The Third AFSS(1998), 663−665

Fuzzy Random Facility Location Problems

Hiroaki Ishii, Takeshi Itoh and Hideki Katagiri

Faculty of Eng., Osaka Univ.

Yamadaoka, Suita, Osaka 565, Japan

Abstract. This paper investigates a facility location problem where there are n possible demand points with demand occuring probabilities and actual deistances between these points and the facility site to be determined are ambiguous. Further we define the fuzzy goal with respect to the maximum value among the actual distances between demand points and the facility. We determine the site of facility maximizing the minimal satisficing degree under the chance constraint. We propose the geometric algorithm to find this optimal site.

Keywords. Facility location, Fuzzy distance, Random demand point

1 Introduction

We investigate the optimal site of an emergency facility under probabilistically occuring demands and fuzzy actual distances. This model is the generalization of the our former model ([5])where there exist n demand points in the plane and a fuzzy goal with respect to the maximal distance among all demands points and the facility site. But in the model [5], each demand oocurs certainly and not probabilistically. Our current model in this paper includes both stochastic and fuzzy elements, that is, demand occurs stochastically and distance is fuzzy. Section 2 formulates the problem. Section 3 presents a procedure to obtain an optimal site of the facility . Section 4 investigates the further research problems including the possibility of other facility models .

2 Problem formulation

We consider the following facility location problem.

1. There are n points $P_1 = (x_1, y_1), P_2 = (x_2, y_2)$, $\cdots, P_n = (x_n, y_n)$ in a plane and demand occurs at each point P_i with probability $p_i > 0, i = 1, \ldots, n$ where $p_1 + p_2 + \cdots + p_n = 1$.
2. Physical distance d_i^0 between each point P_i and the emergency facility site $P = (x, y)$ to be determined is measured by the rectilinear distance, i.e., $|x - x_i| + |y - y_i|, i = 1, 2, \ldots, n$.
3. Actual distance $\tilde{D}_i$ between P_i and the facility $P = (x, y)$ is triangular fuzzy number with the following membership function $\mu_{\tilde{D}_i}(d_i)\ i = 1, 2, \ldots, n$:

$$\mu_{\tilde{D}_i}(d_i) = \begin{cases} 1 - \dfrac{d_i^0 - d_i}{f_i} & (d_i^0 - f_i \leq d_i \leq d_i^0) \\ 1 - \dfrac{d_i - d_i^0}{g_i} & (d_i^0 \leq d_i \leq d_i^0 + g_i) \\ 0 & \text{otherwise} \end{cases}$$

where $f_i \leq d_i^0$.

4. Fuzzy goal G with the following membership function is defined with respect to the maximum value d among actual distances $d_1, d_2, \cdots, d_n$:

$$\mu_G(d) = \begin{cases} 1 & (d \leq d_e) \\ 1 - \dfrac{d - d_e}{d_\ell - d_e} & (d_e \leq d \leq d_\ell) \\ 0 & (d \geq d_\ell) \end{cases}$$

The membership function of the fuzzy goal G represents the satisfaction degree of the maximum actual distance d among $d_1, d_2, \cdots, d_n$ where we assume $d_e > 0$.

5. Under the condition that **probability** {a possibility of satisfaction degree with respect to maximum actual distance d among $d_i, i = 1, 2, \ldots, n$ is less than h} is not less than α, we seek the site of facility maximizing h where $1 \geq \alpha \geq 0$(fixed) and $1 \geq h \geq 0$ to be maximized.

Then we have the following equivalent problem **P** by some transformations.

P:Maximize h

subject to $Pr[(1-h)g_i + d_i^0 \geq d_i \geq d_i^0 - f_i(1-h),$

$d_i \leq (1-h)(d_\ell - d_e) + d_e, i = 1,2,\ldots,n]$

$\geq \alpha, \qquad (1)$

$1 \geq h \geq 0.$

3 Solution Procedure

The condition that

$$(1-h)g_i + d_i^0 \geq d_i \geq d_i^0 - f_i(1-h),$$

$$d_i \leq (1-h)(d_\ell - d_e) + d_e, i = 1,2,\ldots,n$$

in the above Pr is equivalent to the following condition:

$$(1-h)g_i + d_i^0 \geq (1-h)(d_\ell - d_e) + d_e$$

$$\geq d_i^0 - (1-h)f_i.$$

$$\rightarrow (1-h)(d_\ell - d_e + f_i) + d_\ell \geq d_i^0$$

$$\geq (1-h)(d_\ell - d_e - g_i) + d_e. \qquad (2)$$

Now we define $d_i^u(h) = (1-h)(d_\ell - d_e + f_i) + d_e$ and

$d_i^\ell(h) = max\{(1-h)(d_\ell - d_e - g_i) + d_e, 0\}$.
Then the following property holds.

Theorem 1 *From the definition of* $d_i^u(h)$ *and* $d_i^\ell(h)$, *we have*

1. $d_i^u(h)$ *is an decreasing function of* h *and less than* d_ℓ.
2. *If* $d_\ell - g_i \geq d_e$, *then* $d_i^\ell(h)$ *is positive and nonincreasing function of* h.
3. *If* $0 \leq d_\ell - g_i < d_e$, *then* $d_i^\ell(h)$ *is increasing function of* h *and positive* .
4. *If* $0 \geq d_\ell - g_i$, *then* $d_i^\ell(h)$ *is nondecreasing function of* h *and zero when*

   $0 \leq h \leq \frac{g_i - d_\ell}{g_i + d_e - d_\ell}$ *(we denote it with* h_i*)*

   and positive when $h_i < h \leq 1$.
5. $d_i^u(h) - d_i^\ell(h)$ *is a decreasing function of* h.

Proof: Above three facts are directly derived from definitions. Final one is proved from that

$$d_i^u(h) - d_i^\ell(h) = min\{(1-h)(f_i + g_i),$$
$$(1-h)(d_\ell - d_e + f_i) + d_e\},$$

and both terms in the parethesis of min is positive and decreasing function of h. **Q.E.D.**

We assume without loss of generality that
$g_1 \leq g_2 \leq \cdots \leq g_n$
by renumbering if necessary. Further let the rectilear distance between possible demand points i and j be $d_{ij} = |x_i - x_j| + |y_i - y_j|$ for $i \neq j$ and

$$R_i(h) = \{(x,y)|d_i^\ell(h) \leq |x - x_i| + |y - y_i| \leq d_i^u(h)\}.$$

Then we seek the site $(x*, y*)$ of the facility maximizing h under the condition that

$$\textstyle\sum_{i \in I_{(x,y)}(h)} p_i \geq \alpha$$

where $I_{(x,y)}(h) = [i \in \{1,2,\ldots,n\}|(x,y) \in R_i(h)]$.

The condition that intersection region between $R_i(h)$ and $R_j(h)$ is not empty is:

$$d_i^u(h) + d_i^u(h) \geq d_{ij} \qquad (3)$$

and

$$d_{ij} \geq d_i^\ell(h) - d_j^u(h), d_j^\ell(h) - d_i^u(h) \qquad (4)$$

Since $d_i^u(h) > d_j^\ell(h)$ for any i,j from definitions of $d_i^u(h)$ and $d_i^\ell(h)$, the condition (4) holds automatically and so we only consider the condition (3). Condition (3) can be rewritten as follows:

$$(1-h)\{2(d_\ell - d_e) + f_i + f_j\} \geq d_{ij} \qquad (5).$$

From (5), we define

$$h_{ij} = 1 - \frac{d_{ij}}{2(d_\ell - d_e) + f_i + f_j}$$

for $i \neq j$. Then if $h \leq h_{ij}$, $R_i(h)$ and $R_j(h)$ intersect each other. Now we sort $0 \leq h_{ij}$ and let the result be

$$h^0 = 0 < h^1 < \cdots < h^q < h^{q+1} = 1$$

where q is the number of different $0 < h_{ij}$. Under the above preparation, we are ready to propose our solution procedure.

Solution Procedure

Step 1: Set $k_\ell = 0$, $k_u = q+1$ and $k = [\frac{k_\ell + k_u}{2}]$

where $[\cdot]$ is the Gauss notation, i.e., it means the greatest integer not greater than $\cdot$.

Go to Step 2.

Step 2: If there exists a point (x,y) such that

$\sum_{i \in I_{(x,y)}(h_k)} p_i \geq \alpha$,

set $k_\ell = k$, $x(k) = x, y(k) = y$ and go to step 3.

Otherwise, set $k_u = k$ and go to step 3.

Step 3: If $k_u = k_\ell + 1$, then go to step 4. Otherwise,

set $k = \lfloor \frac{k_\ell + k_u}{2} \rfloor$ and return to Step 2.

Step 4: Let Optimal solution of our problem be

$(x^*, y^*) = (x(k_\ell), y(k_\ell))$ and terminate.

Theorem 2 *Above procedure find an optimal site of the facility in at most $O(n^2 log\ n)$ computational time.*

Proof:
(**Validity**) First note that if $h > h_{ij}$, region $R_i(h)$ and $R_j(h)$ does not intersect and so optimal value of the problem is one of h^k. This shows the validity of our procedure.

(**Complexity**) We count computational time of each operation in the solution procedure.

(Calculation of h^k)$\cdots$ Since number of h_{ij} is $O(n^2)$, calculation of h_{ij} takes $O(n^2)$ computational time and sorting of them $O(n^2 log\ n)$ computational time. Totally, it takes $O(n^2 log\ n)$ computational time.

(Iteration number of each step)

Step 1 $\cdots$ $O(1)$, Step 2 and Step 3 $\cdots$

$O(log\ n^2) = O(log\ n)$

since our procedure adopts binary search.

Step 4 $\cdots$ $O(1)$.

(Total Complexity)

Each iteration of Step 2 takes $O(n^2)$ computational time since it checks the situation of intersection among more than 3 regions. So execusion of Step 2 takes $O(n^2 log\ n)$ computational time and it dominates the computational time of execusions of other steps.

Therefore in total, complexity of our solution procedure is at most $O(n^2 log\ n)$. **Q.E.D.**

4 Conclusion

We have considered a facility location problem with random demands and fuzzy distances. Though our model is simple, its solution procedure is complicated and based on geometrically checking the intersection of regions. I think our model is only a stepstone of fuzzy random facility location models and may be first trial of those models. We are now attacking the more general model, which is a random version of our previous fuzzy facility location model ([3]) with fuzzy demand regions based on possibility distribution. Of course there exist many other posibility of fuzzy random facility location model. For example, the model that demand occurs any place of the some region randomly also should be considered.

References

[1] U. Bhattacharya, J. R. Rao and N. R. Tiwari. Fuzzy Multi-criteria Facility Location Problem, Fuzzy Sets and Systems 51 (1992) 277-287.

[2] N. Furukawa. A Parametric Total Order on Fuzzy Numbers and a Fuzzy Shortest Route problem, Optimization 10 (1994) 367-377.

[3] H.Ishii and T. Matutomi, Fuzzy Facility Location Problems: Oral presentation of APORS'97 (1997).

[4] A.Kaufmann, M.M.Gupta, Introduction to Fuzzy Arithmetic: Theory and Applications (Van Nostrand Reinhold, New York, 1985).

[5] T. Matsutomi and H. Ishii. Fuzzy Facility Location Problem with Asymmetric Rectilinear Distance, Journal of the Japanese Fuzzy association 8 (1996) 57-64 (in Japanese).

[6] L. A. Zadeh. Fuzzy Sets, Information and Control 8 (1965) 338-353.