Generalized Convex Combination Strategies for Cross Decomposition Method*

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Abstract

In this research, we propose a new convex combination weight rule for the cross decomposition method which is known to be the most reliable and promising strategy for the large scale optimization problems.

Decomposition methods are multilevel techniques for solving large scale optimization problem and exploit special structure of the problem. Early decomposition methods are Dantzig-Wolfe decomposition and Benders' decomposition, which exploit dual and primal structure of the problem respectively.

Dantzig-Wolfe decomposition algorithm solves successively a dual subproblem and a master problem until the optimum is achieved and verified. Dual subproblem is obtained by taking the Lagrangian relaxation of the original problem relative to some constraints. Each iteration consists of (1) selecting new Lagrangian multipliers by the dual master problem, (2) solving the dual subproblem for the given values of the multipliers.

Dual subproblem denotes if solution of the overall problem can be improved. And master problem improve the solution of the overall problem actually. Dantzig-Wolfe decomposition can find lower bound for the optimal solution, and by using this, we can find approximate solution.

Benders' decomposition solves successively a primal subproblem and a master problem. Primal subproblem is a restriction of the original problem in which some of the primal variables have fixed values. At every iteration, these primal variables are adjusted by the master problem. Master problem is a relaxed problem, and violating constraints are generated by subproblem, and then this constraint is included in master problem and reoptimize it.

It is well known that Benders' decomposition and Dantzig-Wolfe decomposition are dual pairs when they are applied to the pure LP problems.

Cross decomposition method combines the advantages of Benders' decomposition and

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Dantzig-Wolfe decomposition. Its main idea is exploiting the close relationship between primal subproblem and dual master problem and between dual subproblem and primal master problem. This relationship is exploited in such a way that only the subproblems are used as long as they produce a converging sequence of primal and dual solution. But, convergence cannot be guaranteed by the use of subproblems only. Therefore a primal or dual master problem, with all dual or primal solutions generated so far, has to be solved from time to time as the algorithm proceeds. This prevents any economy with respect to the computer memory capacity.

This procedure consists mainly of a subproblem phase, where one iterates between a primal subproblem, where some primal variables are kept fixed, and a dual subproblem, where some constraints are subject to Lagrangian relaxation, and their dual variables are kept fixed. The dual solution of the primal subproblem gives input to the dual subproblem, and the primal solution of the dual subproblem gives input to the primal subproblem. Two subproblems gives upper and lower bound respectively.

The subproblem phase does not yield convergence by itself, so one uses convergence tests that indicates when convergence can no longer be expected, and this happens, one solves a primal or dual master problem, and then returns to the subproblem phase.

The need to solve master problems is perhaps the most important disadvantage of the cross decomposition method, since the master problems usually are harder to solve than the subproblems, and though original problem has some structure, master problem is completely unstructured, so master problem can sometimes become more difficult to solve than the original problem.

So one expect to stay in the subproblem phase as long as possible, and Mean Value Cross Decomposition, inspired by the Kornai-Liptak method, eliminates completely the need for using master problem.

The subproblem phase in cross decomposition can offer very fast convergence temporarily, but it can be may stop converging at all, in which case we have to use a master problem. On the other hand, Kornai-Liptak method only has asymptotic convergence, which often tends to be quite slow towards the end of the procedure. However, no master problems are used in that procedure.

In this paper, we discuss the convergence proof for generalized convex combination weights in the cross decomposition method. Mean Value Cross Decomposition method which is introduced by Holmberg uses the average of all previously obtained solutions of one subproblem as an input to the other subproblem.

By adopting the idea of Sherali and Choi(1996) for the primal recovering strategies, we are proposing a new convex combination weight rule for primal-dual convergence in cross decomposition. With this strategy, we are having more room for selecting convex combination weights depending on the problem structure, and then, ones may choose a rule for either past convergence for getting quick bounds or more accurate solution. Also, we can improve the slow end-tail behavior by using some combined rules.