조화진원에 의한 정밀지하구조탐사 1
(표면조화진원에 의한 반무한 탄성체의 파동장 해석)
Application of Harmonic Seismic Sources to Precision Investigation of Underground Geological Structure, Part 1
(Wave Field Generated by Harmonic Source Attached to the Surface of Elastic Half Space)

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요지

조화진동하는 점진원에 의한 반무한 탄성체의 응답해석을 수행해, 탄성체의 임의의 깊이에서의 응답을 나타내는 계산식을 유도했다. 이 결과는 최근 연구개발이 진행되고 있는 조화진원에 의한 정밀 지하구조탐사에 중요한 이론적 바탕이 된다. 먼저 실용상 중요한 수평가진에 의한 응답을 계산했다. 표면에 큰 응답이 표면파의 형태로 발생하지만, 수평진원의 에너지 방사형태는 지향성을 가지고 있어, 진원아래의 원뿔형의 영역이 크게 여기된다는 사실을 밝혔다.

1. INTRODUCTION

Recently, geophysicists have been discussing the possibility of a new method of active elastic-wave remote sensing of the upper crust of the earth[1]. Artificial seismic sources will emanate continuous harmonic wave of small amplitude. Although signal is considered small, latest high performance seismographs with feedback control mechanism is expected not to miss the signal so long as the sources have practically reasonable ability provided by modern but conventional technologies.

This signal itself appears too small to be separable from the noise. But if the frequency of the emitted wave is kept constant for a very long period as several ten days or several months, we can cancel out the noises by stacking the data. The recent development of the GPS system and electro-mechanical control technologies has made such idea realistic, and models are now under field examination. These sources carry an electric motor and

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rotate an eccentric mass. Being adjusted continually by the time signal from GPS satellites, they have achieved constant frequency with error of less than 10^{-5}.

Another basic idea is the array-deployment strategy of the sources; individual source is regarded as an element of an enormous virtual source aperture. These sources are driven with a common identical frequency while the phase of each source is assigned in an appropriate manner which optimizes the prospecting mission. This method enables a variety of different patterns of wave radiation.

This new concept requires a mathematical theory of compliance of elastic space ; it is necessary for the estimation of the needed power of the sources, the determination of the array geometry and the phase assignment among the sources. Takei and Kumazawa investigated the response of the elastic *full* space to various modes of oscillation[2)] and they manufactured sources based on this knowledge.

The analysis of the response of the elastic half space is needed because the source attached to the surface of the ground will ordinarily generate surface waves. It may induce some secondary modes of oscillation. All these phenomena make worse the efficiency of the source and make the emitted wave dirty[3)]. Analyses without accounting the interference at the surface of the ground are, therefore, insufficient for the new methodology. This is the objective of the present paper.

In this paper, analytical formulae are derived for all possible modes of oscillation. Next, the wave field induced by the lateral source attached to the surface of elastic half space. By using this result, we can design the sources and plan the optimal phased array system of the sources.

2. HISTORY OF THE MATHEMATICAL BASIS

The study of point load in elastic solids has a long history. One of the fundamental results in the theory of elasticity is the Kelvin's solution[4)] for a force applied at a point in a solid of indefinite extent. The classical problem of Boussinesq[4)] dealing with a normal force applied at the plane boundary of a semi-infinite solid is solved by superimposing solutions derived from Kelvin's results and Cerruti[4)] investigated a tangential force case. The Mindlin solution[4)] fills in the gap between the two by giving the displacement and stresses for a case where the force is applied near the surface. The solution is obtained similarly by superposition of a combination of nuclei of strain, which are derived by synthesis from Kelvin's solution.

The generation of elastic waves by the application of concentrated loads on the surface or inside of a half space is known as Lamb's problem[5)] since Lamb was first to treat it. Most fully discussed were the surface motions generated by a line load and a point load applied normally to the surface. Both loads of harmonic time dependence and impulse were considered. Also truly ingenious and skillful introduction of a particular solution to loads inside the half space was shown. All the followers[6),4)] employed this method. Sezawa[6)] extended Lamb's problem to arbitrary asymmetric modes with azimuthal dependency and suggested the general solution.

Since the study of Reissner[7)], the surface source problem was mainly discussed as the response of a harmonically oscillating circular disk attached to the surface of an elastic body. A great many papers have been published while no special attention was paid to the behavior of the internal ground. Higashihara developed a general inversion procedure of the Sezawa's formula in applying it to the circular disk on elastic half space[8)]. It is applicable to the calculation of the response of the internal point.

The response of a homogeneous isotropic elastic full space to the harmonic lateral force is calculated by the following equation[9] that is originally from Kelvin's solution. The force acts in the x direction and u_x is the displacement response to the force direction.

$$u_x = \frac{1}{4\pi\rho} F_0 \left[-e^{\frac{i\omega R}{v_p}} \left\{ \left(\frac{3x^2}{R^3} - \frac{1}{R} \right) \left(\frac{1}{i\omega R v_p} + \frac{1}{\omega^2 R^2} \right) + \frac{x^2}{R^3 v_s^2} \right\} + e^{\frac{i\omega R}{v_s}} \left\{ \left(\frac{3x^2}{R^3} - \frac{1}{R} \right) \left(\frac{1}{i\omega R v_s} + \frac{1}{\omega^2 R^2} \right) + \left(\frac{1}{R v_s^2} - \frac{x^2}{R^3 v_s^2} \right) \right\} \right] \tag{1}$$

where F_0 is the amplitude of the applied force, ρ is density, ω is circular frequency, R is distance, x is horizontal distance, v_s and v_p are S and P wave velocities respectively.

The response of the surface of elastic half space to a lateral excitation at the surface is expressed in the following equation[10],

$$u_x = \frac{F_0}{4\pi\mu} \int_0^\infty \left\{ \left(\frac{k}{\beta} - \frac{k\beta b^2}{F(k)} \right) J_0(kr) + \cos 2\theta \left(\frac{k}{\beta} + \frac{k\beta b^2}{F(k)} \right) J_2(kr) \right\} dk \tag{2}$$

where F_0 is the amplitude of the applied force, $F(k) = (k^2 + \beta^2)^2 - 4k^2\alpha\beta$, $\alpha = \sqrt{k^2 - a^2}$, $\beta = \sqrt{k^2 - b^2}$, $a = \omega/v_p$, $b = \omega/v_s$, r is horizontal distance, μ is rigidity and J_n is Bessel function of the first kind of order n.

Kobayashi reduced the infinite integral into a sum of a term expressing the contribution of the residue and an integral over a finite interval[11].

3. FORMULA FOR THE WAVE IN ELASTIC HALF SPACE INDUCED BY A SUPERFICIAL POINT SOURCE

A detailed response examination of the surface source considering depth dependency is done for understanding underground energy distribution. The new formula is based upon the general solution form given by Sezawa[6] that consists of three components.

$$u_r = \int_0^\infty \left\{ \begin{array}{l} -\dfrac{\partial J_m(kr)}{\partial r} A(k) e^{-\alpha z} \\ + \left[\dfrac{1}{r} J_m(kr) B(k) + \dfrac{\partial J_m(kr)}{\partial r} \beta C(k) \right] e^{-\beta z} \end{array} \right\} dk$$

$$u_\theta = \int_0^\infty \left\{ \begin{array}{l} \dfrac{1}{r} J_m(kr) A(k) e^{-\alpha z} \\ - \left[\dfrac{\partial J_m(kr)}{\partial r} B(k) + \dfrac{\beta}{r} J_m(kr) C(k) \right] e^{-\beta z} \end{array} \right\} dk$$

$$w = \int_0^\infty J_m(kr) \left[\alpha A(k) e^{-\alpha z} - k^2 C(k) e^{-\beta z} \right] dk \tag{3}$$

where z = depth and other symbols are the same as in Eq. (2). The components of displacement in radial, circumferential and vertical direction are $u_r\cos(m\theta)$, $u_\theta\sin(m\theta)$ and $w\cos(m\theta)$ (m = 0, 1, 2, ⋯), respectively.

The response of a homogeneous isotropic elastic half space induced by harmonically vibrating lateral source on its surface is given by the following equation.

$$u_x = \frac{F_0}{4\pi\mu}(A + B\cos 2\theta) \qquad (4)$$

$$u_y = \frac{F_0}{4\pi\mu} B\sin 2\theta$$

$$w = \frac{F_0}{4\pi\mu} C\cos\theta$$

where,

$$A = \int_0^\infty \left[-\frac{2k^3\beta}{F(k)} e^{-\alpha z} + \left(\frac{k}{\beta} + k\beta \frac{k^2+\beta^2}{F(k)}\right) e^{-\beta z} \right] J_0(kr)\,dk$$

$$B = \int_0^\infty \left[\frac{2k^3\beta}{F(k)} e^{-\alpha z} + \left(\frac{k}{\beta} - k\beta \frac{k^2+\beta^2}{F(k)}\right) e^{-\beta z} \right] J_2(kr)\,dk \qquad (5)$$

$$C = \int_0^\infty \left[\frac{2k^2\alpha\beta}{F(k)} e^{-\alpha z} - k^2 \frac{k^2+\beta^2}{F(k)} e^{-\beta z} \right] J_1(kr)\,dk$$

This formula gives a complete wave field at a arbitrary depth, as well as at the surface.

4. WAVE FIELD GENERATED BY THE LATERAL MODE SOURCE

The displacement responses of the force direction of the full space and of the half space induced by laterally vibrating sources are given as contour map in **Fig. 1** and **2**, where 50 Hz excitation, 500 m/sec S wave velocity, and normalized frequencies are used with two different Poisson's ratios.

By looking into the contours in **Fig. 1(a)** and **(b),** we can find the inversely curved parts, dotted lines, in each of the contour lines which make diagonal parts with relatively small amplitude of response. Such a contour shape is from radiation pattern of body wave[13]. The response of the upper part of this diagonal is mainly from the magnitude of the P wave, while the response of the lower part is mainly from the magnitude of the S wave. When the full space is incompressible, this diagonal becomes horizontal.

In the response contours of half space, large response is generated at the free surface of the half space. The responses in the z direction show the same value with different Poisson's ratios as shown in the three contours of **Fig. 2**. We can find the diagonal lines in **Fig. 2(b)** and the inversely curved parts in each of the contour lines in **Fig. 2(a)** as shown in the case of full space. The large response on the surface in **Fig. 2** is considered to be the surface wave motion. The main part of this surface amplification is a free surface Rayleigh wave, the depth of which is not clearly shown here for the convenience of contour drawing. We can find weak diagonal parts in **Fig. 2(a)** as shown in **Fig. 1(a), (b)**. The response of the lower part of this diagonal comes mainly from the magnitude of the S wave and the response of the upper part of this diagonal comes mainly from the magnitude of P waves, but the response of this part is smaller than that of full space. This phenomenon is due to the trapped energy on the free surface. Trapped means that a part of input energy is consumed by surface wave energy. This hinders us from emitting waves into deep underground. The energy propagation of the upper part of the diagonal is mainly caused by P wave motion and much of the energy of that part is moved to the surface. On the other hand, the response of the lower part can be equal with that of full space. This large difference of response of the two parts causes the diagonal lines in **Fig. 2(b)**.

From the above results, we found the directivity of the response of elastic half space under the lateral source. This area of relatively large response has a cone shape whose axis is vertically downward from the source. In spite of the large trapped energy on the surface, the response of half space in this cone is almost the same as that of full space.

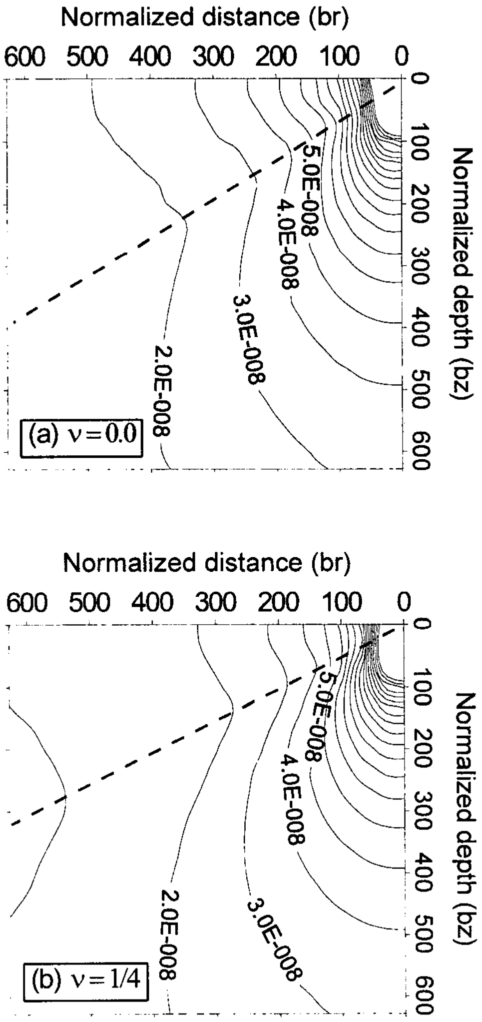

Fig.1 The response contour of full space

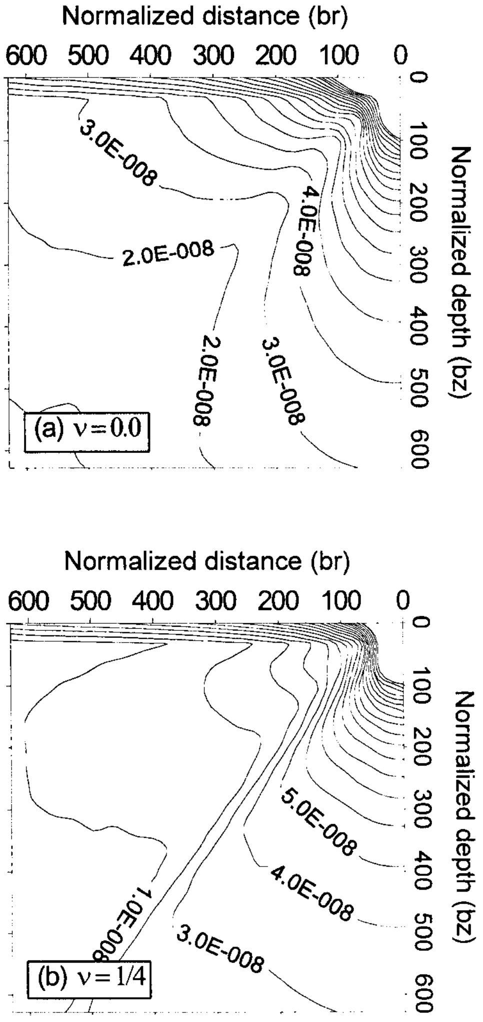

Fig.2 The response contour of half space

5. CONCLUSION

The response of a homogeneous, isotropic, elastic half space induced by a harmonically vibrating lateral point source located on its surface was analyzed, New formula of the response was obtained. This formula provides a useful basis for the evaluation of the performance of a new artificial seismic source which has been developed recently. This source is indispensable for the precision investigation of underground geological structures. Conventional formulae provide the response of the surface only. The present formula gives the wave field at arbitrary depth as well as at the surface. Numerical integration was achieved with high precision; a complex-valued response which contained the information about amplitude and phase was obtained.

The obtained results are summarized as follows:

(1) Large response is generated at the surface in the form of a surface wave. This type of response does not exist in the case of lateral source in full space.

(2) The wave emission displays remarkable directivity. This relatively large response area has a cone shape whose axis is vertically downward from the source.

(3) Owing to this directivity, in spite of the large trapped energy on the surface, the response decay of the lateral surface source with depth in the far field is equal to that of the full space case.

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