Limitation of FDM Weighted Average Grids in Frequency Domain

주파수 영역에서 유한차분 가중평균 격자수의 한계성

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Introduction

The means of reducing central processing unit (CPU) time and direct-access memory requirements in finite-difference methods are to use high order finite-difference approximations to spatial and temporal derivatives (Dablain, 1986) and to use weighted average method which was developed by Jo, et al. (1996). This method is to weight finite difference equation from conventional 5 points and the difference equation from transformed coordinate to 0, 45°. With this method the grid point per minimum wave length reduced to 13, however conventional 5 points method need more than 9 grid points. Shin & Sohn (1995) extended this 9 points weighted average method to 25 points which was transformed coordinate to 0, 45 in 5 points and to 0, 22.5, 45, 62.5 in 25 points. They reduced the grid points per minimum wave length to 3 and show the good accuracy with saving computing time and memory. According to this weighted average methods we can deduce that if we get more finite difference stars we could reduce the grid per minimum wave length keeping accuracy and resolution. Can we reduce the grids to 2 ? In this study we worked 81, 121 and 169 finite difference stars weighted average method and dispersion analysis. And we will show that weighted average methods are optimum between the accuracy and saving computing time and memory.

FDM formulation using weighted average method

In Cartesian coordinate system, the scalar wave equation in frequency domain can be written

$$\frac{\omega^2}{v^2}u + \nabla^2 u = 0 \tag{1}$$

where u is the pressure of wave field, ω is angular frequency and v is velocity of medium. The conventional finite difference expression of equation (1) by the explicit second-order difference scheme can be written

$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\triangle x^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{\triangle z^2} + \frac{\omega^2}{v^2}u_{i,j} = 0 \tag{2}$$

where $u_{i,j}$ is compressional field at $x_i = x_0 + (i-1)\triangle x$, $z_j = z_0 + (j-1)\triangle z$, $\triangle x, \triangle z$ is grid distance and ω is angular frequency. The new Laplacian term by weighted average method is

$$\nabla^2 u = \sum_{k=1}^{N} r_k \nabla_k^2 u \tag{3}$$

where r_k is coefficients of difference stars, N is number of transformed coordinate and ∇_k^2 is Laplace term according to each weighted average method. The mass accelation term of equation (1) can be rewritten by $u = \gamma u^* + (1-\gamma)[u]$. The new mass accelation term by weighted average method is

$$[u_{i,j}^*] = \sum_{k=1}^{n} a_k u_{i,j} \tag{4}$$

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where a_k are weighted average coefficients for the mass term. Substitution equation(3) and equation(4) into equation(??) yields

$$\sum_{k=1}^{n} r_k \nabla_k^2 u + \sum_{k=1}^{n} a_k u = 0 \tag{5}$$

And let's substitute the plane harmonic wave $u = e^{-(K_x x + K_z z)}$, $k_x = k \sin\theta$ and $k_z = k\cos\theta$ into equation (5), where θ is the propagation angle to the normal, then we get the following equation,

$$\frac{\omega^2}{v^2} = -\frac{B}{\triangle^2 A} \tag{6}$$

where $G = \triangle/\lambda$, which is number of grid per wavelength and v is velocity of the medium (Jang, 1999). Since phase velocity V_{ph} is $\frac{\omega}{k}$ and group velocity V_{gr} is $\frac{d\omega}{dk}$, normalized phase velocity for weighted average method is

$$\frac{V_{ph}}{V_0} = \frac{1}{2\pi G}\sqrt{\frac{-B}{A}} \tag{7}$$

and the normalizes group velocity V_{gr} for weighted average method is

$$\frac{V_{gr}}{V_0} = \frac{1}{4\pi G}\frac{V_0}{V_{ph}}\frac{AB' - BA'}{A^2} \tag{8}$$

where A' and B' are the partial derivative of A and B to the wave number and V is velocity of the medium.

Conclusion

We tried to find out optimum weighted average method for solution of scalar wave equation in frequency domain and conclusions are followed:

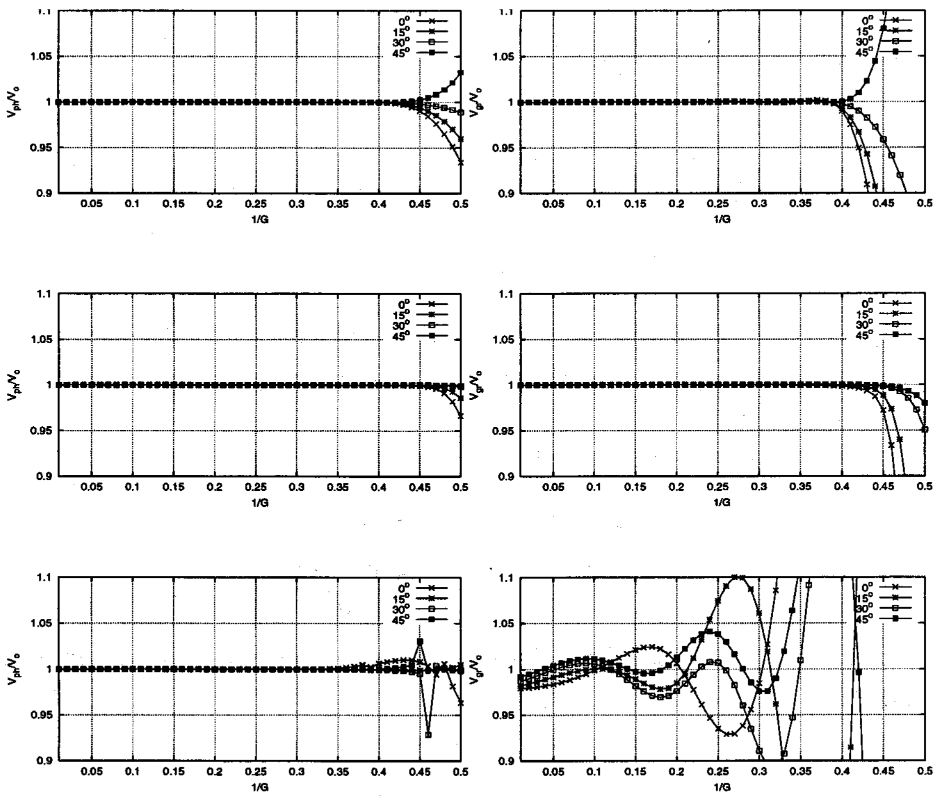
- New weighted average methods, 81, 121 and 169 points difference stars, were studied. The numerical errors in phase velocity less than 1 % for accuracy solution in 81 points could be archived 2.5 grid points per wavelength and in 121 points could be archived 2.3 points. However, in 169 points there was no solution because of oscillation of dispersion curves in group velocities.
- According to dispersion analysis for determination of grid points per wavelength, I have shown that the more rotated finite difference operators, the less grid points. However, the more rotated finite difference operator are needed the more complex difference equation terms.

참고 서적

[1] Dablain, M. A., 1986, The application of high–order differencing to the scalar wave equation: Geophysics, **51**, 54–66.

[2] Jang,S.H., 1999, The use of parallel virtual machine for prestack reverse time migration, KIGAM, internal report.

[3] Jo, C. H., Shin, C. S., and Suh, J. H., 1996, An optimal 9 point finite difference frequency-space 2D acoustic wave extrapolator, Geophysics, **61**, 529–537.



(a) (b)

Figure 1: Normalized phase and group velocity curves for finite-difference solution of 2D scalar wave equation in frequency domain. **left** : phase velocity curves of 81, 121 and 169 points, **right**: group velocity curves of 81, 121 and 169 points

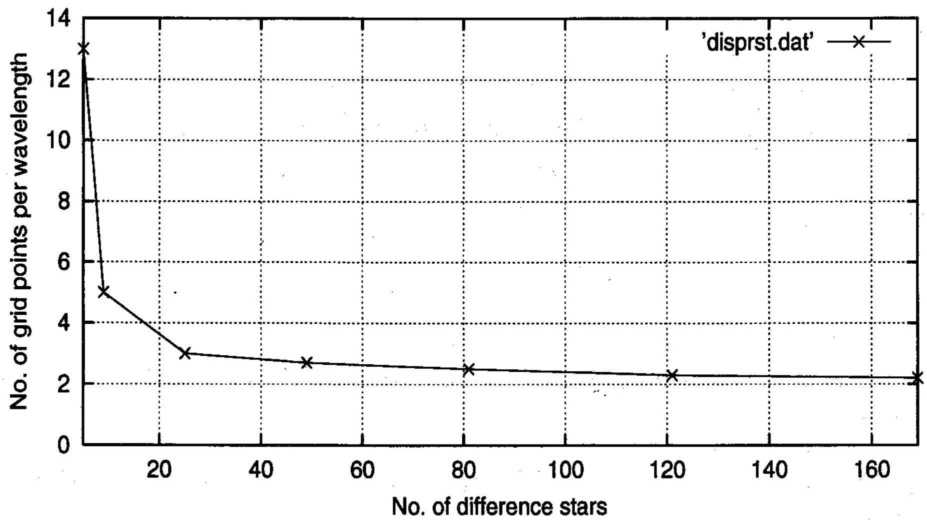


Figure 2: Relationship between number of finite difference stars and grid points per wavelength show that the more finite difference star have the less grid points per wavelength.