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A Fuzzy Vehicle Scheduling Problem

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Abstract In this paper, we consider a bi-objective vehicle routing problem to minimize total distance traveled and to maximize minimum integrated satisfaction level of selecting desirable routes in a fuzzy graph. The fuzzy graph reflects a real delivery situation in which there are a depot, some demand points, paths linking them, and distance and integrated satisfaction level are associated with each route. For solving the bi-objective problem, we introduce a concept of routing vector and define non-dominated solution for comparing vectors. An efficient algorithm involving a selection method of non-dominated solutions based on DEA is proposed for the vehicle routing problem with rigid distance and integrated satisfaction level.
Keywords: Vehicle routing, Bi-objective, Routing vector, DEA

1 Introduction

There are many factors to be considered in practice of Vehicle Routing Problem(**VRP**). In this research, however, we concentrate on two factors, namely multi-objectivity and fuzziness and/or uncertainty in vehicle routing. In the studies reported in the past, especially theoretical ones, a single objective is treated for optimization. In real-world systems, however, there exist many situations that a single objective function is not sufficient to characterize the problem completely. This indicates the necessity to study VRP on with more than two objectives.

Secondly, for many VRP encountered in practice, optimal decisions are usually made under some uncertainties, because practical problems are affected by ambiguous fluctuations of given conditions and/or by lacking necessary and certain information for routing. That is, in some situations, routing parameters are not rigid but fuzzy [1], and some violations of constraints may be accepted.

From these viewpoints, we investigate bi-objective vehicle routing problem **BVRP** with fuzzy parameters in this study. As the fuzzy parameters which are not constants but are decision variables, we introduce a decision maker's integrated satisfaction level for , eg., cost, distance, safety, preference etc, based on his subjectivity in selection of desirable travelling path. Fuzzy graph [2] is convenient to describe the above situations. After selecting one rigid parameter, the remained parameters can be integrated with a total satisfaction level. In this study, we consider the situation that there are a depot, some demand points, paths linking them and rigid delivery distance and integrated satisfaction level selecting travelling path are associated with each route.

Summarizing the above, the main purposes of this study are to propose a BVRP to minimize total distance traveled and to maximize minimum integrated satisfaction level with respect to desirable routes in a fuzzy graph and to develop efficient procedure for solving the problem. For solving the multi-objective problem, we introduce a concept of routing vector of which consists of two objectives as it's elements, and define non-dominated solution for comparing vectors. We propose an efficient algorithm to find non-dominated solutions and a selection method of more reasonable non-dominated solutions based on Cook and Kress's voting model fully utilizing DEA(Data Envelopment Analysis)[3].

2 Problem Formulation

(1) There are the depot $i=0$, n demand points $i=1,2,\cdots,n$ and arcs (i,j) $(i,j=0,1,\cdots,n, i\neq j)$ linking them.

(2) Each demand point i has a demand w_i.

(3) Total amount delivered by a single vehicle cannot exceed Q.

(4) Each demand point must be served by a single vehicle.

(5) rigid delivery distance d_{ij} and integrated satisfaction level μ_{ij} selecting travelling path are associated with each (i,j).

(6) The objective is to find a set of routes for the vehicles of minimal length to minimize total travelling distance and to maximize minimum integrated satisfaction level in the above fuzzy graph N.

We do not solve directly this problem **BVRP** but solve a series of subproblems defined as below.

Sorting different μ_{ij} in nonincreasing order, let

$$1 > \mu^0 > \mu^1 > \cdots > \mu^k > 0$$

here, k denotes number of different μ_{ij}. Thus the best solution among optimal solutions of each subproblem $\mathbf{BVRP_k}$ is an optimal solution of **BVRP** and the corresponding route is an optimal route.

Note that for any μ^c $(c = 0, 1, 2, \cdots, k)$, the route, to minimize total travelling distance, can be constructed by the classic Savings Algorithm [4].

3 Solution Procedure

The Modified Savings Algorithm

Step 0 : Sorting different μ_{ij} in nonincreasing order on fuzzy graph N, let

$$1 = \mu^0 > \mu^1 > \cdots > \mu^k = 0$$

here, k denotes number of different μ_{ij}.

Step 1 : Set $c = 0$, $DS = \{\emptyset\}$ and construct N^c for each arc $(i, j) \in N$ such that

$$d_{ij} = \begin{cases} d_{ij} & \mu_{ij} = \mu^c \\ 0 & i = j. \\ +\infty & \text{otherwise} \end{cases}$$

Step 2 : Start with the solution that has each customer visited by separate vehicle on N^c.

Step 3 : Calculate the savings $s_{ij} = d_{0i} + d_{j0} - d_{ij} \geq 0$ for all pairs of customers i and j.

Step 4 : Sort the savings in nonincreasing order.

Step 5 : Find the first feasible arc (i, j) in the savings list where
(1)i and j are on different routes.
(2)both i and j are either the first or last visited on their respective routes.
(3)the sum of demands of routes i and j i no more than Q.
Add arc (i, j) to the current solution and delete arcs $(0, i)$ and $(j, 0)$. Delete arc (i, j) from the savings list.

Step 6 : Repeat step 5 until no more arcs satisfy the conditions.

Step 7 : let obtained routes be R^c and routing vector

$$(TD^c = \sum_{(i,j)\in R^c} d_{ij}, \mu^c))$$

be DS^c.

Step 8 : Compare DS^c with DS^{c-1}. If DS^c dominate DS^{c-1},

$$DS = DS - DS^{c-1} + DS^c$$

Step 9 : if ck, $c = c + 1$ and go to step 1. Otherwise stop.

First of all, we sort all of satisfaction levels in nonicreasing order on graph N and construct a graph N^c consists with arcs with highest satisfaction level which weighted by d_{ij}. Next, we find shortest path matrix based on rigid distance on N^c by Warshall-Floyd Algorithm [5]. By solving the classic Savings Algorithm, and optimal routes and total distance traveled are obtained. Then Let (total distance traveled, satisfaction level) be routing vector as solution for subproblem. Next, we compare it and routing vector obtained after next iterating. For the comparing, we define routing vector V^π as a vector consisting of two elements, that is, TD^c and μ^c in some feasible route R^c. That is, $(TD^c = \sum_{(i,j)\in R^c} d_{ij}, \mu^c))$. For two vectors

$$V^1 = (v_1^1, v_2^1), \quad V^2 = (v_1^2, v_2^2),$$

such that

$$|V^1| = -w_1 v_1^1 + w_2 v_2^1, \quad |V^2| = -w_1 v_1^2 + w_2 v_2^2,$$

where, w_1, w_2 are positive weights, we call V_1 **dominate** V_2 when $|V^1| \geq |V^2|$.

From definition of $|V^1| \geq |V^2|$, we obtain

$$w_1(v_1^2 - v_1^1) + w_2(v_2^1 - v_2^2) \geq 0. \tag{1}$$

The condition of satisfying inequality (1) is that

$$v_1^1 \leq v_1^2,\ v_2^1 \geq v_2^2 \text{ and } |V^1| \neq |V^2|$$

Without loss of generality, we suppose that z nondominated solutions $x (= 1, 2, \cdots, z)$ are obtained by the above dominated definition and s elements $v_y (y = 1, 2, \cdots, s)$ of routing vector V are ranked in decreasing order of importance by decision maker. The problem to select reasonable non-dominated solutions is likely to obtain overall desirability index $\theta_{x_o x_o}$ for each nondominated solutions x_o and is formulated as follows.

$$\textbf{SM}: \quad \begin{array}{lll} \theta_{x_o x_o}(\epsilon) = & \text{Maximize} & \sum_{y=1}^{s} w_y v_{x_o y} \\ & \text{subject to} & \sum_{y=1}^{s} w_y v_{xy} \leq 1 \\ & & (x = 1, 2, \cdots, z) \\ & & w_y - w_{y+1} \geq d(y, \epsilon) \\ & & (y = 1, 2, \cdots, s-1) \\ & & w_s \geq d(s, \epsilon) \end{array} \tag{2}$$

Where, w_y, $d(y, \epsilon)$ and v_{xy} denote weight reflecting importance objective, a nonnegative function to be nondecreasing in ϵ(discrimination intensity function) and objective v_y's value for schedule x, respectively.
Note that this problem is equivalent to the well known DEA-Assurance Region Model. See [3].

4 Conclusion

we proposed a selection method to find reasonable non-dominated routes based on Cook and Kress's voting model in VRP.

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