The Third AFSS(1998), 568—572

THE CONSTRUCTIVE METHOD OF FUZZY RULES OF A CLASS OF DATA

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Abstract
This paper defines Fuzzy Logic Units(FLUs) which are piece wise finite elements in multi-dimension Euclidean space, and redefines triangular membership functions which are different from those defined in traditional literature[4]. By analyzing FLUs, this paper gives a constructive method of fuzzy rules in fuzzy logic systems based on finite element method. The simulation results of single-machine to infinite bus system show the effectiveness of the proposed method in this paper.
Keywords Membership Function, Fuzzy Control, Fuzzy Model, Interpolation, Excitation Control

1. Introduction

Fuzzy controller whose kernel is fuzzy rules plays a decisive role in fuzzy control systems, therefore, how to generate fuzzy rules, from experiment data and information given by the skilled workers, is a key problem concerned by engineers in fuzzy control field. A lot of authors have made contribution to fuzzy rules[1][2][3][4][5][6][7][8]. [6] puts forward fuzzy rules with one modified factor, [7] fuzzy rules with two modified factors, [8] fuzzy rules with self-tuning. [4] proposed a promising and creative method generating fuzzy rules from the given data and information about the studied plant. However, the algorithm needs long training or learning time and the obtained results have bad accuracy. Especially, for some specified problem, the number of fuzzy rules, the type of membership function, the partition of fuzzy interval and so on, have little fundamental theory, are still problems to be solved. This paper gives a constructive method of fuzzy rules in fuzzy logic systems based on finite element method. The simulation results of single-machine to infinite bus system shows the effectiveness of the proposed method in this paper.

2 Basic Concepts

2.1 Basic Definition

Support, Center, and Fuzzy Singleton: The **support** of a fuzzy set F is the crisp set of all points $u \in U$ such that $\mu_F(u)>0$. The **center** of a fuzzy set F is the points(s) $u \in U$ at which $\mu_F(u)$ achieves its maximum value. If the support of a single point in U at which $\mu_F = 1$, the F is called a **fuzzy singleton**.

Regular Fuzzy set: a Fuzzy set with only one singleton is called **regular fuzzy set**, that is, **one-humped fuzzy set**.

Let $A = \{A_i\}_{(i=0,1,\ldots,n)}$ be a class of regular fuzzy sets defined on X, Their singletons: x_i $(i = 0, 1, \ldots, n)$ (that is the points of $A_i(x)=1$), A is called fuzzy partition, if A satisfies the condition: (1) $(\forall\ i,j)(x_i \neq x_j)$; (2) $(\forall x \in X)(\sum_{i=0}^{n} A_i(x) = 1)$, then every A_i is called a basic element of A, A is called basic element group. Especially, A is called a two-phase basic element group, if for any given $x \in X$, there exists a pair of two neighbor basic elements A_i and A_{i+1} of A such that $A_i(x) \neq 0 \neq A_{i+1}(x)$.

2.2 The Method of Fuzzy Interval Partition and Membership Function Selection

Fuzzy interval partition for one input should satisfies the following conditions:

(1) For a given close interval [a,b], the given data are arranged in the order of $a = x_1 < x_2 < \ldots < x_n = b$, $x_0, x_1, x_2, \ldots, x_n$ forms the centers corresponding to the conventional fuzzy sets;

(2) Two fuzzy sets, whose membership functions are triangular, forming a pair of two-phase basic elements, are defined in every interval of [a,b] as shown in Fig.1. Note that triangular membership function here is different from one in almost all ordinary literature.

Triangular membership functions are defined in traditional method: the first $\Delta a.x_0.x_1$, the second $\Delta x_0.b.x_2$, the third $\Delta x_1.c.x_3$, ... Triangular membership functions are defined in the method in this paper: the first $\Delta a.x_0.x_1$, the second $\Delta x_0.b.x_1$, the third $\Delta x_1.b.x_2$, ...

The regular fuzzy sets satisfying the above conditions can guarantee simple defuzzification as shown in the following section.

3 The Construction of Fuzzy Rules

3.1 The Construction of FLU Fuzzy Rules of One-dimension input System

FLUs are defined as shown in Fig.1 in the case of single-input and single-output system, here, we analyze a FLU, which obtained from the partition in Fig.1, as shown in Fig.2. Given two points: $(x_0; f_0)$ and $(x_1; f_1)$. Lagrange interpolation formula of the segment between points x_0 and x_1 is expressed as follows :

$$f = \frac{x-x1}{x0-x1} f_0 + \frac{x-x0}{x1-x0} f_1 = \mu(A_0(x)) f_0 + \mu(A_1(x)) f_1 = F_0(x) + F_1(x) \qquad (1)$$

A_0 and A_1 are two fuzzy subsets , $\mu(A_0(x))$ and $\mu(A_1(x))$ are membership function of fuzzy subsets A_0 and A_1 respectively , thus two fuzzy rules can be obtained according to Equ.(1):

R^1: if $x = A_0$ then $f = f_0$
R^2: if $x = A_2$ then $f = f_1$ (2)

We have defuzzification formula (3) in terms of the gravity method :

$$f = (\mu(A_0(x))f_0 + \mu(A_1(x))f_1) / (\mu(A_0(x)) + \mu(A_1(x))) \qquad (3)$$

Because A_0 and A_1 are two-phase basic elements , that is : $\mu(A_0(x)) + \mu(A_1(x)) = 1$, Equ. (1) is the same equation as Equ. (3).

3.2 The Construction of FLU Fuzzy Rules of Two-dimension Inputs System

For $f = F(x, y)$, given four points $(x_0, y_0; f_{00})$, $(x_0, y_1; f_{01})$, $(x_1, y_0; f_{10})$ and $(x_1, y_1; f_{11})$, Lagrange interpolation formula within the rectangular as shown in Fig.3 is expressed as follows :

$$f = \frac{x-x1}{x0-x1}\frac{y-y1}{y0-y1} f_{00} + \frac{x-x1}{x0-x1}\frac{y-y0}{y1-y0} f_{01} + \frac{x-x0}{x1-x0}\frac{y-y1}{y0-y1} f_{10} + \frac{x-x0}{x1-x0}\frac{y-y0}{y1-y0} f_{11}$$
$$= F_1(x, y, xy) + F_2(x, y, xy) + F_3(x, y, xy) \qquad (4)$$

According to fuzzy partition in Fig.3 and interpolation calculation of Equ. (4), we can obtain the following 4 fuzzy rules :

R^1: if x is A_0 and y is B_0 then $f = f_{00}$
R^2: if x is A_0 and y is B_1 then $f = f_{01}$
R^3: if x is A_1 and y is B_0 then $f = f_{10}$
R^4: if x is A_1 and y is B_1 then $f = f_{11}$
(5)

Defuzzification results of Equ.(5) are the same equation as Equ.(4) in terms of the gravity method, because for any x, y defined in the rectangular $\frac{x-x1}{x0-x1}\frac{y-y1}{y0-y1} + \frac{x-x1}{x0-x1}\frac{y-y0}{y1-y0} + \frac{x-x0}{x1-x0}\frac{y-y1}{y0-y1} + \frac{x-x0}{x1-x0}\frac{y-y0}{y1-y0} = 1$ always exists.

3.3 The Construction of FLU Fuzzy Rules of Three-dimension Inputs System

For $f = F(x, y, z)$, given eight points $(x_0, y_0, z_0; f_{000})$, $(x_0, y_0, z_1; f_{001})$, $(x_0, y_1, z_0; f_{010})$, $(x_0, y_1, z_1; f_{011})$, $(x_1, y_0, z_0; f_{100})$, $(x_1, y_0, z_1; f_{101})$, $(x_1, y_1, z_0; f_{110})$, and $(x_1, y_1, z_1; f_{111})$. Lagrange interpolation formula in the defined hyper rectangular is :

$$f = \frac{x-x1}{x0-x1}\frac{y-y1}{y0-y1}\frac{z-z1}{z0-z1} f_{000} + \frac{x-x1}{x0-x1}\frac{y-y1}{y0-y1}\frac{z-z0}{z1-z0} f_{001} + \frac{x-x1}{x0-x1}\frac{y-y0}{y1-y0}\frac{z-z1}{z0-z1} f_{010} + \frac{x-x1}{x0-x1}\frac{y-y0}{y1-y0}\frac{z-z0}{z1-z0} f_{011} +$$

$$\frac{x-x0}{x1-x0}\frac{y-y1}{y0-y1}\frac{z-z1}{z0-z1}f_{100}+\frac{x-x0}{x1-x0}\frac{y-y1}{y0-y1}\frac{z-z0}{z1-z0}f_{101}+\frac{x-x0}{x1-x0}\frac{y-y0}{y1-y0}\frac{z-z1}{z0-z1}f_{110}+\frac{x-x0}{x1-x0}\frac{y-y0}{y1-y0}\frac{z-z0}{z1-z0}f_{111}=F_0(x,y,z,xy,xz,yz,xyz)\,F_1(x,y,z,xy,xz,yz,xyz)+F_2(x,y,z,xy,xz,yz,xyz)+F_3(x,y,z,xy,xz,yz,xyz)+F_4(x,y,z,xy,xz,yz,xyz)+F_5(x,y,z,xy,xz,yz,xyz)+F_6(x,y,z,xy,xz,yz,xyz)+F_7(x,y,z,xy,xz,yz,xyz) \quad (6)$$

According to fuzzy partition in Fig.4 and interpolation calculation of Equ. (5), we can obtain the following 8 fuzzy rules :

R^1: if x is A_0 and y is B_0 and z is C_0 then $f = f_{000}$
R^2: if x is A_0 and y is B_0 and z is C_1 then $f = f_{001}$
R^3: if x is A_0 and y is B_1 and z is C_0 then $f = f_{010}$
R^4: if x is A_0 and y is B_1 and z is C_1 then $f = f_{011}$
R^5: if x is A_1 and y is B_0 and z is C_0 then $f = f_{100}$
R^6: if x is A_1 and y is B_0 and z is C_1 then $f = f_{101}$
R^7: if x is A_1 and y is B_1 and z is C_0 then $f = f_{110}$
R^8: if x is A_1 and y is B_1 and z is C_1 then $f = f_{111}$

(6)

Defuzzification results of Equ.(6) are the same equation as Equ.(5) in terms of the gravity method, because for any x, y, z defined in hyper rectangular,

$$\frac{x-x1}{x0-x1}\frac{y-y1}{y0-y1}\frac{z-z1}{z0-z1}+\frac{x-x1}{x0-x1}\frac{y-y1}{y0-y1}\frac{z-z0}{z1-z0}+\frac{x-x1}{x0-x1}\frac{y-y0}{y1-y0}\frac{z-z1}{z0-z1}+\frac{x-x1}{x0-x1}\frac{y-y0}{y1-y0}\frac{z-z0}{z1-z0}+\frac{x-x0}{x1-x0}\frac{y-y1}{y0-y1}\frac{z-z1}{z0-z1}+\frac{x-x0}{x1-x0}\frac{y-y1}{y0-y1}\frac{z-z0}{z1-z0}+\frac{x-x0}{x1-x0}\frac{y-y0}{y1-y0}\frac{z-z1}{z0-z1}+\frac{x-x0}{x1-x0}\frac{y-y0}{y1-y0}\frac{z-z0}{z1-z0}=1$$

always exists 。

For multi-dimension inputs hyper rectangular, fuzzy rules is constructed according to above same method, here it is omitted 。

3.4 The Number of Fuzzy Rules

The fuzzy rues obtained from one, two and three-dimension inputs system show that the essence of the fuzzy rules in the fuzzy system is piece wise interpolation. For a specified problem, the first step is to form many FLUs finite element according to the given data; the second step is to construct fuzzy rules by the triangular membership functions defined in this paper for every FLU; the last step is to defuzzy a fuzzy rules obtained above. We have the following discussion about the number of fuzzy rules of a studied system :

3.4.1 Single-input and single-output system

$y = f(x)$, for $x \in [a,b]$, singletons are: x_0, x_1, x_2, ..., x_n, the number of FLU contained in a given system is n, every FLU contains 2 fuzzy sets, so, the number of fuzzy rules generated from the given data is $2^1 \times n$ 。

3.4.2 Two-input and single-output system :

$y = f(x, y)$, the number of singletons of x is n+1, the number of singletons of y is m+1, the number of fuzzy rules generated from the given data : $2^2 \times n \times m$ 。

3.4.3 Three-input and single-output system :

the number of singletons of x is n+1, the number of singletons of y is m+1, the number of singletons of z is k+1, the number of fuzzy rules generated from the given data : $2^3 \times n \times m \times k$ 。

3.4.4. Multi-input and single-output system :

$y = f(x_1, x_2, ..., x_n)$ the number of singletons of x_1 is m_1+1, the number of singletons of x_2 is m_2+1, ··· the number of singletons of x_n is m_n+1, the number of fuzzy rules generated from the given data is $2^n \times m_1 \times m_2 \times \cdots \times m_n$ 。

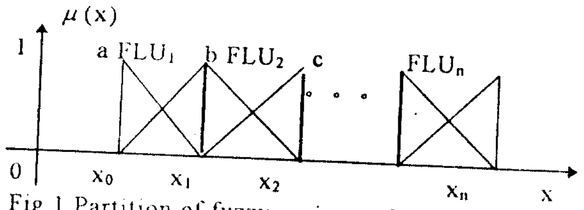

Fig.1 Partition of fuzzy region and selection of membership function

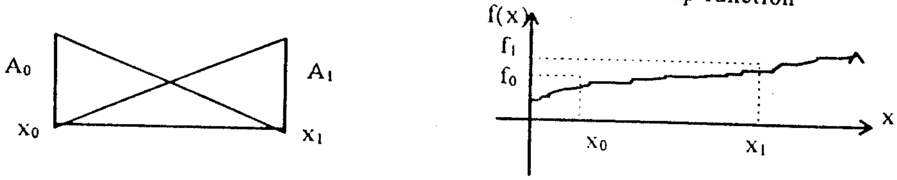

Fig.2 One-dimension FLU Unit

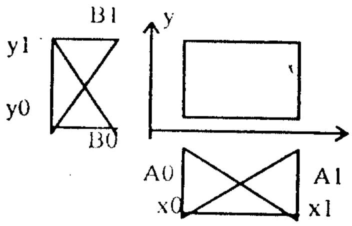

图 3 Two-dimension FLU Unit

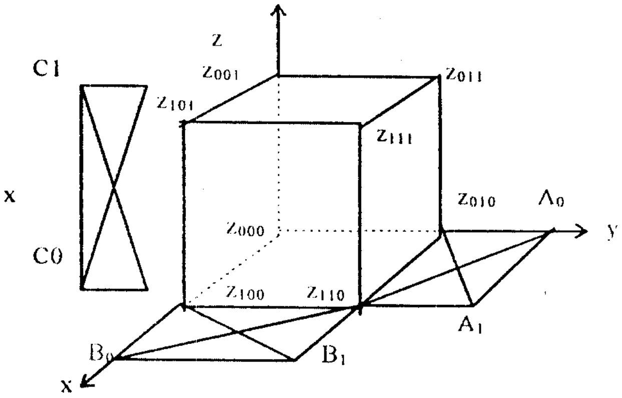

Fig.4 Three-dimension FLU Unit

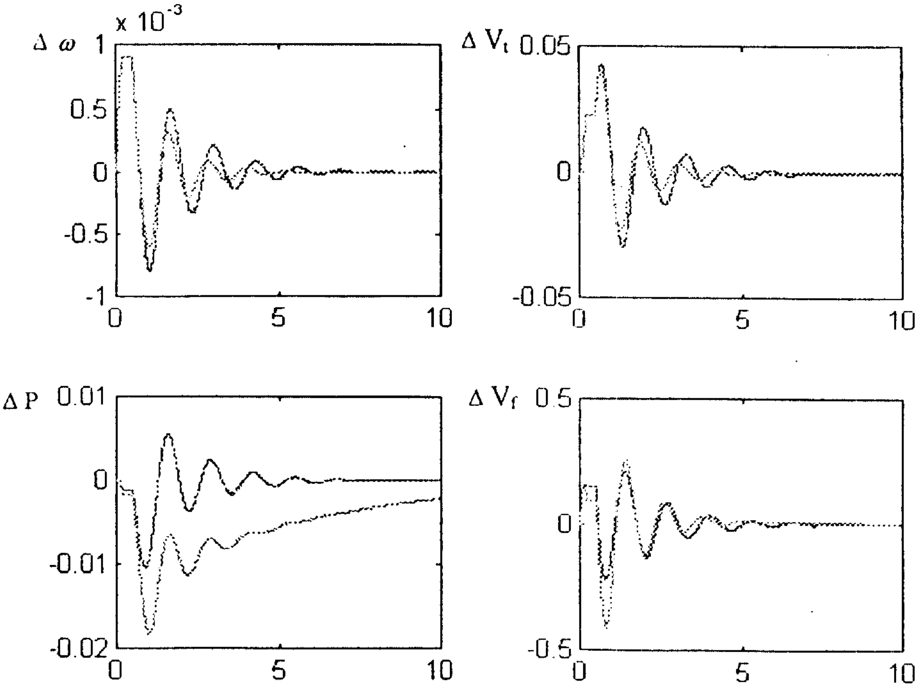

Fig.5 Dynamic response comparison of two excitation controller

4 The Simulation Example

This paper employs single-machine infinite bus system to simulate the fuzzy excitation controller, 48 FLUs and 192 fuzzy rules are constructed using Table-1 sample data [9] obtained in terms of optimal control theory. The simulation of fuzzy controller is compared with those of optimal controller in two kinds of operation conditions, that is, sample condition and no sample condition. The results show that two kinds of controller are same on the sample condition, and optimal controller is better than or close to fuzzy controller as shown in Fig.5.

5 conclusions

The constructive method of fuzzy rules of a class of data based on finite element method has the following advantages: (1) fuzzy model don't loss any information in the process of defuzzificaon; (2) contradiction fuzzy rules don't appear; (3) the fuzzy rules are uniqueness and the number of fuzzy rules is constant for the fuzzy partition of the given data in term of Polynomial Interpolation Theorem

The generalization ability of fuzzy rule-based method is determined by the given data and their partition method.

The example of single-machine to infinite bus system shows the proposed method constructing fuzzy rules in this paper is effective.

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