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무한요소를 사용한 지반-구조물 상호작용계의 시간영역 지진응답해석

Time Domain Soil-Structure Interaction Analysis for Earthquake Loadings Based on Analytical Frequency-Dependent Infinite Elements

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ABSTRACT

This paper presents a time domain method for soil-structure interaction analysis for seismic loadings. It is based on the finite element formulation incorporating analytical frequency-dependent infinite elements for the far-field soil. The dynamic stiffness matrices of the far-field region formulated in frequency domain using the present method can be easily transformed into the corresponding matrices in time domain. Hence, the response can be analytically computed in time domain. Example analysis has been carried out to verify the present method for an embedded block in a multi-layered half-space. The present method can be easily extended to the nonlinear analysis, since the response analysis is carried out in time domain.

INTRODUCTION

Most of the well-known computer software packages for soil-structure interaction analysis are based on the frequency domain analysis, and utilize the equivalent linearization technique to consider the material nonlinearity of the soil medium. However, in recent years, several time domain methods have been proposed to study nonlinear behaviors of the soil medium, effects of pore water, and nonlinear conditions along the interface between soil and structure. One method is the coupling of the boundary and the finite element methods(Karabalis & Beskos 1985). In this method the structure and the near-field soil region are modeled using finite elements, while the far-field soil region is represented using boundary elements. However, it is generally difficult to derive fundamental solutions in layered soils and to couple the boundary elements with the finite elements. Another method is the one using the transformation of the dynamic stiffness matrix into the terms in time

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domain(Wolf 1988; Hayashi & Katukura 1990). However, the dynamic stiffness matrix for the far-field region is usually obtained numerically at each frequency by conventional methods. Therefore, the transformation has to be carried out numerically using discrete Fourier transform or discrete *z*-transform, which requires tremendous computational time and huge computer-memory for realistic problems.

In this paper, a time domain method for soil-structure interaction analysis is proposed, incorporating the analytical frequency-dependent infinite elements for the far-field soil which are recently developed by the present authors(Yun *et al.* 1999). In this formulation, the dynamic stiffness matrices of the far-field soil region can analytically transformed into terms in time domain, hence the analysis can be easily carried out in time domain.

ANALYTICAL FREQUENCY-DEPENDENT INFINITE ELEMENTS

The soil-structure interaction system in two dimensional space is modeled using the finite elements and the analytical frequency-dependent infinite elements as in Figure 1. The structure and the near-field soil region are modeled using 9-node plane strain finite elements, and the remaining far-field soil region is represented using horizontal, vertical, left corner, and right corner infinite elements; i.e., HIE, VIE, LCIE, and RCIE. The present horizontal and vertical infinite elements have three nodes each on the interface(Γ_e) between the near- and the far-field soil regions, while the corner infinite elements have one node.

The dynamic stiffness matrix of the far-field region formulated by the present infinite elements can be obtained in an analytical form of the exciting frequency and the constant matrices as(Yun *et al.* 1999)

$$\mathbf{S}_{ee}^{F}(\omega) = \mathbf{S}_0^F + i\omega\mathbf{S}_1^F + \frac{1}{a+i\omega}\mathbf{S}_2^F + \frac{1}{(a+i\omega)^2}\mathbf{S}_3^F \tag{1}$$

where ω is the circular frequency; a is a positive constant for the geometric attenuation of the stress waves in the far-field soil region; and $\mathbf{S}_0^F$, $\mathbf{S}_1^F$, $\mathbf{S}_2^F$, and $\mathbf{S}_3^F$ are real-valued constant matrices if the material damping in the soil is ignored.

EARTHQUAKE RESPONSE ANALYSIS IN TIME DOMAIN

At first, the equivalent earthquake force $\mathbf{f}_e^f$ is evaluated along Γ_e from the free-field responses using the rigid exterior boundary method as(Zhao & Valliappan, 1993)

$$\mathbf{f}_e^f(\omega) = \mathbf{S}_{ee}^F(\omega)\mathbf{u}_e^f(\omega) - \mathbf{A}\mathbf{t}_e^f(\omega) \tag{2}$$

where $\mathbf{u}_e^f$ and $\mathbf{t}_e^f$ are the displacement and the traction along Γ_e for the free-field soil medium subjected to the earthquake excitation; and $\mathbf{A}$ is the matrix to transform the traction into the force.

Then, the equation of motion for the soil-structure interaction system subjected to $\mathbf{f}_e^f(\omega)$ can be written in frequency domain as(Wolf 1988)

$$\begin{bmatrix} \mathbf{S}_{nn}(\omega) & \mathbf{S}_{ne}(\omega) \\ \mathbf{S}_{en}(\omega) & \mathbf{S}_{ee}(\omega)+\mathbf{S}_{ee}^F(\omega) \end{bmatrix} \begin{Bmatrix} \mathbf{u}_n(\omega) \\ \mathbf{u}_e(\omega) \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_e^f(\omega) \end{Bmatrix} \tag{3}$$

where subscript n stands for the nodes of the structure and the near-field soil region, while e denotes those on the interface Γ_e. For the computational convenience, equation (3) can be rewritten as

$$\begin{bmatrix} \mathbf{S}_{nn}(\omega) & \mathbf{S}_{ne}(\omega) \\ \mathbf{S}_{en}(\omega) & \mathbf{S}_{ee}(\omega) \end{bmatrix} \begin{Bmatrix} \mathbf{u}_n(\omega) \\ \mathbf{u}_e(\omega) \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_e^f(\omega) + \mathbf{f}_e(\omega) \end{Bmatrix} \tag{4}$$

$$\mathbf{f}_e(\omega) = -\mathbf{S}_{ee}^F(\omega)\mathbf{u}_e(\omega) \tag{5}$$

where $\mathbf{f}_e(\omega)$ may be defined as the interaction force which depends on the response of the interface Γ_e with the far-field soil region.

For time domain analysis, the interaction force $\mathbf{f}_e(\omega)$ can be transformed as

$$\mathbf{f}_e(t) = -\int_0^t \mathbf{S}_{ee}^F(t-\tau)\mathbf{u}_e(\tau)d\tau \tag{6}$$

where $\mathbf{S}_{ee}^F(t)$ is the inverse Fourier transform of $\mathbf{S}_{ee}^F(\omega)$, which can be obtained from equation (1) in an analytical form as

$$\mathbf{S}_{ee}^F(t) = \mathbf{S}_0^F\delta(t) + \mathbf{S}_1^F\dot{\delta}(t) + \mathbf{S}_2^F e^{-at}H(t) + \mathbf{S}_3^F te^{-at}H(t) \tag{7}$$

where $H(t)$ is unit step function; and $\delta(t)$ and $\dot{\delta}(t)$ are Dirac-delta function and its derivative respectively. From equations (6) and (7), the interaction force can be also obtained analytically as

$$\mathbf{f}_e(t) = -\mathbf{S}_0^F\mathbf{u}_e(t) - \mathbf{S}_1^F\dot{\mathbf{u}}_e(t) - \int_0^t \{\mathbf{S}_2^F + (t-\tau)\mathbf{S}_3^F\}e^{-a(t-\tau)}\mathbf{u}_e(\tau)d\tau \tag{8}$$

Finally, the time domain equation of motion for the soil-structure interaction system can be derived from equations (4) and (8) as

$$\begin{bmatrix} \mathbf{M}_{nn} & \mathbf{M}_{ne} \\ \mathbf{M}_{en} & \mathbf{M}_{ee} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_n(t) \\ \ddot{\mathbf{u}}_e(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{S}_1^F \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{u}}_n(t) \\ \dot{\mathbf{u}}_e(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{nn} & \mathbf{K}_{ne} \\ \mathbf{K}_{en} & \mathbf{K}_{ee} + \mathbf{S}_0^F \end{bmatrix} \begin{Bmatrix} \mathbf{u}_n(t) \\ \mathbf{u}_e(t) \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{f}_e^f(t) + \bar{\mathbf{f}}_e(t) \end{Bmatrix} \tag{9}$$

where $\mathbf{M}_{nn}$, $\mathbf{M}_{ne}$, $\mathbf{M}_{en}$, $\mathbf{M}_{ee}$, $\mathbf{K}_{nn}$, $\mathbf{K}_{ne}$, $\mathbf{K}_{en}$, and $\mathbf{K}_{ee}$ are the conventional mass and stiffness matrices for the structure and the near-field soil; and $\bar{\mathbf{f}}_e(t)$ is the third term of the interaction force $\mathbf{f}_e(t)$ in equation (8) as

$$\bar{\mathbf{f}}_e(t) = -\int_0^t \{\mathbf{S}_2^F + (t-\tau)\mathbf{S}_3^F\}e^{-a(t-\tau)}\mathbf{u}_e(\tau)d\tau \tag{10}$$

The new interaction force $\bar{\mathbf{f}}_e(t)$ in equation (10) can be decomposed as

$$\bar{\mathbf{f}}_e(t) = \bar{\mathbf{f}}_{e1}(t) + \bar{\mathbf{f}}_{e2}(t) + \Delta\bar{\mathbf{f}}_e(t) \tag{11}$$

where

$$\bar{\mathbf{f}}_{e1}(t) = -\int_0^{t-\Delta t} \mathbf{S}_2^F e^{-a(t-\tau)}\mathbf{u}_e(\tau)d\tau \tag{12a}$$

$$\bar{\mathbf{f}}_{e2}(t) = -\int_0^{t-\Delta t}(t-\tau)\mathbf{S}_3^F e^{-a(t-\tau)}\mathbf{u}_e(\tau)d\tau \tag{12b}$$

and
$$\Delta\bar{\mathbf{f}}_e(t) = -\int_{t-\Delta t}^{t}\{\mathbf{S}_2^F + (t-\tau)\mathbf{S}_3^F\}e^{-a(t-\tau)}\mathbf{u}_e(\tau)d\tau \tag{12c}$$

Then, the equation (9) incorporating the above equations can be approximately rewritten into discrete time forms at $t = n\Delta t$ as(Kim 1999)

$$\begin{bmatrix}\mathbf{M}_{nn} & \mathbf{M}_{ne}\\ \mathbf{M}_{ns} & \mathbf{M}_{ee}\end{bmatrix}\begin{Bmatrix}\ddot{\mathbf{u}}_n(t)\\ \ddot{\mathbf{u}}_e(t)\end{Bmatrix}+\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{S}_1^F\end{bmatrix}\begin{Bmatrix}\dot{\mathbf{u}}_n(t)\\ \dot{\mathbf{u}}_e(t)\end{Bmatrix}+\begin{bmatrix}\mathbf{K}_{nn} & \mathbf{K}_{ne}\\ \mathbf{K}_{en} & \mathbf{K}_{ee}+\mathbf{S}_0^F+\alpha_2\mathbf{S}_2^F+\alpha_3\mathbf{S}_3^F\end{bmatrix}\begin{Bmatrix}\mathbf{u}_n(t)\\ \mathbf{u}_e(t)\end{Bmatrix}$$
$$=\begin{Bmatrix}\mathbf{0}\\ \mathbf{f}_e^f(t)+\bar{\mathbf{f}}_{e1}(t)+\bar{\mathbf{f}}_{e2}(t)+(\beta_2\mathbf{S}_2^F+\beta_3\mathbf{S}_3^F)\mathbf{u}_e(t-\Delta t)\end{Bmatrix} \tag{13}$$

where α_2, α_3, β_2 and β_3 are real constants, which depend on a and Δt. It is noteworthy that the convolution integrals for $\bar{\mathbf{f}}_{e1}(t)$ and $\bar{\mathbf{f}}_{e2}(t)$ can be evaluated recursively as finite sums of a few past terms of $\bar{\mathbf{f}}_{e1}(t)$, $\bar{\mathbf{f}}_{e2}(t)$, and $\mathbf{u}_e(t)$. Hence the present time domain formulation based on the analytical frequency-dependent infinite elements is very straightforward and computationally very efficient in comparison with the methods using numerical transforms such as discrete Fourier transform or discrete *z*-transform, which usually require huge computational efforts. In this study, Newmark direct integration procedure is used to solve equation (13).

NUMERICAL EXAMPLE AND DISCUSSION

For the verification of the present earthquake response analysis procedure, dynamic analysis was carried out for a layered soil medium with and without an embedded block as in Figure 1. The block and the near-field soil region are discretized with plane strain finite elements and the remaining far-field soil region is modeled by analytical frequency-dependent infinite elements. The properties of the soil layers and the block are shown in Table 1. A horizontal acceleration record was used as the input control motion on the ground surface, which is the NS-component of an earthquake measured at Hualien, Taiwan on January 20, 1994(Ohsaki Research Institute, 1994). The peak ground acceleration is 0.0318*g*, and the time history is shown in Figure 2.

At first, the equivalent earthquake force was computed along the interface(Γ_e) based on the free-field responses obtained using the conventional method(Zhao and Valliappan, 1993). Then, for the case without block, the earthquake responses were calculated at two locations in the soil medium (B1 and C1 in Figure 1) using the present time domain method. The time history of the acceleration history and the response spectra are compared with those of the free-field analysis in Figures 2 and 3, which show excellent agreements. For the case with the block, the earthquake response were calculated at two locations in the block(A1 and B1). Then the computed time histories and response spectra are compared with those obtained solving equation (3) in frequency domain in Figure 4-6, which also show very good comparisons. In the present time domain formulation of equation (13), the mass, damping, and stiffness matrices of the total soil-structure interaction system are real-valued constants, because only the geometric damping of the unbounded soil medium is introduced excluding the material damping which is customarily expressed in complex form. However, the damping matrix may remain real-valued, if the material damping is approximated as equivalent viscous damping.

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REFERENCES

Y. Hayashi and H. Katukura, "Effective time-domain soil-structure interaction analysis based on FFT algorithm with casaulity condition", *Earth. Engrg. Str. Dyn.*, 19, 693-708, 1990.

D.L. Karabalis and D.E. Beskos, "Dynamic response of 3-D flexible foundations by time domain BEM and FEM", *Soil Dyn. Earthquake Eng.*, 4, 91-101, 1985.

D.K. Kim, "Analytical Frequency-Dependent Infinite Elements for Soil-Structure Interaction Analysis in Time Domain", *Ph.D. Dissertation*, Korea Advanced Institute of Science and Technology, Korea, August, 1999.

J.P. Wolf, *Soil-Structure-Interaction Analysis in Time Domain*, Prentice Hall, 1988.

C.B. Yun, D.K. Kim, and J.M. Kim, "Analytical Frequency-Dependent Infinite Elements for Soil-Structure Interaction Analysis in Two-Dimensional Medium", *Engineering Structure, in press*, 1999.

C.B. Yun, J.M. Kim, and H.C. Hyun, "Axisymmetric infinite elements for multi-layered halfspace," *Int. J. Num. Meth. in Eng.*, 38, 3723-3743, 1995.

Ohsaki Research Institute, Blind Prediction Analysis of 1/20/94 Earthquake, *Hualien LSST Meeting*, Palo Alto, CA, December, 1994.

C. Zhao and S. Valliappan, "An efficient wave input procedure for infinite media," *Communications in Numerical Methods in Eng.*, 9, 407-415, 1993.

Table 1 Material properties of soil layers and block

Soil Region	Layer Depth (m)	c_s (m/sec)	ρ (kg/m^3)	Poisson's ratio
Layer 1	2.0	133	1.69	0.38
Layer 2	3.0	231	1.93	0.48
Half Space	∞	333	2.42	0.47
Block	-	1800	2.57	0.17

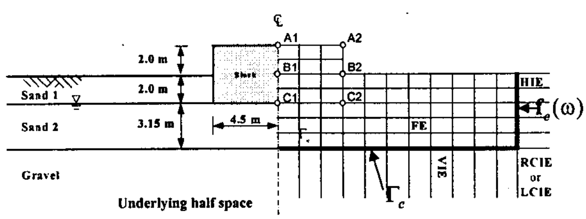

Figure 1 Finite and infinite element mesh for earthquake response analysis using $\mathbf{f}_e^f(\omega)$

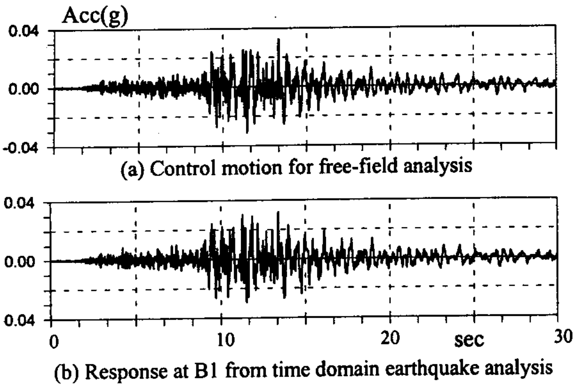

Figure 2 Free-field ground accelerations

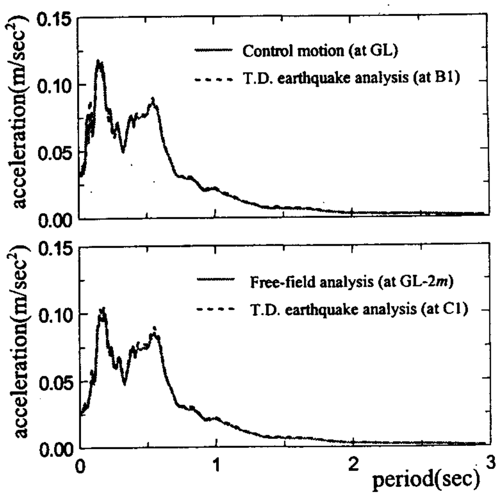

Figure 3 Response spectra for free-field motions

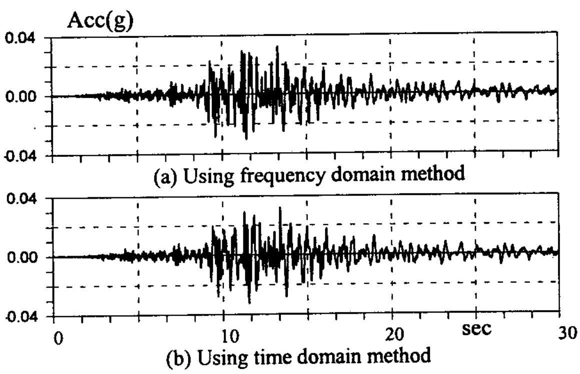

Figure 4 Responses at A1 in a block

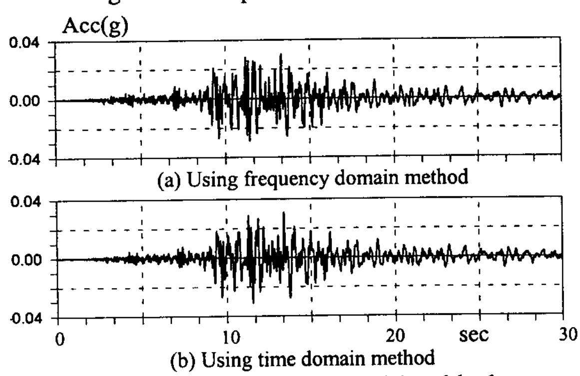

Figure 5 Responses at B1 in a block

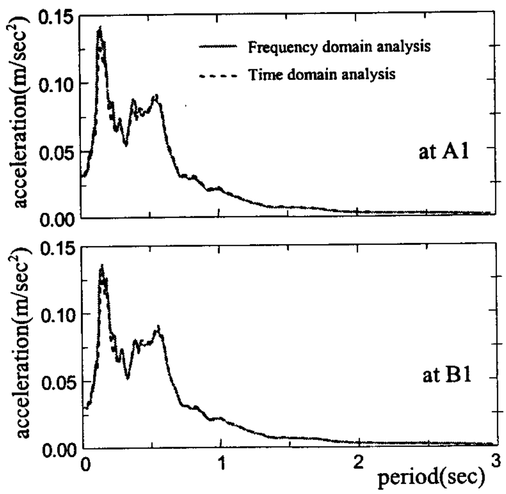

Figure 6 Response spectra for cases with a block