Optimality Analysis for Large Scale LP Problems using Metamodelling with Application to Air Force Transportation

Chan Seok Jeong* · Young Hae Lee*

* Department of Industrial Engineering, Hanyang University

Abstract

Linear programming (LP) is a tool for solving optimization problems. Since the development of the simplex algorithm for solving LP, and later the development of interior point methods, LP has been used to solve many optimization problems in industries and government. As the size of LP problem increases, the difficulty of optimality analysis, as described below, increases. Although, today we are able to solve the large size LP problem with millions of variables and hundred thousands of constraints by using the developed computer packages, we still need efficient optimality analysis methods.

The light-hand-side (RHS) vectors of an LP problem may be changed for a variety of reasons: We may want to conduct a sensitivity analysis to check the preciseness of the RHS vectors, and whether or not it matters if the RHS vectors are perturbed. Also, we may want to update the LP problem when additional (reduced) resources become available (unavailable). In this case, optimality analysis comes in to play. Optimality analysis is performed to determine the effect on the optimal solution when the right-hand-side vector (or the objective function coefficient) is changed. We also refine the geographical areas and use the Hot Start technique for making the large-scale LP runs required by the experimental design. The Hot Start method is very helpful for this research because we could save time and effort on the computer experiments. We use the converter programs to support the Hot Start method and create the files for Hot Start by the programs successive use. The Hot Start method gives us high efficiency for reducing running time of large scale LP model.

Optimality analysis generally involves multiple critical regions with different optimal bases. This

differs from "post-optimality analysis" and "sensitivity analysis, since those analyses deal with only one critical region. Optimality analysis of large scale LP problem across multiple critical regions is more difficult than situations dealing with only one critical region. Thus, identifying which critical region contains a particular right-hand-side vector creates a burden for the analyst.

Metamodels using response surface methodology (RSM) are used for the optimality analysis of LP problem. They have the form of a simple polynomial, and predict the optimal objective function value of an LP problem for various levels of the constraints. The metamodelling techniques for optimality analysis of LP can be applied to large-scale LP models. What is needed is some large-scale application of the techniques to verify how accurate they are. In this paper, we use the strategic transport optimal routing model (STORM) for solving a kind of the large scale air force transportation LP model. The developed metamodels of the large scale LP can provide some useful information.

We intend to create metamodels with only first order polynomials unless the higher order polynomials (such as second order) are needed. We basically apply 2^k-p factional and 2^k full fractional designs to create metamodels by least square regression. The problem is to verify whether or not this technique is valid for large scale LP problem. For the evaluation of metamodels, two primary measures are used : mean squared error (MSE) and mean absolute percentage error (MAPE). Since metamodels are really time and effort savers, the analyst will be able to observe the response of the optimal objective function value of a particular LP very easily and efficiently when the levels of the constraints of this LP are changed.

By using this approach, the purpose is to analyze the accuracy of the metamodels and to describe the key relationships among the optimal objective function value, the objective function coefficient vectors and the right-hand-side vectors of interest. It is possible to predict the response of the optimal objective function value easily and efficiently when the levels of the factors are changed.