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Fuzzy similarity measure in Hypergraph

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Abstract

For a fuzzy system modeled by a fuzzy hypergraph, two fuzzy similarity measures are proposed: one for the fuzzy similarity between fuzzy sets and the other between elements in fuzzy sets. The proposed measures can represent the realistic similarities which can not be given by the existing measures. With an example, it is shown that it can be used in the behavior analysis in an organization.

Keywords : Fuzzy sets; similarity measure; behavior analysis

1. Introduction

Many measures of similarity between fuzzy sets have been proposed in the literature and some measures have been used in system analysis and linguistic approximation[2,4,5,6]. Especially, in [1], the similarity measures between fuzzy sets and between elements have been introduced. However, it has been pointed out that all of these measures give scalar similarities, but not fuzzy. Therefore, they could not be used when the similarity should be measured by a fuzzy value.

The hypergraph has been used as a useful modeling tool and was extended to the fuzzy hypergraph by Lee-Kwang[9]. The fuzzy hypergraph is defined as follows:

$$\tilde{H} = (X, E)$$
$$X = \{(x_i, \mu_v(x_i)) \mid \mu_v(x_i) > 0, \quad i = 1 \ldots n\}$$
$$E = \{A_1, A_2 \ldots A_m\}$$
$$A_j = \{(x_i, \mu_j(x_i)) \mid \mu_j(x_i) > 0, \quad i = 1 \ldots n\}$$
$$A_j \neq \phi, \quad j = 1 \ldots m$$
$$\mu_X(x_i) = \max_j[\mu_j(x_i)], \quad i = 1 \ldots n$$

Let us consider an example of behavior analysis. There is a class in which five students $(x_1, x_2, x_3, x_4, x_5)$ are. In this class, there are three activity groups (A_1, A_2, A_3). Each student paricipates in one or more group, and the participation degrees of students to the groups are represented by a fuzzy hypergraph Fig 1. and its incidence matrix in Table 1.

Table 1.

	A_1	A_2	A_3
x_1	0.4	0.6	0
x_2	0.8	0.3	0.4
x_3	0.9	0	0.8
x_4	0	0.5	1
x_5	0.5	0	0.5

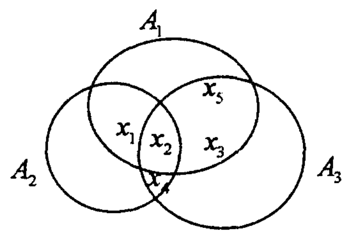

Fig. 1 Fuzzy hypergraph

From the example, we can have two types of questions from the above example.

- (Type-1) " At what degree can the groups A_1 and A_2 be cooperated?" or "What is the gauranteed minimum level of cooperation between groups A_1 and A_2?"
- (Type-2) "At what degree can x_1 and x_2 be in the same group?", or "What is the level of their friendship?"

For these questions, we have proposed similarities measures in [1]. But if the questions need fuzzy values, we have no measure for the fuzzy similarities. Therefore, this paper will propose the fuzzy similarity measrues between fuzzy sets and between elements. We will follow the notations of fuzzy theory in [3,7,8]

2. Fuzzy hypergraph

In Table 1, for example, the student x_1 participates in group A_1 and A_2 with the degrees 0.4 and 0.6 respectively. The student x_2 is involved in groups A_1, A_2 and A_3 with 0.8, 0.3 and 0.4 respectively. There is no constraint such that the sum of participation degrees of a student should be 1. Therefore, we know the participation degree follows the possibility theory, and thus we can interprete the groups as fuzzy sets.

From the above observation, we can see that the student x_1 participates in A_2 more actively than A_1; x_2 participates in A_1 more actively than others. x_1's maximum participation degree is 0.6 and x_2's maximum degree is 0.8. Therefore we can say that x_2 is more active student than x_1. The participation degree of x_1 in this class becomes 0.6 and that of x_2 is 0.8. We can thus summarize the participation possibilities of students by using fuzzy set notations as follows:

$$X = \{(x_1,0.6),(x_2,0.8),(x_3,0.9),(x_4,1),(x_5,0.5)\}$$

The participation possibility of a student is his maximum participation degree in the class. This set corresponds to the vertex set X of the fuzzy hypergraph.

We can also see that the group A_1 has four members and its most active member is x_3 (degree = 0.9); the group A_2 has three members with the most active member x_1 (degree = 0.6); A_3 has four members with the most active member x_4 (degree = 1). We can say that the group A_1 has higher activity level than A_2 because A_1's most active member x_3 (degree = 0.6) has higher degree than A_2's member x_1 (degree = 0.6). Again, the group A_3 has the highest activity level with 1. Therefore, if we take the degree of the most active member, we can have activity level or membership value of each group in the class as follows.

$$\Omega = \{(A_1,0.9),(A_2,0.6),(A_3,1)\}$$

3. Fuzzy similarity measure

The similarity measures given in [1] are as follows and we call them scalar similarity measures.

- Scalar similarity $S(A_i, A_j)$ between fuzzy sets A_i and A_j

$$S(A_i,A_j) = \max_{x\in X} \min[\mu_{A_i}(x), \mu_{A_j}(x)]$$

- Scalar similarity $S_e(x,y)$ between elements x and y

$$S_e(x,y) = \max_i \min[\mu_{A_i}(x), \mu_{A_i}(y)]$$

In order to measure the similarities with fuzzy values, we define the fuzzy measures as follows:

- Fuzzy similarity $\tilde{S}(A_i, A_j)$ between fuzzy sets A_i and A_j

$$\tilde{S}(A_i,A_j) = \{(\sigma_{i,j}, \mu_{\tilde{S}}(\sigma_{i,j})) \mid \sigma_{i,j} = \min[\mu_i(x), \mu_j(x)],$$
$$\mu_{\tilde{S}}(\sigma_{i,j}) = \max_{\sigma_{i,j}=\min[\mu_i(x),\mu_j(x)]} [\mu_X(x)], x \in X\}$$

- Fuzzy similarity $\tilde{S}_e(x,y)$ between elements x and y

$$\tilde{S}_e(x,y) = \{(\gamma_{x,y}, \mu_{\tilde{S}_e}(\gamma_{x,y})) \mid \gamma_{x,y} = \min_i[\mu_i(x), \mu_i(y)],$$
$$\mu_{\tilde{S}_e}(\gamma_{x,y}) = \mu_\Omega(A_i)\}$$

For example, let's calculate some similarities in the above example.

- Scalar similarity between fuzzy sets A_1 and A_2

$$S(A_1,A_2) = \max[\min(0.4,0.6), \min(0.8,0.3), \min(0.9,0), \min(0,0.5), \min(0.5,0)]$$
$$= \max[0.4,0.3,0,0,0] = 0.4$$

- Scalar similarity between elements x_1 and x_2

$$S_e(x_1,x_2) = \max[\min(0.4,0.8), \min(0.6,0.3), \min(0,0.4)]$$
$$= \max[0.4,0.3,0] = 0.4$$

- Fuzzy similarity between fuzzy sets A_1 and A_2

$$\tilde{S}(A_1,A_2)=\{(\min(0.4,0.6),0.6),(\min(0.8,0.3),0.8),(\min(0.9,0),0.9),(\min(0,0.5),1),(\min(0.5,0),0.5)\}$$
$$=\{(0.4,0.6),(0.3,0.8),(0,0.9),(0,1),(0,0.5)\}$$
$$=\{(0.4,0.6),(0.3,0.8),(0,1)\}$$

For example, the similarity 0.4 is given when x_1 is contained in both A_1 and A_2. But the membership degree of x_1 in X is 0.6, that is, $\mu_X(x_1)=0.6$. Therefore, the possibility of similarity 0.4 is 0.6, that is, $\mu_{\tilde{S}}(0.4)=0.6$.

· Fuzzy similarity between elements x_1 and x_2

$$\tilde{S}_e(x_1,x_2)=\{(\min(0.4,0.8),0.9),(\min(0.6,0.3),0.6),(\min(0,0.4),1)\}$$
$$=\{(0.4,0.9),(0.3,0.6),(0,1)\}$$

For example, the similarity 0.4 is given when A_1 contains the both two elements x_1 and x_2. But the membership degree of A_1 in Ω is 0.9, that is, $\mu_\Omega(A_1)=0.9$. Therefore, the possibility of similarity 0.4 is 0.9, that is, $\mu_{\tilde{S}_e}(0.4)=0.9$.

With the similarity calculation in the previous section, we can now answer to the questions given in section 1.

· Answer to type 1 question : the groups A_1 and A_2 cooperate
  - with the scalar degree 0.4
  - with the fuzzy degree $\{(0.4,0.6),(0.3,0.8),(0,1)\}$

· Answer to type 2 question : the students x_1 and x_2 are in the same group
  - with the scalar degree 0.4
  - with the fuzzy degree $\{(0.4,0.9),(0.3,0.6),(0,1)\}$

4. Conclusion

In conclusion, we have proposed two fuzzy measures; one measures the fuzzy similarity between fuzzy sets and the other between elements in fuzzy sets. The proposed mothod can be used when a system is represented by a fuzzy hypergraph. We have seen that the proposed measures can represent the features which can not analyzed by the scalar similarity measures. The proposed measures could be extended to the continuous fuzzy sets in the future work.

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