1. Reliability-based Optimization for Rock Slopes

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ABSTRACT

The stability condition of rock slopes is greatly affected by the geometry and strength parameters of discontinuities in the rock masses. Rock slopes involving movement of rock blocks on discontinuities are failed by one or combination of the three basic failure modes – plane, wedge, and toppling. In rock mechanics, practically all the parameters such as the joint set characteristics, the rock strength properties, and the loading conditions are always subject to a degree of uncertainty. Therefore, a reasonable assessment of the rock slope stability has to include the excavation of the multi-failure modes, the consideration of uncertainties of discontinuity characteristics, and the decision on stabilization measures with favorable cost conditions.

This study was performed to provide a new numerical model of the deterministic analysis, reliability analysis, and reliability-based optimization for rock slope stability. The sensitivity analysis was carried out to verify proposed method and developed program; the parameters needed for sensitivity analysis are design variables, the variability of discontinuity properties (orientation and strength of discontinuities), the loading conditions, and rock slope geometry properties. The design variables to be optimized by the reliability-based optimization include the cutting angle, the support pressure, and the slope direction.

The variability in orientations and friction angle of discontinuities, which can not be considered in the deterministic analysis, has a greatly influenced on the rock slope stability. The stability of rock slopes considering three basic failure modes is more influenced by the selection of slope direction than any other design variables. When either plane or wedge failure is dominant, the support system is more useful than the excavation as a stabilization method. However, the excavation method is more suitable when toppling failure is dominant. The case study shows that the developed reliability-based optimization model can reasonably assess the stability of rock slopes and reduce the construction cost.

1. INTRODUCTION

The stability condition of rock slopes is mostly affected by the geometry and strength parameters of discontinuities in rock masses. Rock slopes involving movement of rock blocks on discontinuities tend to fail through a combination of three basic failure modes - plane, wedge, and toppling. In rock mechanics, practically all parameters, such as joint set characteristics, rock strength properties, and loading conditions, are subject to a degree of uncertainty. Therefore, a reasonable assessment of rock slope stability should include the evaluation of the multi-failure modes, the consideration of uncertainties of discontinuity characteristics, and the decision on stabilization measures with favorable cost conditions.

Engineering design requires risk assessment and a regard for cost constraints in order to balance safety with economy before conclusive decisions are made. It is now generally recognized that geotechnical engineering problems are nondeterministic and, consequently, that optimum design must cope with uncertainties. Clearly the proper tool for the assessment and analysis of such uncertainties requires the concepts of reliability. Therefore, it is not an overstatement to claim that the combination of reliability-based design procedures and optimization techniques are the only means of providing a practical optimum design solution.

Frangopol(1985) presented an overview of concepts and methods used in reliability-based optimization which obtained the proper performance criteria of a structure requiring an adequate safety margin (i.e., sufficient small probability of failure) against the occurrence of different limit states. In the case of rock engineering, many papers on the reliability analysis of rock slopes have been published (Scavia, et al., 1990; Leung and Quek, 1995). The optimization techniques of rock slopes, however, have received limited attention.

This study aims to formulate a new numerical model for the reliability-based optimization of rock slopes that is capable of evaluating the optimum values of slope direction, cutting angle, and support pressure that corresponds to the three basic failure modes. This model may well improve the quality of rock slope stability analysis and reduce construction costs.

2. STABILITY ANALYSIS OF ROCK SLOPES

2.1 Failure modes of rock slopes

The analysis of rock slope stability is fundamentally a two-part process: the kinematic analysis and the stability analysis. The first step is to analyze the discontinuities of a rock mass to determine whether the orientation of the discontinuities could result in instability of the slope. Once it has been assessed that a kinematically possible failure mode exists, the second step requires a limit equilibrium stability analysis to compare the resisting forces with the forces causing failure.

2.2 Plane and wedge failures

The method of vector analysis provides relatively simple formulations for all the quantities related to block morphology including the volume of each joint block, the areas of block faces, the positions of its vertexes, and the positions and attitudes of its faces and edges. The use of vectors also permits kinematic and static equilibrium analysis of key blocks under various loading conditions.

From a rock slope design perspective, the most important characteristic of a discontinuity is its orientation, which is best defined by two parameters; dip(α) and dip direction(β). The unit normal vector of the discontinuity plane, $\hat{n}$, is given as

$$\hat{n} = (\sin\alpha\sin\beta, \sin\alpha\cos\beta, \cos\alpha) \quad (1)$$

(1) Kinematic analysis

Three major components of block theory are as follows (Goodman and Shi, 1985):

(1) Finiteness analysis determines whether the rock joints and excavation surfaces contribute to the formation of an isolated rock block that is separate from the rest of the rock mass.

(2) Removability analysis determines whether the isolated block has a shape that allows the rock block to move into the excavation without movement of any other part of the rock mass. The steps of finiteness and removability analyses are commonly entitled kinematic analysis.

(3) Stability analysis determines whether driving forces acting on the block are sufficient to undermine resisting forces.

(2) Stability analysis

If the kinematic analysis indicates that some removable blocks are present, the rock slope stability for plane and wedge failures must be evaluated by a limit equilibrium analysis, which considers the friction force and cohesion strength along the failure surface and the resultant force. Vector analysis facilitates the analysis of block stability under gravity force, water pressure, seismic force, support force, friction, and cohesion.

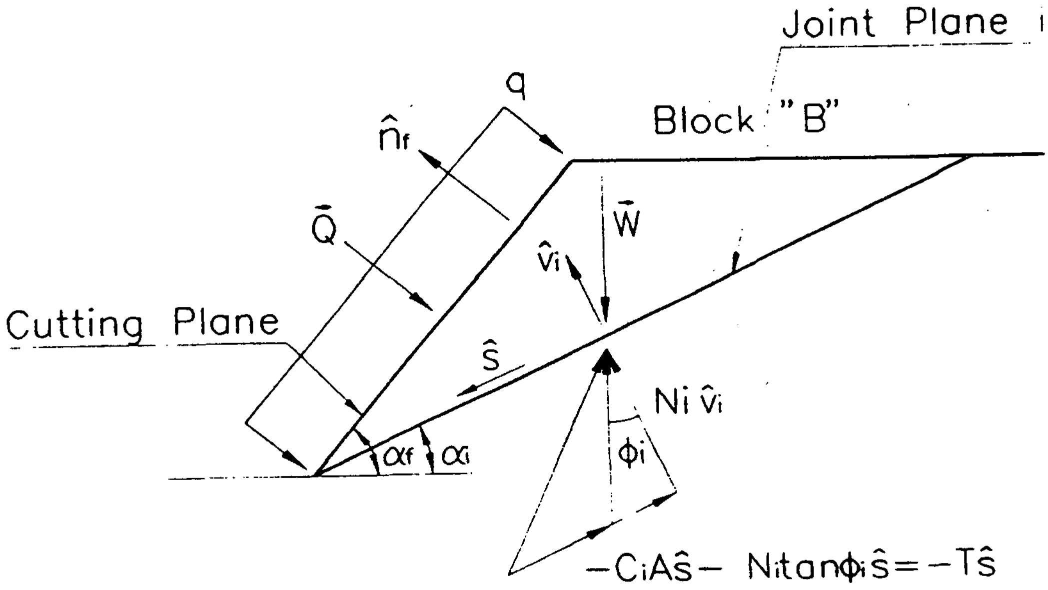
The condition of equilibrium for a potential or real key block B, described in Figure 1, is given as

$$\vec{r} + \sum_i N_i \hat{v}_i - T\hat{s} = 0 \qquad (2)$$

where, N_i = normal reaction force of joint plane i, $\hat{v}_i$ = unit vector normal to joint plane i, T = resultant of the tangential frictional force, $\hat{s}$ = unit vector of sliding direction, and $\vec{r}$ = resultant force. The resultant ($\vec{r}$) of all forces is given as

$$\vec{r} = \vec{W} + \vec{U} + \vec{F}_d + \vec{Q} \qquad (3)$$

where, $\vec{W}$ = gravity force, $\vec{U}$ = hydrostatic force, $\vec{F}_d$ = seismic force, and $\vec{Q}$ = support force.



$\hat{n}_f$: unit vector of cutting plane
α_f : dip angle of cutting plane
α_i : dip angle of joint plane i
C_i : cohesion of joint plane i
ϕ_i : friction angle of joint plane i
q : uniformly distributed stabilizing pressure
(normal to cutting plane)

Figure 1: Limit equilibrium on block B

2.3 Stability analysis for toppling failure

Toppling failure most commonly occurs in rock masses containing a large number of slabs or columns formed by a set of fractures that strike approximately parallel to the slope face and dip steeply into the face. Toppling failure is characterized by significant horizontal movement at the crest and very little movement at the toe. To accommodate this difference in movement between the toe and crest, interlayer movement must occur. Thus, the shear strength between layers is crucial to the stability of slopes structurally susceptible to toppling.

An analysis method of toppling failure presented by Goodman and Bray(1976) provide the fundamentals for the stability of rock slopes susceptible to toppling. A numerical analysis method for toppling analysis is developed in this study, modifying and combining the analytical method by Goodman and Bray(1976) and the numerical method by Zanbak(1983).

(1) Kinematic Analysis

The necessary kinematic conditions for the occurrence of toppling failure can be summarized into two parts: One being the direction of the toppling plane and the rock slope face; and the other being the dip angle of the joint plane and rock slope face, and the friction angle of the joint plane. The strike of the joint plane must be approximately parallel to the rock slope face. The dip of the joint plane must extend into the rock slope face. In order for interlayer slip to occur, the normal to the toppling plane must have a plunge less than the inclination of the slope face and less than the friction angle of the surface. The condition can be formulated as (Goodman and Bray, 1976)

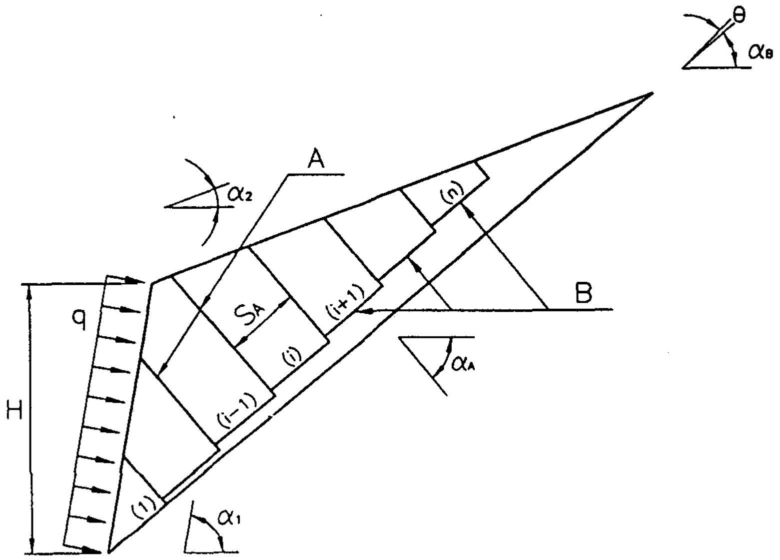
$$(90^\circ - \alpha_A) \leq (\alpha_1 - \phi_A) \qquad (4)$$

where, α_A = dip angle of discontinuity A,

ϕ_A = friction angle of discontinuity A,

and α_1 = cutting angle of rock slope.

(2) Stability Analysis



H : slope height

q : uniformly distributed stabilizing pressure

α_1, α_2: cutting angle of slope and upper slope surface

α_A, α_B: dip angles of discontinuities A and B

S_A : spacing of discontinuity system A

θ : step angle

Figure 2: Idealized slope generating with a postulated failure surface

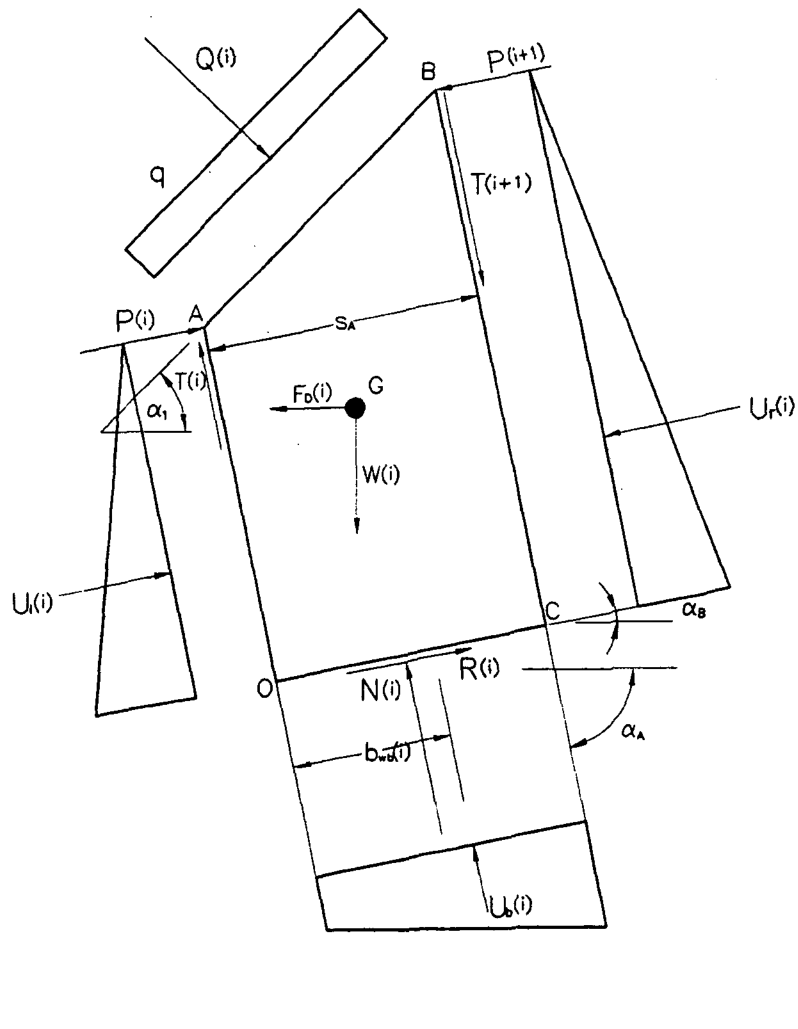


Figure 3: Forces applied to slice (i)

Figure 2 shows a cross section of the rock slope with a system of blocks on a stepped failure surface. Two discontinuity systems A and B with 100% persistence are assumed present in the rock mass. For the total volume of rock blocks in the slope resting on the estimated stepped failure surface, the n blocks are numbered in sequence starting from the toe as shown in Figure 2. For the (i)th block shown in Figure 3, the force P(i) transferred to the (i-1)th block is calculated from the limiting equilibrium condition. When considering the limiting equilibrium condition of a typical block (say i, on Figure 2), the following additional forces are acting on the block :

- the weight W(i) ;
- the shear forces T(i), T(i+1) on the sides and R(i) on the base (these forces are related to the friction angle ϕ_A, at the sides, and ϕ_B, at the base);
- the hydrostatic forces $U_l(i)$, $U_r(i)$ at the sides and $U_b(i)$ at the base, resulting from the water pressure distribution;
- the seismic force $F_D(i)$, which is as usual given by K×W(i), with K being the seismic coefficient ;
- the force P(i+1), transferred from the (i+1)th block, taken as normal to the side and applied at point B ; and
- the force Q(i) resulting from a stabilizing pressure distribution.

The force P(i) is assumed to be normal to the side of the (i)th block and applied at discontinuity A. The greater value of the calculated $P_t(i)$, the toppling resistance of the (i)th block, and $P_s(i)$, the shear resistance of the (i)th block, will be the P(i) force exerted on the (i-1)th column. The overall stability of the toppling slope is defined by the P_1 force exerted on the first column at the toe (Figure 2). The slope is considered unstable if the resultant P_1 force applied on the toe column is greater than zero. When $P_1 > 0$, then the magnitude of P_1 is the retaining force in the base plane obtained from the limit equilibrium condition.

3. RELIABILITY-BASED ANALYSIS

3.1 *General*

The failures of rock masses are mainly controlled by discontinuities in the rock mass. Therefore, it is important to know features of discontinuities. These features are classified into three groups: (1) orientations of discontinuities; (2) strength properties of discontinuities; and (3) size and spacing of discontinuities.

In this study, the reliability-based analysis of rock slopes is accomplished by the Monte Carlo simulation based on independently generated random numbers. Some basic features and assumptions are given below:

- The kinematic analysis and the stability analysis for plane and wedge failures of rock slopes are accomplished by block theory.
- The kinematic analysis and the stability analysis for toppling failures of rock slopes are performed by a newly developed 2-D numerical method.
- The orientations of discontinuities are random variables assumed to be Fisher's distribution assessed by the clustering technique.
- The friction angles of discontinuities are random variables assumed to be normal distribution.
- This study does not consider any time related aspects such as creep or weathering.
- Correlations between parameters are neglected.

3.2 *Evaluation of failure probability*

In probabilistic reliability analysis, the safety margin (SM) of a rock slope is defined as the difference between the force resisting the sliding down the plane (R) and the force causing the sliding to occur (L). Failure is defined by the event SM<0. The probability of this event is:

(1) for plane and wedge failures,

$$P_f = P(SM = (R-L) \leq 0); \qquad (5)$$

(2) and for toppling failure,

$$P_f = P(SM = -P_1 \leq 0) \qquad (6)$$

where P_f is the probability of failure.

By generating numerous combinational sets of random variables, one can estimate the relative chance of each failure mode, P_{fp}, P_{fw}, and P_{ft}, and finally P_f:

$$P_{fp} = \frac{N_p}{N}, \quad P_{fw} = \frac{N_w}{N}, \quad P_{ft} = \frac{N_t}{N}, \quad P_f = \frac{N_f}{N} \tag{7}$$

where, P_{fp} = probability of plane failure, P_{fw} = probability of wedge failure, P_{ft} = probability of toppling failure, P_f = probability of overall failure, N_p = number of plane failure, N_w = number of wedge failure, N_t = number of toppling failure, N_f = number of overall failure irrespective of failure modes $= N_p \cup N_w \cup N_t$, and N = total number of sets analyzed.

4. RELIABILITY-BASED OPTIMIZATION

4.1 General

This study aims to formulate a new numerical model for the reliability-based optimization of rock slopes that is capable of evaluating the optimum values of slope direction, cutting angle, and support pressure corresponding with the three basic failure modes.. This model may well improve the quality of rock slope stability analysis and reduce construction costs.

The essential characteristics of a reliability-based optimization problem for rock slopes are:

- random variables - orientation and the friction angle of discontinuities;
- deterministic parameters - cohesion and spacing of discontinuities and physical properties of rock mass;
- design variables - dip direction and cutting angle (dip angle) of rock slopes and support pressure;
- load conditions - gravity force, water pressure, and seismic force; and
- failure modes - plane, wedge, and toppling failure.

Some basic features and assumptions are given below:

- The orientations of discontinuities are random variables assumed to follow Fisher's distribution assessed by the clustering technique;
- The friction angles of discontinuities are random variables assumed to follow normal distribution;
- This study does not consider any time-related aspects such as creep or weathering; and
- Correlations between parameters are neglected.

4.2 Formulations

This study focuses on the reliability-based optimization for rock slopes to minimize construction costs with a prescribed overall failure probability level. It is assumed that the cost consists of three parts: initial cost; cost of stabilization measures; and damage cost as shown in Figure 4. The initial cost indicates the construction cost needed in preparing necessary space. The damage cost represents the expenses needed for repairing the failed slopes. In general, the types of stabilization measures are classified into three categories; removal of rock mass from an unstable zone, reinforcement of slope using rock bolts or anchors, and protection measures through placement of fences or nets. Two methods out of the above mentioned are considered in the feasibility study in regard to cost; the rock removal and the slope reinforcement.

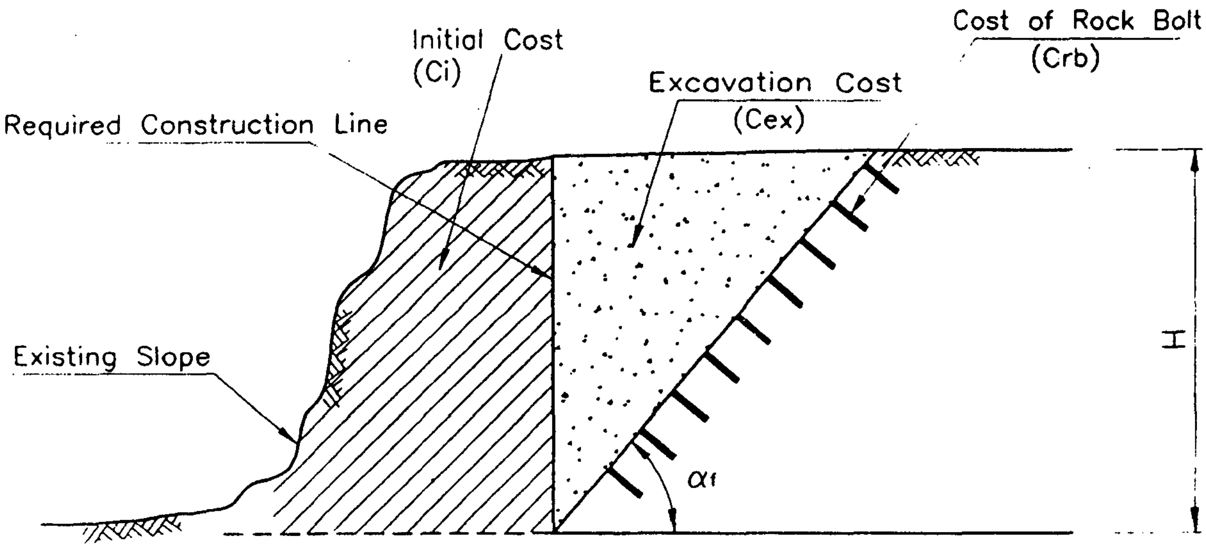


Figure 4: Definition of initial cost, excavation cost, and cost of rock bolt

The reliability-based optimization for rock slopes is a constrained minimization specified as one of the following forms:

- Objective function is

$$\textbf{Min} \quad : \quad \mathbf{TC(\{D\}) = C_i + C_{ex} + C_{rb} + C_d \times P_f} \tag{8}$$

where, **TC** = total cost per unit length of a slope (₩/m), **{D}** = design variables = $\{\alpha_f, \beta_f, q\}$, α_f = cutting angle of rock slopes, β_f = dip direction of the rock slope, q = stabilized load uniformly distributed (ton/m²), $\mathbf{C_i}$ = initial cost per unit length (₩/m), $\mathbf{C_{ex}}$ = excavation cost per unit length (₩/m) = $c_{ex} \times (H^2/2\tan\alpha_f)$, $\mathbf{C_{rb}}$ = cost of rock bolts per unit length (₩/m) = $c_{rb} \times (Hq/\sin\alpha_f)$, $\mathbf{C_d}$ = damage cost per unit length (₩/m), $\mathbf{P_f}$ = the overall probability of failure of the rock slope, c_{ex} = excavation cost per unit volume (₩/m³), c_{rb} = cost of rock bolt per unit ton (₩/ton) , and 1$ = 1500 ₩.

- Constraint is defined as

$$P_f(\{D\}) \leq P_f^0 \tag{9}$$

where, P_f = the overall probability of failure of the rock slope, and P_f^0 = the prescribed allowable failure probability of the rock slope. Both the total cost and the constraint of the failure probability are usually implicit functions of the design variables {D}. Total cost is calculated by Eqn. 8. Constraint conditions are the overall failure criterion represented by Eqn. 9.

5. APPLICATION OF THE PROPOSED MODEL

5.1 General

The proposed model for the reliability-based optimization of rock slopes was applied to a hypothetical site. This site is assumed to have a system of four discontinuity sets as shown in Table 1. The four discontinuity sets are numbered 1, 2, 3, and 4. Their mean values of dips and dip directions are shown in Table 1. Fisher's constants are assumed to be 100. The friction angles of the discontinuities have a mean value of 30° with a standard deviation of 3°. The spacing of discontinuities is considered to be 3m. The cohesion of the discontinuities is taken to be zero.
The design variables to be optimized are either two or three: the cutting angle of the rock slope; the support pressure; and optionally, the dip direction of the rock slope. The deterministic parameters for the study are: the height of the rock slope, H = 30m; the unit weight of rock mass, $\gamma = 2.5t/m^3$; and the angle of upper slope surface, $\alpha_2 = 0$. First of all, the probability of failure of the example rock slope is obtained for the full range of the design variables: the dip direction of the rock slope, β ranging from 0° to 360°; the cutting angle of the rock slope, α ranging from 30° to 90°; and the support pressure, q ranging from 0 to $10t/m^2$.

TABLE 1

SUMMARY OF INPUT PARAMETERS

	Orientation			Friction Angle		S (m)
	E(α)	E(β)	K	E(ϕ)	σ(ϕ)	
1	35°	20°	100	30°	3°	3
2	15°	125°	100	30°	3°	3
3	60°	220°	100	30°	3°	3
4	75°	300°	100	30°	3°	3

For the optimization of the rock slope, the following assumptions are made. The initial cost is neglected. The excavation cost is assumed to be ₩30,000 per cubic meter. And, the cost of rock bolts is assumed to be ₩10,000 per ton. In the case where damage costs are included, they are assumed to be ₩100,000,000 per unit length. The sensitivity of design parameters on the calculated results of the reliability-based optimization is studied using the program 'RBO-RSSA' developed in this study. The parameters analyzed in the sensitivity analysis include orientation and strength of discontinuities, loading conditions, and spacing of discontinuities, etc.

5.2 Results and Discussion

The stability of rock slope including plane, wedge, and toppling failure modes was found to be more influenced by the selection of dip direction of cutting rock face than any other design variables as shown Figure 4. The toppling failure is more sensitive to the variation of cutting angle of rock slope than any other failure modes as shown Figure 5. Figure 6 shows that both of plane and wedge failures are more sensitive to the variation of support pressure than toppling failure is.

Figure 7 shows that the failure probability is highly dependent on the Fisher's constant(dispersion of discontinuity orientation). Figure 8 also shows that the increase of coefficient of variation of friction angle of discontinuities gives an increase of failure probability. It means that variability in orientations and friction angles of discontinuity set can lead to failures which would not be predicted by just performing deterministic analysis using mean orientations and mean friction angles.

The minimum cost and optimum design variables are obtained by using the constraint condition of prescribed allowable failure probability. The two dimensional design space shown in Figure 10 and 11 has three P_f constraint curves; $P_f^0 = 0.001$, 0.01, and 0.1. The design space inside the P_f constraint boundaries denoted by hatched lines is the feasible zone, and all the designs located in this region are acceptable. The minimum total cost must be decided within the feasible zone defined by the iso-curve of the specified allowable failure probability. Because the iso-curve of the prescribed failure probability is not a smooth curve, the optimum value which satisfies the failure probability constraints can be obtained using the DP algorithm sorting technique. Figure 10 shows that the optimum cost is 12,800,000 ₩/m, and the optimum design variables are a cutting angle of 55° and a support pressure of $9t/m^2$ within an allowable failure probability 0.01 (1%). When the toppling failure mode controls (as in the case when $\beta = 20°$), the optimum cutting angle is distributed in the range between 35° and 60° as shown in Figure 10. On the other hand, when the dip direction of the rock slope is equal to 210°, either of plane or wedge failure is dominant, and the optimum cutting angle is distributed in the range between 70° and 90° as shown in Figure 11. This indicates that the appropriate stabilization measure for toppling failure is rock removal by excavation rather than the support system.

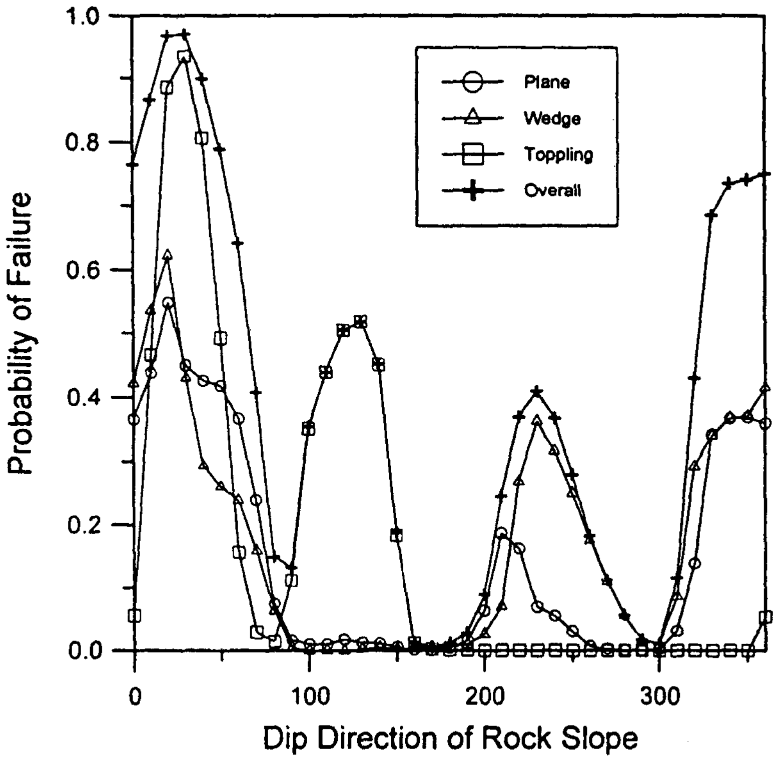


Figure 5: P_f versus β_f (α_f =70° and q = 0 t/m^2)

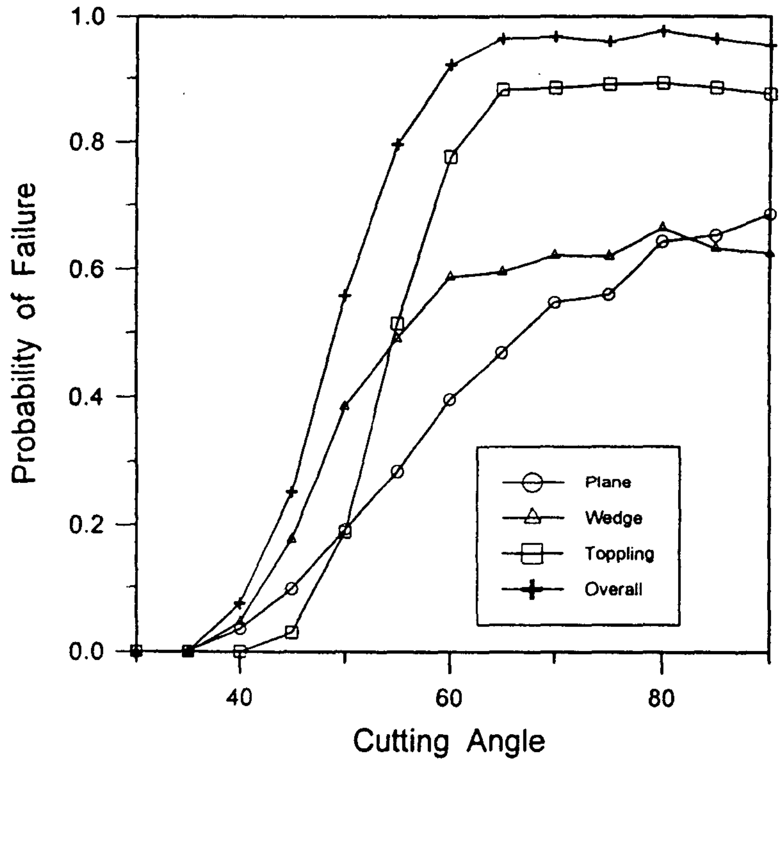


Figure 6: P_f versus α_f (β_f =20° and q = 0 t/m^2)

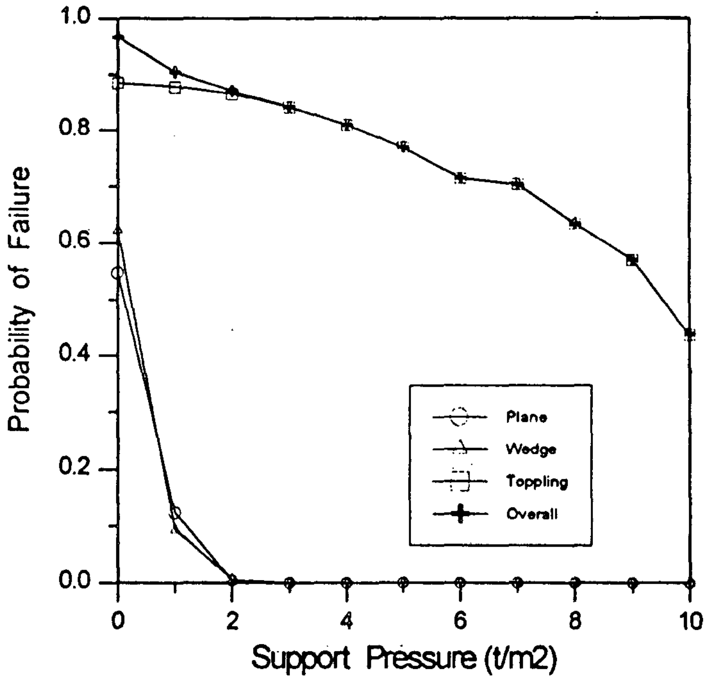


Figure 7: P_f versus q (β_f =20° and α_f =70°)

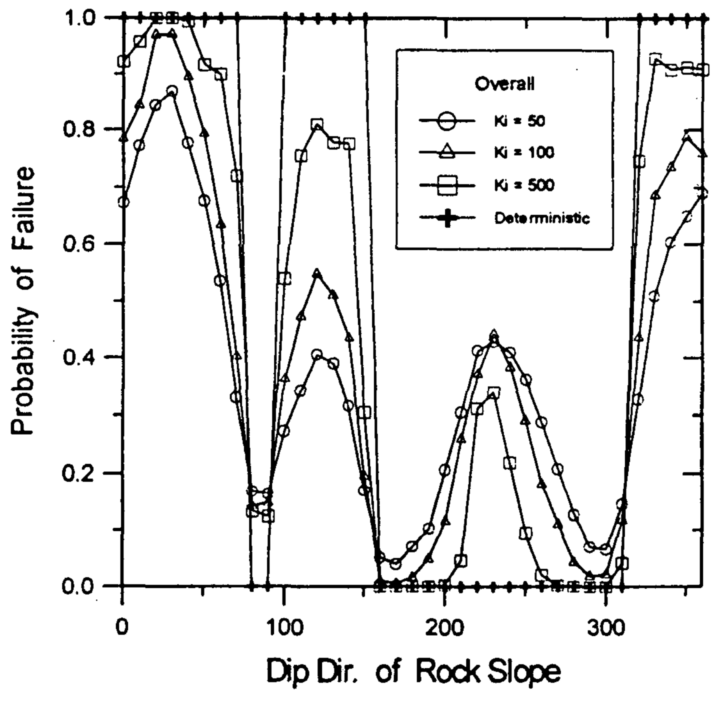


Figure 8: P_f versus α_f with the variation of K

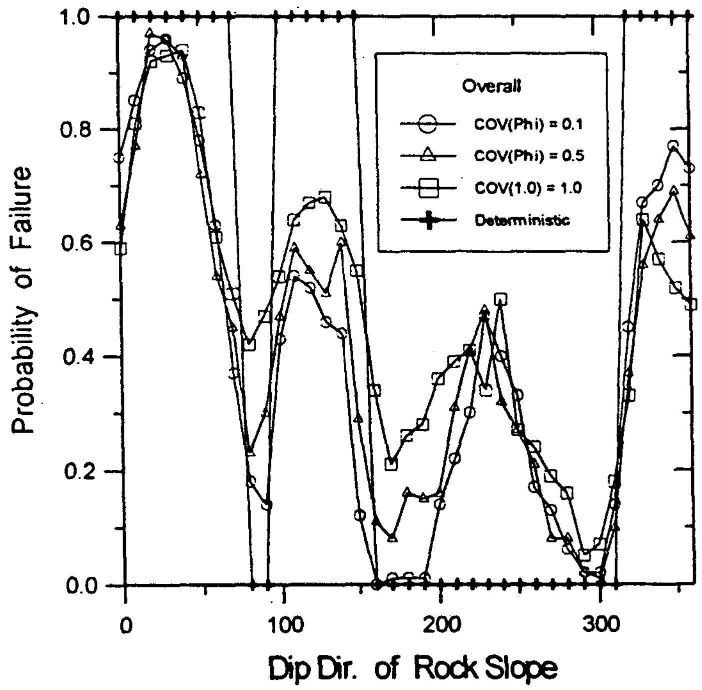


Figure 9: P_f versus α_f with the variation of coefficient of variation of ϕ

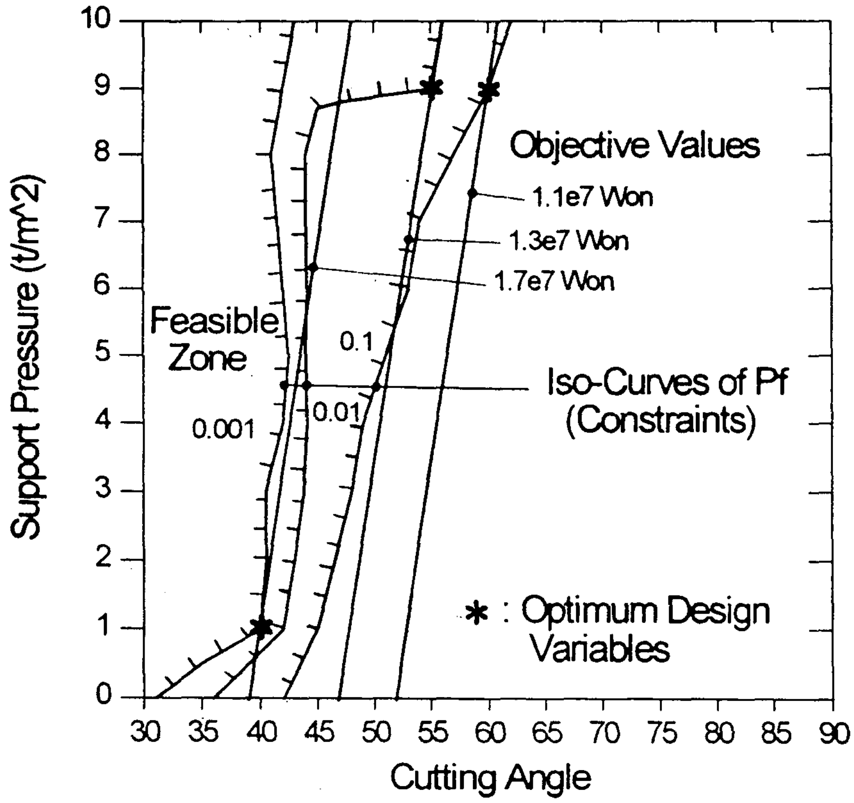


Figure 10: Sensitivity of Optimum Solution to the Change of the Prescribed Failure Probability (when β_f =20°)

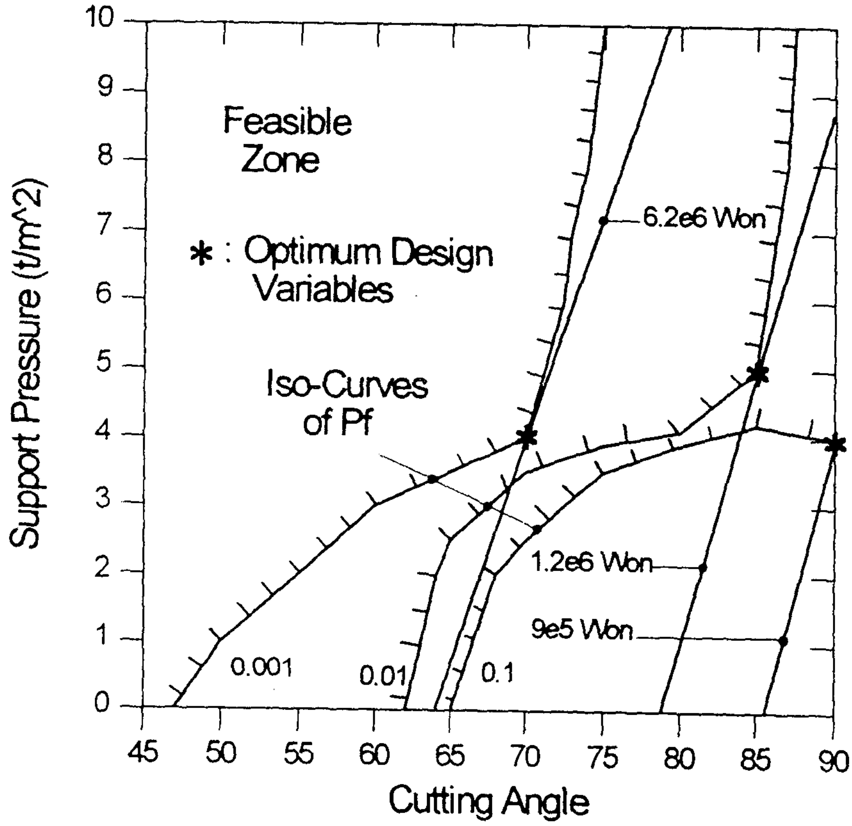


Figure 11: Sensitivity of Optimum Solution to the Change of the Prescribed Failure Probability (when β_f =210°)

The sensitivity analysis was performed for the reliability-based optimization of the example rock slope. The parameters studied are Fisher's constant of discontinuity orientation, the coefficient of variation and the mean value of the friction angles of discontinuities, cohesion of discontinuities, H/t ratio, and loading conditions. The optimum cutting angle of the rock slope decreases with the increase of variability in the orientations and friction angles (decrease of the Fisher's constant and increase of the coefficient of variation of friction angle) of discontinuities as shown in Table 2. This means that the deterministic optimization methodology may at times give erroneous results. In general, the optimum cutting angle increases, i.e. the excavation quantity decreases, as either the friction angle or the cohesion of discontinuities increases and the H/t ratio decreases as shown in Table 3-5. The increase of porewater pressure and seismic force would also lead to a decrease of the optimum cutting angle and, in turn, an increase of the minimum cost. The optimum cost was the most sensitive to the change of the dip direction of the rock slope.

TABLE 2

RESULTS OF SENSITIVITY ANALYSIS (β_f=20°)

Analysis Cases		Prescribed Failure Probability P_f^0	Formulation A			Formulation B		
			Minimum Cost TC^* (Won/m)	Optimum Design Variables		Minimum Cost TC^* (Won/m)	Optimum Design Variables	
				α^*(°)	q^*(t/m²)		α^*(°)	q^*(t/m²)
Deterministic Analysis		-	9.27×10^6	65	9	9.27×10^6	65	9
Fisher's Constant	k_i= 50	0.001	1.98×10^7	35	1	1.98×10^7	35	1
		0.01	1.66×10^7	40	1	1.69×10^7	40	1
		0.1	1.09×10^7	60	9	1.43×10^7	55	10
	k_i= 100	0.001	1.66×10^7	40	1	1.66×10^7	40	1
		0.01	1.28×10^7	55	9	1.36×10^7	55	9
		0.1	1.13×10^7	60	10	1.36×10^7	55	9
	k_i= 500	0.001	1.44×10^7	45	2	1.44×10^7	45	2
		0.01	1.28×10^7	55	9	1.34×10^7	55	10
		0.1	1.13×10^7	60	10	1.34×10^7	55	10
COV of Friction Angle	$\Omega(\phi)$ =0.1	0.001	1.66×10^7	40	1	1.66×10^7	40	1
		0.01	1.28×10^7	55	9	1.36×10^7	55	9
		0.1	1.13×10^7	60	10	1.36×10^7	55	9
	$\Omega(\phi)$ =0.5	0.001	2.29×10^7	35	7	2.29×10^7	35	7
		0.01	1.94×10^7	40	7	2.01×10^7	40	7
		0.1	1.48×10^7	45	3	1.93×10^7	40	3
	$\Omega(\phi)$ =1.0	0.001	2.94×10^7	30	10	2.94×10^7	30	10
		0.01	2.58×10^7	30	4	2.63×10^7	30	4
		0.1	1.70×10^7	40	2	2.30×10^7	35	5

TABLE 3

RESULTS OF SENSITIVITY ANALYSIS ($\beta_f = 20°$, $P^0_f = 0.001$)

Analysis Condition		Formulation A			Formulation B			Remarks
		Minimum Cost TC^* (Won/m)	Optimum Design Variables		Minimum Cost TC^* (Won/m)	Optimum Design Variables		
			α (°)	q^* (t/m²)		α (°)	q^* (t/m²)	
Friction Angle	ϕ=25°	1.70×10^7	40	2	1.71×10^7	40	2	C=0 t/m² U=0 K=0 H/t=10
	ϕ=30°	1.61×10^7	40	1	1.66×10^7	40	1	
	ϕ=35°	1.28×10^7	55	9	1.28×10^7	55	9	
Cohesion	C=0 t/m²	1.66×10^7	40	1	1.66×10^7	40	1	ϕ=30° U=0 K=0 H/t =10
	C=0.5 t/m²	1.61×10^7	40	0	1.61×10^7	40	0	
	C=1 t/m²	1.61×10^7	40	0	1.61×10^7	40	0	
Porewater Pressure	U=0	1.66×10^7	40	1	1.66×10^7	40	1	ϕ=30° C=0t/m² K=0 H/t =10
	U=0.5	1.98×10^7	40	8	1.98×10^7	40	8	
	U=1.0	-	-	-	-	-	-	
Seismic Force	K=0	1.66×10^7	40	1	1.66×10^7	40	1	ϕ=30° C=0 t/m² U=0 H/t =10
	K=0.1	1.70×10^7	40	2	1.70×10^7	40	2	
	K=0.2	1.70×10^7	40	2	1.70×10^7	40	2	
H/t Ratio	H/t=5	1.13×10^7	60	10	1.14×10^7	60	10	ϕ=30° C=0 t/m² U=0 K=0
	H/t=10	1.66×10^7	40	1	1.66×10^7	40	1	
	H/t=20	1.66×10^7	40	1	1.66×10^7	40	1	

TABLE 4

RESULTS OF SENSITIVITY ANALYSIS ($\beta_f = 20°$, $P^0_f = 0.01$)

Analysis Condition		Formulation A Minimum Cost TC^* (Won/m)	Formulation A Optimum Design Variables α (°)	Formulation A Optimum Design Variables q^* (t/m^2)	Formulation B Minimum Cost TC^* (Won/m)	Formulation B Optimum Design Variables α (°)	Formulation B Optimum Design Variables q^* (t/m^2)	Remarks
Friction Angle	ϕ=25°	1.66×10^7	40	1	1.71×10^7	40	2	C=0 t/m^2 U=0 K=0 H/t=10
	ϕ=30°	1.28×10^7	55	9	1.36×10^7	55	9	
	ϕ=35°	1.09×10^7	60	9	1.15×10^7	60	10	
Cohesion	C=0 t/m^2	1.28×10^7	55	9	1.36×10^7	55	9	ϕ=30° U=0 K=0 H/t =10
	C=0.5 t/m^2	1.28×10^7	55	9	1.36×10^7	55	9	
	C=1 t/m^2	1.09×10^7	60	9	1.36×10^7	55	10	
Porewater Pressure	U=0	1.28×10^7	55	9	1.36×10^7	55	9	ϕ=30° C=0t/m^2 K=0 H/t =10
	U=0.5	1.89×10^7	40	6	1.95×10^7	40	6	
	U=1.0	2.34×10^7	30	0	2.41×10^7	30	0	
Seismic Force	K=0	1.28×10^7	55	9	1.36×10^7	55	9	ϕ=30° C=0 t/m^2 U=0 H/t =10
	K=0.1	1.66×10^7	40	1	1.70×10^7	40	2	
	K=0.2	1.70×10^7	40	2	1.70×10^7	40	2	
H/t Ratio	H/t=5	9.61×10^7	65	10	1.04×10^7	65	10	ϕ=30° C=0 t/m^2 U=0 K=0
	H/t=10	1.28×10^7	55	9	1.36×10^7	55	9	
	H/t=20	1.66×10^7	40	1	1.66×10^7	40	1	

TABLE 5

RESULTS OF SENSITIVITY ANALYSIS ($\beta_f = 20°$, $P^0_f = 0.1$)

Analysis Condition		Formulation A: Minimum Cost TC^* (Won/m)	Formulation A: Optimum Design Variables α (°)	Formulation A: Optimum Design Variables q^* (t/m²)	Formulation B: Minimum Cost TC^* (Won/m)	Formulation B: Optimum Design Variables α (°)	Formulation B: Optimum Design Variables q^* (t/m²)	Remarks
Friction Angle	ϕ=25°	1.44×10^7	45	2	1.71×10^7	40	2	C=0 t/m² U=0 K=0 H/t=10
	ϕ=30°	1.13×10^7	60	10	1.36×10^7	55	9	
	ϕ=35°	8.11×10^7	70	10	1.09×10^7	65	10	
Cohesion	C=0 t/m²	1.13×10^7	60	10	1.36×10^7	55	9	ϕ=30° U=0 K=0 H/t =10
	C=0.5 t/m²	1.13×10^7	60	10	1.36×10^7	55	9	
	C=1 t/m²	1.09×10^7	60	9	1.36×10^7	55	10	
Porewater Pressure	U=0	1.13×10^7	60	10	1.36×10^7	55	9	ϕ=30° C=0t/m² K=0 H/t =10
	U=0.5	1.13×10^7	60	10	1.57×10^7	55	10	
	U=1.0	1.93×10^7	35	0	2.40×10^7	35	1	
Seismic Force	K=0	1.13×10^7	60	10	1.36×10^7	55	9	ϕ=30° C=0 t/m² U=0 H/t =10
	K=0.1	1.44×10^7	45	2	1.70×10^7	40	2	
	K=0.2	1.44×10^7	45	2	1.70×10^7	40	2	
H/t Ratio	H/t=5	7.79×10^7	70	9	1.04×10^7	65	10	ϕ=30° C=0 t/m² U=0 K=0
	H/t=10	1.13×10^7	60	10	1.36×10^7	55	9	
	H/t=20	1.39×10^7	45	1	1.60×10^7	45	1	

6. CONCLUSIONS

In this paper, a methodology for the reliability-based optimization of rock slopes was proposed, and a sensitivity analysis was performed to figure out the most influencing design parameters and the most cost-effective stabilization measure. The ratio of the optimum excavation cost to the optimum total cost for plane and wedge failure is smaller than that of toppling failure. Therefore, when either plane or wedge failure is dominant, the support system is more effective than excavation as a stabilization method, although the rock removal by excavation is more suitable when toppling failure is dominant.

The optimum cutting angle of the rock slope decreases with an increase of variability in the orientations and friction angles of discontinuities. It means that the deterministic optimization approach may at times give erroneous results. The optimum cost is the most sensitive to the change of the dip direction of rock slopes. Engineering design requires risk assessment and a regard to cost constraints in order to balance safety with economy before conclusive decisions are taken. The new numerical model of the reliability-based optimization for rock slopes proposed in this paper can evaluate the multi-failures modes, consider the uncertainty and variability of discontinuities in rock masses, and decide upon stabilization measures with the most favorable cost conditions. Therefore, the reliability-based optimization model may well improve the quality of rock slope stability analysis and reduce construction costs.

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