of degree less than 5. But Interval subdivision is faster than Implicitization for curve of degree 5.

In this paper, an algorithm for intersecting Bezier curves is provided and is extended to an algorithm for intersections between NURBS curves. The algorithm characterizes both curves to be intersected and approximates them in lower degree curves. Implicitization technique is applied to the intersections between approximated low degree curves for locating initial solution. Then a good initial solution is obtained and Newton-Raphson iteration converge to a true intersection quickly and robustly. Tangential case and overlapping case are not considered in the pro+posed algorithm.

2. Outline of the proposed algorithm(Bezier curve case)

The core part of the proposed algorithm is to locate the good initial seed for Newton-Raphson iteration. Newton-Raphson iteration-wise method shows a good convergence rate in case that a good initial seed is given. Otherwise, it provides a wrong solution or diverges.

In first step, the algorithm characterizes both the curves to be intersected, that is to find the characteristic points; the cusp, inflection point and point with conjugate tangent vectors. The reason why the algorithm characterizes the curve is that we devide each curve into simple curve segments and subdivide. The definition of simple curve is given section 3. Then each simple curve segment is approximated by rational quadratic Bezier curve.

In second step, the initial seed for Newton-Raphson iteration is obtained by intersecting both rational quadratic Bezier curve using implicitization method.

In third step, Newton-Raphson iteration is performed.

3. Curve characterization, approximation and Implicitization

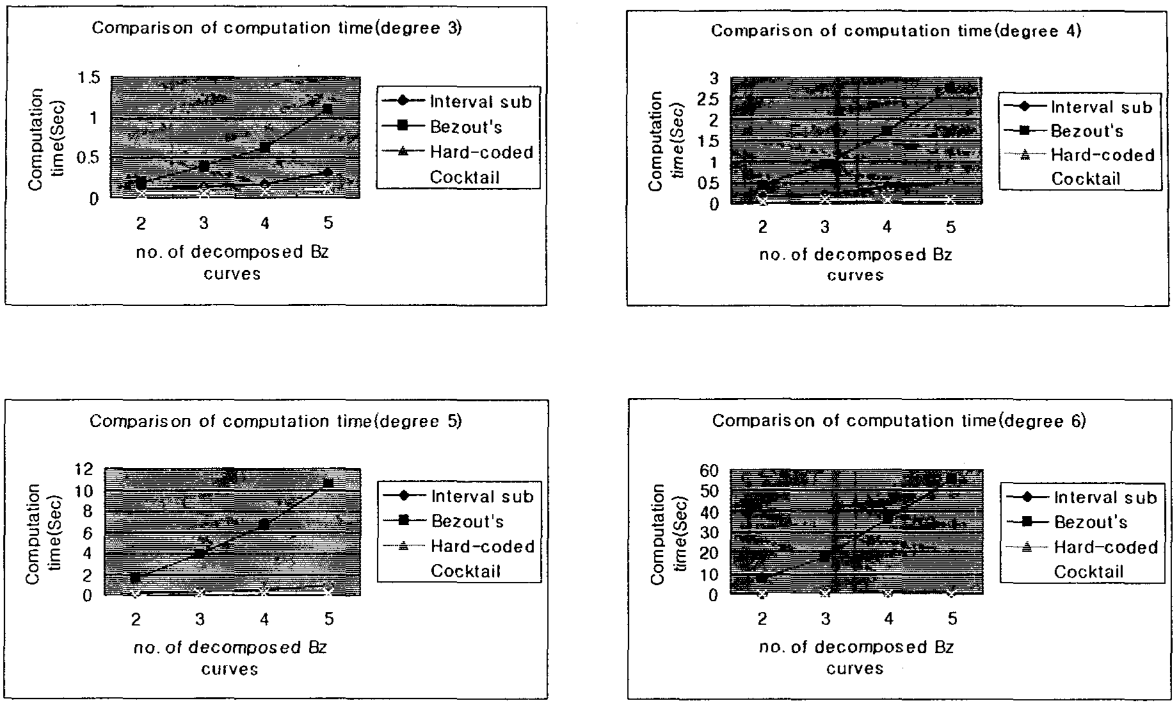
In the first step of the algorithm, each curve is analysed and divided into simple curve segments by finding the characteristic points. If a curve has no cusps and no inflection points and the angle of tangent cone of the curve does not exceed π, the curve is defined as simple curve[4]. Now, the each simple curve is approximated by rational quadratic Bezier curve. Then the interections between the approximated rational quadratic Bezier curves is located by implicitization method.

4. Extension to NURBS curve

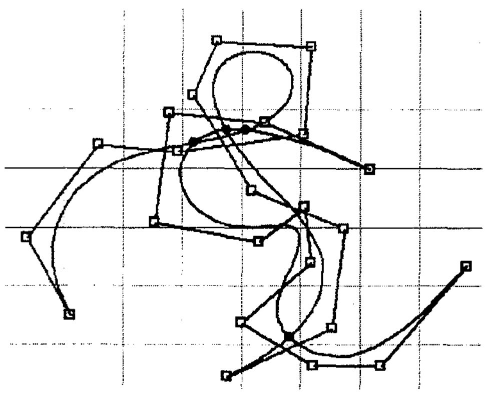
To apply the proposed algorithm into the intersections between NURBS curves, the several preprocessing operations alone is sufficient in the initial step of algorithm. To characterize and approximate the NURBS curve, the information for NURBS curve is necessarily obtained before the proposed algorithm is applied into the intersections between NURBS curves. Once this preprocessing operation is performed, the algorithm is applicable into the intersections problem between NURBS curves the same as Bezier curve case.

5. Assessment and conclusion

To appreciate the efficiency of proposed algorithm, comparison with other algorithms is performed. Interval subdivision, Implicitization method using hard-coded resultant and Implicitization method using Bezouts resultant are used to be compared with the proposed algorithm. According to the increase in the number of decomposed rational Bézier curves in a given degree, the computation time of each algorithm is compared. As we can verify the comparative performance from the result of the figures 1~4, the proposed algorithm prevails on other algorithms in all(degree 3, 4, 5 and 6)degree. In addition, the computation time of the proposed algorithm is relatively constant regardless of the number of decomposed rational Bézier curves. Because of run-time inefficiency, implicitization method using Bezouts resultant is inferior to any other algorithm.



[Fig. 1~4] Comparison of computation time in each degree



[Fig. 5] The intersections between NURBS curves(5 degree)

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