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AN INTERPOLATIVE FUZZY INFERENCE METHOD AND ITS APPLICATION

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Abstract

This paper deals with our proposed fuzzy inference method, in which the fuzzy relation is represented by the membership functions of the antecedent and consequent parts, it is not used any fuzzy composition. The strong point of this method is that the membership function of an inferred conclusion has a simple shape and thus its meaning can be interpreted easily. Firstly, the proposed method is explained, and then it is applied to fuzzy modeling of distributed data.

Keywords: Fuzzy inference, Fuzzy relation, Fuzzy modeling, Distributed data.

1. Introduction

In usual fuzzy inference methods[1-5], an inferred conclusion will have a complex shape and it is difficult to interpret its meaning as it stands. A representative value of its membership function (which is normally calculated by the center-of-gravity method, etc.) is generally used as the inference output. Therefore, the inferred conclusion using such methods handles a real number only, not a fuzzy set.

To cope with such problems, there are some approaches, which use interpolative technique. Kóczy and Hirota proposed a fuzzy inference method called a linear interpolative method [6, 7], which linearly interpolates a conclusion using distance of input variable space. Especially, this method mentioned to a sparse fuzzy rule base. When membership functions of fuzzy rules and input fuzzy sets are triangular type, in this method, the inferred conclusion will be also triangular type. However, it was found that the inferred conclusions by this method are sometimes abnormal fuzzy sets [8, 9].

We proposed an interpolative fuzzy inference method [10], which doesn't use a fuzzy implication to transform fuzzy rules to a fuzzy relation. The proposed method assumes that the fuzzy sets in the consequent part of fuzzy rules are defined by membership functions, which depend on some parameters. The fuzzy relation is defined by these membership functions with antecedent part membership functions. The inferred conclusion using the proposed method has a membership function of simple shape. Hence, its meaning can be easily interpreted. If the fuzzy sets in the consequent part of the fuzzy rules represent possibility distributions, this fuzzy inference method will be able to perform as a possibility distribution model [11].

Firstly, the proposed method is explained, and then it is applied to fuzzy modeling of distributed data. This modeling result shows the effectiveness of this method.

2. Proposed fuzzy inference method

In this paper, the proposed fuzzy inference method is explained by means of n fuzzy rules, m inputs and one output:

$$R_i:\ \textit{IF } x_1 \textit{ is } A_{i1} \textit{ and} \cdots \textit{and } x_m \textit{ is } A_{im} \quad \textit{THEN } y \textit{ is } B_i \quad (i=1,\ldots,n),$$

where $x_1,\ldots,x_m$ are input variables and y is an output variable. $A_{i1},\ldots,A_{im},B_i$ represent fuzzy sets. The fuzzy sets B_i $(i=1,\ldots,n)$ are defined by the following parameters: r position parameters and s height parameters. They are expressed in this way:

$$B_i=\{p_{i1},\ldots,p_{ir},h_{i1},\ldots,h_{is}\} \qquad (i=1,\ldots,n). \quad (1)$$

The fuzzy sets B_i are characterized by a membership function f:

$$\mu_{B_i}(y)=f(p_{i1},\ldots,p_{ir},h_{i1},\ldots,h_{is},y) \qquad (i=1,\ldots,n). \quad (2)$$

It is necessary that the consequent parts of all rules are defined by the same membership function f. The membership function f is called a definition membership function.

As an example, a triangular type membership function $f_\Delta(p_1, p_2, p_3, h_1, y)$ which depends on three position parameters and one height parameter is shown in Fig. 1.

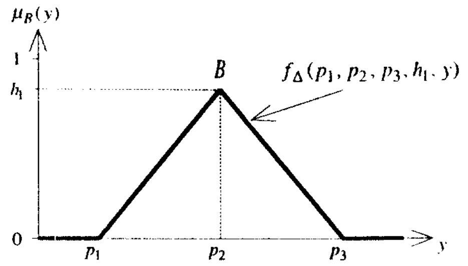


Fig. 1 Example of the definition membership function, triangular type, $B = \{p_1, p_2, p_3, h_1\}$.

In this method, the fuzzy relation R is defined by the definition membership function f. The fuzzy relation membership function depends on r weighted averages of each position parameter and s weighted sums of each height parameter, with compatibility degrees of membership functions in antecedent parts for each fuzzy rule as weights. Let the input variables set be $\mathbf{x} = \{x_1, \ldots, x_m\}$, then the fuzzy relation R is expressed by the following membership function $\mu_R(\mathbf{x}, y)$:

$$\mu_R(\mathbf{x}, y) = f(p_1^*(\mathbf{x}), \ldots, p_r^*(\mathbf{x}), h_1^*(\mathbf{x}), \ldots, h_s^*(\mathbf{x}), y) \wedge 1 \tag{3}$$

$$p_k^*(\mathbf{x}) = \frac{\sum_{i=1}^{n} w_i(\mathbf{x}) \cdot p_{ik}}{\sum_{i=1}^{n} w_i(\mathbf{x})} \qquad (k = 1, \ldots, r),$$

$$h_\ell^*(\mathbf{x}) = \sum_{i=1}^{n} w_i(\mathbf{x}) \cdot h_{i\ell} \qquad (\ell = 1, \ldots, s),$$

$$w_i(\mathbf{x}) = \prod_{j=1}^{m} \mu_{A_{ij}}(x_j),$$

where $\mu_{A_{ij}}(x_j)$ are membership functions which characterize the fuzzy sets in the antecedent part of the fuzzy rules.

Given the input variables x_j $(j = 1, \ldots, m)$ by fuzzy sets A'_j as inputs, the output B' is obtained by sup-min composition ($\circ$) of the inputs A'_j $(j = 1, \ldots, m)$ and the fuzzy relation R:

$$B' = (A'_1 \times \cdots \times A'_m) \circ R. \tag{4}$$

Its membership function can be expressed as following:

$$\mu_{B'}(y) = \sup_{x_1, \ldots, x_m} \left[\left(\bigwedge_{j=1}^{m} \mu_{A'_j}(x_j) \right) \wedge \mu_R(\mathbf{x}, y) \right]. \tag{5}$$

3. Fuzzy inference process

In this section, the fuzzy inference process is explained for two cases: real numbers input case and fuzzy sets input case.

3.1 *Real numbers input case*

Let's consider that the input value of each variable x_j $(j = 1, \ldots, m)$ is a real number x'_j. The real number x'_j is expressed by a singleton type membership function:

$$\mu_{A'_j}(x_j) = \begin{cases} 1 & \text{if } x_j = x'_j \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Let the input real numbers be $\mathbf{x}' = \{x'_1, \ldots, x'_m\}$, then the output B' is obtained from Eq. (5) as following:

$$\begin{aligned} \mu_{B'}(y) &= \sup_{x_1, \ldots, x_m} \left[\left(\bigwedge_{j=1}^{m} \mu_{A'_j}(x_j) \right) \wedge \mu_R(\mathbf{x}, y) \right] \\ &= \mu_R(\mathbf{x}', y) \\ &= f(p_1^*(\mathbf{x}'), \ldots, p_r^*(\mathbf{x}'), h_1^*(\mathbf{x}'), \ldots, h_s^*(\mathbf{x}'), y) \wedge 1, \end{aligned} \tag{7}$$

$$B' = \left\{ p_1^*(\mathbf{x}'), \ldots, p_r^*(\mathbf{x}'), h_1^*(\mathbf{x}'), \ldots, h_s^*(\mathbf{x}') \right\}, \tag{8}$$

where

$$p_k^*(\mathbf{x}') = \frac{\sum_{i=1}^{n} w_i(\mathbf{x}') \cdot p_{ik}}{\sum_{i=1}^{n} w_i(\mathbf{x}')} \qquad (k = 1, \ldots, r),$$

$$h_\ell^*(\mathbf{x}') = \sum_{i=1}^{n} w_i(\mathbf{x}') \cdot h_{i\ell} \qquad (\ell = 1, \ldots, s),$$

$$w_i(\mathbf{x}') = \prod_{j=1}^{m} \mu_{A_{ij}}(x'_j).$$

In this case, the inferred conclusion can be obtained easily. Moreover, the inferred conclusion is expressed by the definition membership function f, and its shape doesn't become complicated. Hence, its meaning can be easily interpreted.

3.2 *Fuzzy sets input case*

Let's consider that the input variables x_j $(j = 1, \ldots, m)$ take fuzzy sets A'_j as input values. Let the input variables set be $\mathbf{x} = \{x_1, \ldots, x_m\}$, then the membership function $\mu_R(\mathbf{x}, y)$ of the fuzzy relation R will be expressed as following, using resolution identity:

$$\mu_R(\mathbf{x}, y) = \max_{\alpha \in (0,1]} \left[\alpha \wedge \chi_{R_{\overline{\alpha}}}(\mathbf{x}, y)\right], \tag{9}$$

$$\chi_{R_{\overline{\alpha}}}(\mathbf{x}, y) = \begin{cases} 1 & \text{if } y \in R_{\overline{\alpha}}(\mathbf{x}) \\ 0 & \text{otherwise,} \end{cases} \tag{10}$$

$$R_{\overline{\alpha}}(\mathbf{x}) = \{y \mid \mu_R(\mathbf{x}, y) \geq \alpha\} \tag{11}$$

where $R_{\overline{\alpha}}(\mathbf{x})$ is an α-level set of the membership function $\mu_R(\mathbf{x}, y)$ at some α ($\alpha \in (0, 1]$). $\chi_{R_{\overline{\alpha}}}(\mathbf{x}, y)$ is a characteristic function of the α-level set $R_{\overline{\alpha}}(\mathbf{x})$. The α-level set $R_{\overline{\alpha}}(\mathbf{x})$ depends on the shape of the definition membership function f and the input variables $\mathbf{x}$.

The output B' is obtained as following, using resolution identity:

$$\mu_{B'}(y) = \max_{\alpha \in (0,1]} \left[\alpha \wedge \chi_{B'_{\overline{\alpha}}}(y)\right], \tag{12}$$

$$\chi_{B'_{\overline{\alpha}}}(y) = \begin{cases} 1 & \text{if } y \in B'_{\overline{\alpha}} \\ 0 & \text{otherwise,} \end{cases} \tag{13}$$

where $B'_{\overline{\alpha}}$ is an α-level set of the membership function $\mu_{B'}(y)$ at some α ($\alpha \in (0, 1]$). $\chi_{B'_{\overline{\alpha}}}(y)$ is a characteristic function of the α-level set $B'_{\overline{\alpha}}$. The α-level set $B'_{\overline{\alpha}}$ is defined as following:

$$\begin{aligned} B'_{\overline{\alpha}} &= \left\{ \bigcup R_{\overline{\alpha}}(\mathbf{x}) \,\middle|\, \alpha \leq \bigwedge_{j=1}^{m} \mu_{A'_j}(x_j) \right\} \\ &= \left\{ \bigcup R_{\overline{\alpha}}(\mathbf{x}) \,\middle|\, \forall j,\ \alpha \leq \mu_{A'_j}(x_j) \right\}. \end{aligned} \tag{14}$$

In this case, the process to obtain the inferred conclusion is more complicated than in the real numbers input case. However, the inferred conclusion can be obtained discretely, since it is defined by α-level sets.

4. Simple example

As a simple example, let's consider both cases of real number input and fuzzy set input, by means of the following two rules:

R_1: *IF x is* "about 10" *THEN y is* "about 20",
R_2: *IF x is* "about 20" *THEN y is* "about 30",

where the fuzzy numbers in the rules are defined by triangular type membership functions shown in Fig. 1 as follows:

"about 10" = { 0, 10, 20, 1 },
"about 20" = { 10, 20, 30, 1 },
"about 30" = { 20, 30, 40, 1 }.

These membership functions are shown in Fig. 2.

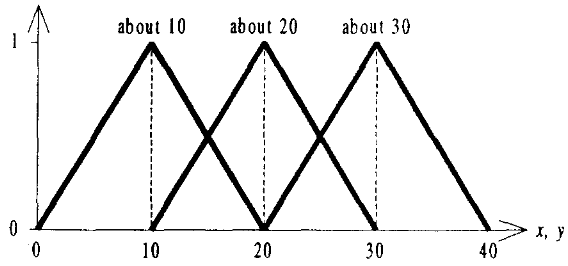


Fig. 2 Membership functions of the fuzzy numbers.

The membership function $\mu_R(x, y)$ that represents the fuzzy relation R for the two rules is defined from Eq. (3) as following:

$$\mu_R(x, y) = f(p_1^*(x), p_2^*(x), p_3^*(x), h_1^*(x), y) \wedge 1 \tag{15}$$

$$p_1^*(x) = \frac{w_1(x) \cdot 10 + w_2(x) \cdot 20}{w_1(x) + w_2(x)}$$

$$p_2^*(x) = \frac{w_1(x) \cdot 20 + w_2(x) \cdot 30}{w_1(x) + w_2(x)}$$

$$p_3^*(x) = \frac{w_1(x) \cdot 30 + w_2(x) \cdot 40}{w_1(x) + w_2(x)}$$

$$h_1^*(x) = w_1(x) \cdot 1 + w_2(x) \cdot 1$$

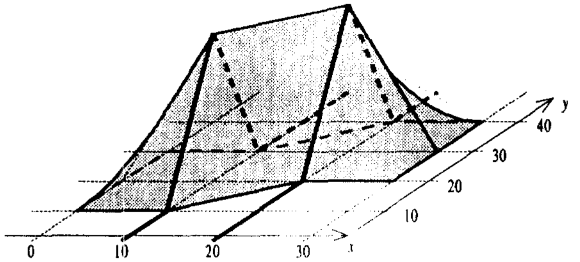
$$w_1(x) = \mu_{A_1}(x), \quad w_2(x) = \mu_{A_2}(x),$$

where $\mu_{A_1}(x), \mu_{A_2}(x)$ are membership functions that characterize the fuzzy numbers "about 10", "about 20" in the given fuzzy rules. Fig. 3 shows the fuzzy relation R.

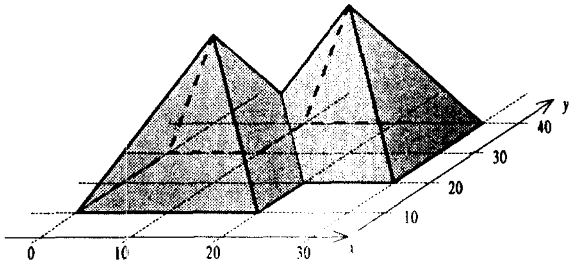
4.1 Real number input case

Let a given input real number be $x = 14$. It can be defined as a fuzzy set that is characterized by the following singleton type membership function:

$$\mu_{A'}(x) = \begin{cases} 1 & (x = 14) \\ 0 & (x \neq 14) \end{cases} \tag{16}$$



(a) The proposed method.



(b) Mamdani's method, $a \to b = \min(a, b)$.

Fig. 3 Fuzzy relation R of the two fuzzy rules.

By sup-min composition, the output B' can be obtained as following:

$$\mu_{B'}(y) = \sup_x [\mu_{A'}(x) \wedge \mu_R(x, y)]$$
$$= \mu_R(14, y) \qquad (17)$$
$$= f_\Delta(14, 24, 34, 1, y)$$

$$B' = \{14, 24, 34, 1\} \qquad (18)$$

This output can be interpreted as a fuzzy number "about 24" (Fig. 4 (a)).

4.2 *Fuzzy set input case*

Let a given input fuzzy set be "about 14", which is defined by a triangular type membership function:

"about 14" = { 10, 14, 18, 1 }.

The membership function $\mu_R(x, y)$ of fuzzy relation R is expressed as following, using resolution identity:

$$\mu_R(x, y) = \max_{\alpha \in (0,1]} \left[\alpha \wedge \chi_{R_{\bar{\alpha}}}(x, y)\right], \qquad (19)$$

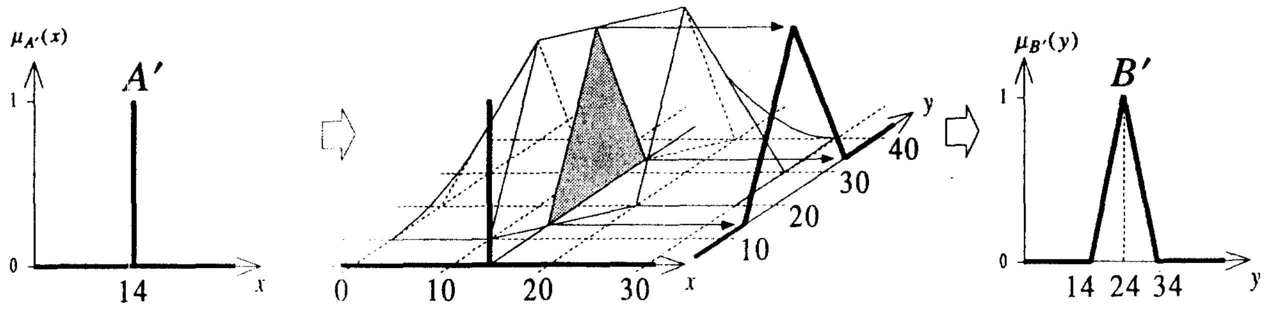
$$\chi_{R_{\bar{\alpha}}}(x, y) = \begin{cases} 1 & \text{if } y \in R_{\bar{\alpha}}(x) \\ 0 & \text{otherwise,} \end{cases} \qquad (20)$$

where the α-level set $R_{\bar{\alpha}}(x)$ is defined as following:

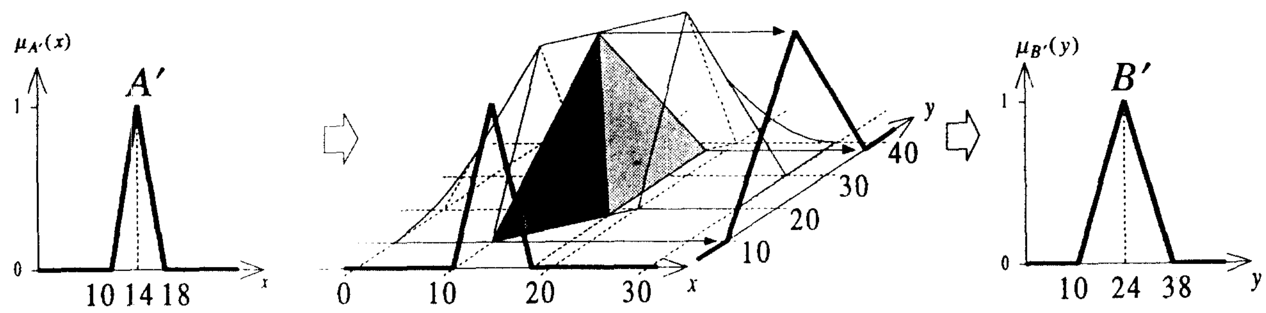
$$R_{\bar{\alpha}}(x) = \left\{ y \,\middle|\, y \in [R_{lower}, R_{upper}] \right\}, \qquad (21)$$

$$R_{lower} = \frac{p_2^*(x) - p_1^*(x)}{h_1^*(x)} \alpha + p_1^*(x),$$

$$R_{upper} = \frac{p_2^*(x) - p_3^*(x)}{h_1^*(x)} \alpha + p_3^*(x).$$



(a) Real number input case, x=14.



(b) Fuzzy set input case, $A' = \{10, 14, 18, 1\}$.

Fig. 4 Fuzzy inference processes using the proposed method.

Let's calculate α-level sets $B'_{\overline{\alpha}}$ at each discrete α-level: α=0.0⁺, 0.25, 0.5, 0.75 and 1.0. Firstly, for α=0.0⁺, the α-level set $A'_{\overline{0.0^+}}$ of the input A' is

$$A'_{\overline{0.0^+}} = \left\{ x \mid x \in [10.0, 18.0] \right\}. \tag{22}$$

Therefore, from Eq. (14), the α-level sets $B'_{\overline{0.0^+}}$ of the output B' comes to be:

$$\begin{aligned} B'_{\overline{0.0^+}} &= \left\{ \cup R_{\overline{0.0^+}}(x) \,\middle|\, 0.0^+ \le \mu_{A'}(x) \right\} \\ &= \left\{ \cup R_{\overline{0.0^+}}(x) \,\middle|\, x \in A'_{\overline{0.0^+}} \right\} \\ &= \left\{ y \mid y \in [10.0, 38.0] \right\}. \end{aligned} \tag{23}$$

In the same way, the α-level sets $B'_{\overline{0.25}}$, $B'_{\overline{0.5}}$, $B'_{\overline{0.75}}$ and $B'_{\overline{1.0}}$ come to be:

$$\begin{aligned} B'_{\overline{0.25}} &= \left\{ y \mid y \in [13.5, 34.5] \right\}, \\ B'_{\overline{0.5}} &= \left\{ y \mid y \in [17.0, 31.0] \right\}, \\ B'_{\overline{0.75}} &= \left\{ y \mid y \in [20.5, 27.5] \right\}, \\ B'_{\overline{1.0}} &= \left\{ y \mid y \in [24.0, 24.0] \right\}. \end{aligned} \tag{24}$$

Fig. 5 shows these α-level sets $B'_{\overline{\alpha}}$. From resolution identity, the output B' becomes a triangular membership function:

$$\mu_{B'}(y) = f_{\Delta}(10, 24, 38, 1, y), \tag{25}$$

$$B' = \{ 10, 24, 38, 1 \}. \tag{26}$$

This output B' can be also interpreted as a fuzzy number "about 24", but the support interval of the membership function is wider than in the real number input case. This means an increase of the fuzziness (Fig. 4 (b)).

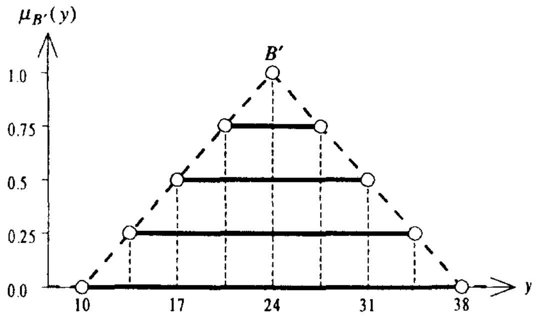


Fig. 5 Discrete inferred conclusion in the fuzzy set input case.

5. Fuzzy modeling of distributed data

5.1 Distributed Modeling Data

The proposed fuzzy inference method is applied to fuzzy modeling of the distributed data as shown in Fig. 6. The modeling data is based on the following non-linear function, one input and one output, Fig. 7:

$$y = \begin{cases} -x & (-1 \le x < 0) \\ x^2 & (0 \le x \le 1). \end{cases} \tag{27}$$

The modeling data follows Gaussian distribution (average is 0.0, variance is 0.052). This means that the distribution has an extent of about ±0.15.

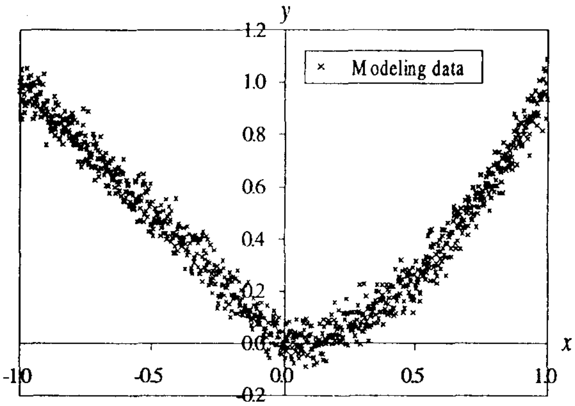


Fig. 6 Distributed modeling data.

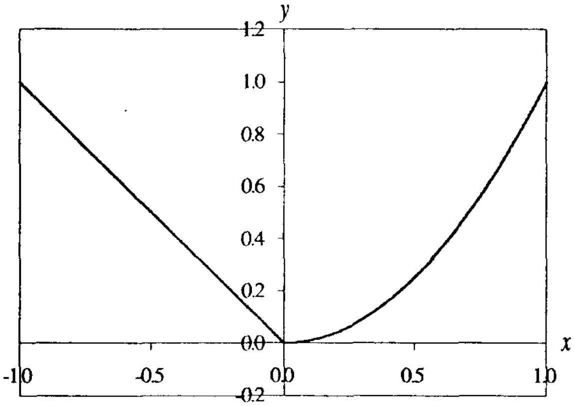


Fig. 7 Based non-linear function.

5.2 Fuzzy modeling

The space of the input variable x is divided to nine fuzzy divisions as shown in Fig. 8, hence the fuzzy model consists the following nine fuzzy rules:

$$R_i : IF\ x\ is\ A_i\ THEN\ y\ is\ B_i \qquad (i = 1, \ldots, 9), \tag{28}$$

$$B_i = \{ p_{i1}, p_{i2}, p_{i3}, h_{i1} \} \qquad (i = 1, \ldots, 9). \tag{29}$$

Membership functions of the consequent part of the fuzzy rules are of triangular type (as shown in Fig. 1). The parameters of the membership functions are identified from modeling data by using the Δ-rule method, although parameters h_{i1} $(i = 1,\ldots,9)$ are fixed to 1.0.

Under these conditions, a fuzzy model is made using a thousand sets of the modeling data. The modeling result is shown in Fig. 9. Consequently, the error variance of the fuzzy model was found to be the almost same as the variance of the modeling data. Moreover, 98.4% of the modeling data were included in the support interval of the fuzzy model output.

From the results, it was found that this fuzzy model using the method proposed in this paper, could represent not only the non-linear structure of the function but also the uncertainness of the distributed modeling data.

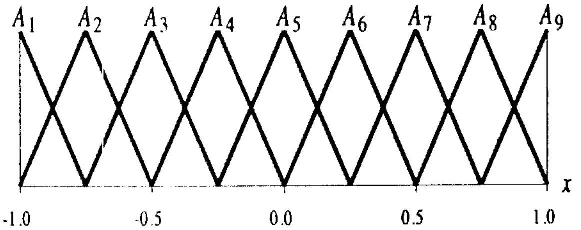


Fig. 8 Fuzzy divisions in the antecedent part.

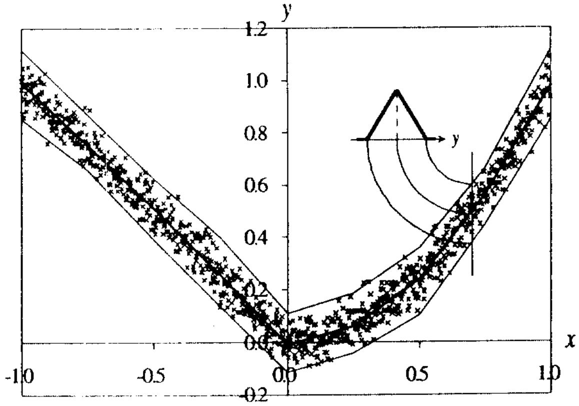


Fig. 9 Modeling result.

6. Conclusion

In this paper, our proposed fuzzy inference method was explained. The strong point of the proposed method is that the membership function, which represents an inferred conclusion fuzzy set, comes to have a simple shape, thus it can be easily interpreted qualitatively.

From the results of the fuzzy modeling of a distributed function, it was found that the fuzzy model using our proposed method could represent not only the non-linear structure of the function but also the uncertainness of the distributed modeling data. If the fuzzy sets in the consequent part of the fuzzy rules represent possibility distributions, this fuzzy inference method will be able to perform as a possibility distribution model.

The proposed fuzzy inference method is expected to be effective on a human supervised system, in which a human being takes any action according to the interpretation of the inferred conclusion. Future works will consider its application for such cases, in order to confirm its effectiveness.

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