Annual Meeting of Korean Society of Coastal and Ocean Engineers
Ajou University, 19 September 1998

Diffraction-Induced Wave Setup inside Harbor

Oh, Tae-Myoung

Introduction

This paper identifies diffraction-induced wave setup when waves enter into a harbor of constant depth through a breakwater gap narrower than one wave length. It is well known, for this case, that the waves in the lee of the breakwater propagate as if from a point source and the wave crest lines are approximated well by semicircular arcs. Wave height decreases in accordance with conservation of energy; thereby resulting in an appreciable setup of the surface. Thompson and Hamon (1980) estimated diffraction-induced wave setup in a harbor as a function of the distance from the center of the gap and the phase of the tide in addition to the wave height at the center of the gap. They indicated that 3m waves would result in 0.65m of setup inside 10m deep harbor, which seemed to be unrealistic. Their results were strongly supported by McDougal and Slotta (1981). In this paper, however, it was shown that their results were overestimated by a factor of 3 because they did not consider the divergence of wave rays to calculate the radiation stresses and only used radiation stresses per unit width based on the energy flux conservation. Bernoulli equation and Longuet-Higgins and Stewart's results (1964) were simply applied to clarify this problem; which was also clarified by employing cylindrical coordinate system for this case.

Mechanism

Consider long waves normally entering into a harbor of constant depth through a breakwater gap narrower than one wave length, as shown in Figure 1. Assuming no reflection inside the harbor, the waves in the lee of the breakwater propagate as if from a point source so that the wave crests become semicircular arcs and the wave heights decrease as $r^{-1/2}$ with distance from the center of the gap in accordance with energy conservation. Balancing the forces, then, the following differential equation results

$$\frac{\partial \bar{\eta}}{\partial x} = -\frac{1}{\rho g(h+\bar{\eta})}\frac{\partial S_{xx}}{\partial x} \tag{1}$$

where, $\bar{\eta}$ is the water level setup, x the direction of wave propagation, ρ the water density, g the gravitational acceleration, h the water depth and S_{xx} is the onshore-directed

Marine Environment Research Team, Samwoo Environmental Consultant Corp.,
6th Fl., HengUn Building, 1056-17, Namhyun-Dong, Kwanak-Ku, Seoul

momentum flux. Using the shallow water asymptotes for the radiation stress terms, assuming $h+\bar{\eta} \approx h$, and expressing the wave height inside the harbor as $a^2 = a_0^2 \frac{W}{\pi x} \quad (x \geq \frac{W}{2})$ based on the energy conservation, where W is the width of the gap and a_0 is the wave amplitude at the gap, then the resulting setup is

$$\bar{\eta} = \frac{3a_0^2}{4h} \left(1 - \frac{W}{\pi x}\right) \quad \left(x \geq \frac{W}{2}\right) \tag{2}$$

As waves propagate far into the harbor (increasing distance from the gap), the water level approaches to the maximum setup value

$$\bar{\eta} = \frac{3a_0^2}{4h} \tag{3}$$

This maximum value would result in 0.65m of setup when 3m waves enter a 10m deep harbor, which seems to be unrealistic. Thompson and Hamon (also McDougal and Slotta) indicated that this might be rather extreme but storm waves might indeed put spikes in sea level records from tide gauges in the harbor.

Let's check the above results by applying Bernoulli equation. From the linear progressive wave theory, the mean sea level is

$$\bar{\eta} = -\frac{a^2 k}{2\sinh 2kh} + C \tag{4}$$

where k is the wave number. From the given condition that $a = a_0$ and $\bar{\eta} = 0$ at $x = 0$ (gap center), the mean sea level is given as

$$\bar{\eta} = \frac{a_0^2 k}{2\sinh 2kh} \left(1 - \frac{a^2}{a_0^2}\right) \tag{5}$$

With shallow water assumption ($\sinh 2kh \approx 2kh$) as waves propagate far into the harbor ($a \to 0$), the water level approaches

$$\bar{\eta} = \frac{a_0^2}{4h} \tag{6}$$

which is different from the mean water level given in Equation (3) by a factor of 3 and looks more realistic. This can be supported by using the results given by Longuet-Higgins and Stewart. From

$$\bar{\eta} = -\frac{1}{2g}\left(\overline{u^2} - \overline{w^2}\right) + C \tag{7}$$

where u and w are horizontal and vertical velocities of water particles, respectively. Substituting for u and w for the linear progressive waves and using the condition at $x = 0$ (gap center), we have

$$\bar{\eta} = \frac{a_0^2}{4h}\left(1-\frac{a^2}{a_0^2}\right) \tag{8}$$

which yields the same results as Equation (6) as waves propagate far into the harbor ($a \to 0$). Hence, we can conclude that the results given by Thompson and Hamon have mistakes.

For deriving Equation (2), Thompson and Hamon obtained the wave height along the axis of harbor by considering energy flux conservation and also calculated the radiation stresses by using those wave heights variations as

$$S_{xx} = \frac{3}{4}\rho g a_0^2 \frac{W}{\pi x} \tag{9}$$

However, the radiation stress is defined as the excess flux of momentum past a section in the fluid domain. Hence, we should consider the change in sectional area to get the radiation stress. Actually, Thompson and Hamon (also McDougal and Slotta) did not consider the divergence of wave rays with increasing distance from the center of the gap to calculate radiation stress terms; which is actually equal to the decrease in the momentum transfer zone after wave heights calculations. This results in unrealistically large value of wave setup at the boundary since large change in the momentum flux induces large change in the mean water level. In order to support the above argument, let's employ cylindrical coordinate system which seems naturally to consider the change in sectional area as radius increases.

Radiation Stresses in Cylindrical Coordinate

Considering cylindrical coordinate system, defined in Figure 1, radiation stresses can be expressed as follows:

$$\begin{aligned} \frac{\partial}{\partial r} S_{rr} + M_r + \frac{1}{r}\frac{\partial}{\partial \theta} S_{r\theta} &= -\rho g (h+\bar{\eta})\frac{\partial \bar{\eta}}{\partial r} \\ \frac{\partial}{\partial r} S_{r\theta} + M_\theta + \frac{1}{r}\frac{\partial}{\partial \theta} S_{\theta\theta} &= -\rho g \frac{(h+\bar{\eta})}{r}\frac{\partial \bar{\eta}}{\partial \theta} \end{aligned} \tag{10}$$

where,

$$\begin{aligned} S_{rr} &= E\,[\,n\{\cos^2(\theta-\psi)+1\} - \frac{1}{2}\,] \\ S_{r\theta} &= -E\; n\; \cos(\theta-\psi)\; \sin(\theta-\psi) \\ S_{\theta\theta} &= E\,[\,n\{\sin^2(\theta-\psi)+1\} - \frac{1}{2}\,] \\ M_r &= \frac{E}{r}\,[\; n-\sin^2(\theta-\psi)\;] \\ M_\theta &= -\frac{E}{r}\; n\; \sin 2(\theta-\psi) \end{aligned}$$

and ψ is the wave propagation angle from x-axis. Assuming shallow water asymptotes, $\theta=\psi$ for simplicity and $h \gg \bar{\eta}$, and using $a^2 = \frac{W}{\pi r} a_0^2$ with the condition at the gap ($a=a_0$ and $\bar{\eta}=0$ at $r=0$), then we have

$$\bar{\eta} = \frac{a_0^2}{4h}\left(1-\frac{a^2}{a_0^2}\right) \tag{11}$$

which is the same as Equation (8), as excepted. For θ-direction, $\frac{\partial \bar{\eta}}{\partial \theta}=0$; hence, wave setup is independent of θ-direction.

Discussions

It was shown, when waves enter normally into a harbor of constant depth through a breakwater gap narrower than one wave length, that previous researches overestimated the diffraction-induced wave setup by a factor of 3 by missing the change in the sectional area as waves propagate; which is actually equal to the decrease in the momentum transfer zone after wave heights calculations. These differences were simply clarified by applying Bernoulli equation for wave setup and Longuet-Higgins and Stewart's results for the radiation stress terms. When directly employed was cylindrical coordinate system which seemed to be natural for this problem, it could be seen that energy spreading itself did not change the momentum flux between two sections unless forces were applied. For this problem, wave setup turned out to be independent of θ-direction, as expected. Finally, it would be very important to employ appropriate coordinate system depending on the problem.

References

Longuet-Higgins, M.S. and Stewart, R.W. (1964), ''Radiation Stresses in Water Waves; a Physical Discussion with Applications'', Deep Sea Res., Vol. 2, pp. 529-563.

McDougal, W.G. and Slotta, L.S. (1981), ''Comment on 'Wave Setup of Harbor Water Levels' by R.O.R.Y. Thompson and B.V. Hamon'', J. Geophys. Res., Vol. 86, pp. 4309-4310.

Thompson, R.O.R.Y. and Hamon, B.V. (1980), ''Wave Setup of Harbor Water Levels'', J. Geophys. Res., Vol. 85, pp. 1151-1152.

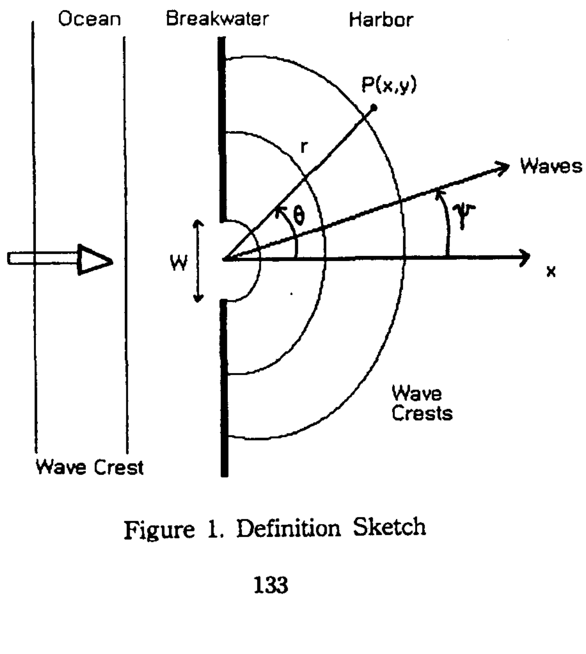

Figure 1. Definition Sketch