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Multiple Linear Goal Programming Using Scenario Approach to Obtain Fuzzy Solution

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Abstract

Fuzzy mathematical programming (FMP) can be treated an uncertainty condition using fuzzy concept. Further, it can be extended to the multiple objective (or goal) programming problem, naturally. But we feel that FMP have some shortcomings such as the fuzzy number in FMP is the one dimensional possibility set, so it can not be represented the relationship between them, and, in spite of FMP includes some (uncertainty) fuzzy parameters, many algorithms are only obtained a crisp solution. In this study, we propose a method of FMS. Our method use the scenario approach (or fuzzy random variables) to represent the relationship between fuzzy numbers, and can obtain the fuzzy solution.

Keyword: Fuzzy Mathematical Programming, Fuzzy Solution

1. Introduction

In a management planning problem, we have many uncertainty factors when we plan it. For example, consumers' demand in future and process time of new product. It is very important to consider these uncertainty factors when we plan. Fuzzy mathematical programming (FMP) is one of the optimizing methods which consider the these uncertainty conditions.

FMP [7, 11] includes some fuzzy parameters and/or goals. When we obtain the optimal solution, we must interpret these fuzzy parameters by some meanings. Many FMP approaches is obtained only a crisp solution, in spite of considering the fuzzy conditions. In the management decision, we have long time till carrying out the plan. So, it is very useful information the ambiguous (fuzzy) solution to the decision maker, because it can show the possibility of the range of solution. Some of method to obtain the fuzzy solution is suggested [3, 5, 8].

Ordinal fuzzy number is the possibility distribution on one dimensional real number line. Hence, it can not be directly represented the relation between plural fuzzy numbers. But for example, if business condition becomes bad, then the consumers' demand is down. So the firm should be short entire working hours. In such way, there are some trend among the uncertainty factors. It is valuable that we should consider such the trend (relationship). Some idea to represent the relationship between the parameters in FMP is suggested. Inuiguchi et al. [1] has suggested a method using the canonical fuzzy number. Ohta et al.[4] has represented the relationship between the fuzzy coefficients using the scenario approach which is appeared in the two-stage problem of stochastic programming. The other, there are some approaches to represent the relationship among the fuzzy numbers, for example T-norm approach and quadratic-form membership function.

In this study, we suggest a method to the multi-objective fuzzy linear programming problem using the scenario approach to obtain fuzzy solution.

2. Formulation

We treat the fuzzy multiple objective programming problem formulation as follow. It is the symmetric form which is equated the objective functions with constrains. So, there is not difference between them.

[P1]

$$\max \quad z_k(\tilde{\mathbf{X}}) = \sum_{j=1}^{n} \tilde{A}_{kj}\tilde{X}_j, \quad k \in I_1 \tag{1}$$

$$\min \quad z_k(\tilde{\mathbf{X}}) = \sum_{j=1}^{n} \tilde{A}_{kj}\tilde{X}_j, \quad k \in I_2 \tag{2}$$

$$\text{s.t.} \quad \tilde{X}_j \geq 0, \quad j = 1, ..., n \tag{3}$$

where $\tilde{A}_{kj}$ is the kth objective's the jth fuzzy coefficients, $\tilde{\mathbf{X}} = [\tilde{X}_j]$ is the fuzzy variables vector and I_1 and I_2 is the set of subscripts of maximizing and minimizing objective functions, respectively. We assume that all the fuzzy variables are in positive. The set of subscripts of all objective is I, i.e. $I = I_1 \cup I_2$.

3. Algorithm

3.1. Fuzzy Goal

First, we put fuzzy goal to each objective function in [P1]. Assume that $\tilde{G}_i$ is a fuzzy goal for the ith objective functions of [P1]. Using these fuzzy goals, [P1] is rewritten as following [P2].

[P2]

$$\text{Goal} \quad z_k(\tilde{\mathbf{X}}) = \sum_{j=1}^{n} \tilde{A}_{kj}\tilde{X}_j \gtrsim \tilde{G}_k, \quad k \in I_1 \qquad (4)$$

$$z_k(\tilde{\mathbf{X}}) = \sum_{j=1}^{n} \tilde{A}_{kj}\tilde{X}_j \lesssim \tilde{G}_k, \quad k \in I_2 \qquad (5)$$

$$\text{s.t.} \quad \tilde{X}_j \geq 0, \quad j = 1, ..., n \qquad (6)$$

The sign "$\gtrsim$" and "$\lesssim$" means *almost large* and *almost small*, respectively. If there is an *equal* objective (goal), then we can treat into divide it to two objective function, i.e.,

$$\tilde{G}_k^- \lesssim \sum_{j=1}^{n} \tilde{A}_{kj}\tilde{X}_j \lesssim \tilde{G}_k^+, \quad k \in \text{equaling goal} (7)$$

3.2. Scenario Approach

In this study, we use the scenario approach to consider the relationship between fuzzy parameters. *Scenario* is a discrete probability distribution using the two stage problem of stochastic programming [9]. We assume that it is constructed some finite estimate values (scenarios). If the number of scenario is Q and the probability of the qth scenario is p_q, then [p2] is formulated as follow.

[P3] $(q = 1, ..., Q)$

$$\text{Goal} \quad z_{kq}(\tilde{\mathbf{X}}) = \sum_{j=1}^{n} \tilde{A}_{kjq}\tilde{X}_{jq} \gtrsim \tilde{G}_{kq}, \quad k \in I_1 \ (8)$$

$$z_{kq}(\tilde{\mathbf{X}}) = \sum_{j=1}^{n} \tilde{A}_{kjq}\tilde{X}_j \lesssim \tilde{G}_{kq}, \quad k \in I_2 \ (9)$$

$$\text{s.t.} \quad \tilde{X}_j \geq 0, \quad j = 1, ..., n \qquad (10)$$

where $\sum_{q=1}^{Q} p_q = 1$.

3.3. Integration of objective functions

There are I objective functions for each scenario. Next, we integrate these objective function. As the same sense of flexible programming [10], we introduce the minimum operator λ. We assign a linear membership function $\mu_{\tilde{G}_k}$ for each fuzzy goal $\tilde{G}_k$. g_k^L and g_k^M is the bound the membership value of $\mu_{\tilde{G}_k}$ being 0 and 1, respectively. The membership function for the kth objective function of qth scenario is given $\mu_{z_{qk}}(\tilde{\mathbf{X}})$. Then, [P3] can be rewritten in [P4].

[P4] $(q = 1, ..., Q)$

$$\max \quad \mu_{z_{kq}}(\tilde{\mathbf{X}}) = \mu_{z_{qk}}\left[\sum_{j=1}^{n} \tilde{A}_{kjq}\tilde{X}_{jq}\right], \quad k \in (11)$$

$$\text{s.t.} \quad \tilde{X}_j \geq 0, \quad j = 1, ..., n \qquad (12)$$

There are I objective functions for each scenario. Next, we integrate these objective functions. As the same sense of flexible programming [10], we introduce the minimum operator λ. Now, we integrate I objective functions about each scenario.

$$\lambda_q = \min(\mu_{z_{q1}}(\tilde{\mathbf{X}}), \mu_{z_{q1}}(\tilde{\mathbf{X}}), ..., \mu_{z_{q,\#I}}(\tilde{\mathbf{X}})) \qquad (13)$$

Using the minimum operator for each scenario, we can formulate from [P4] to [P5].

[P5] $(q = 1, ..., Q)$

$$\max \quad \lambda_q \qquad (14)$$

$$\text{s.t.} \quad \mu_{z_{kq}}\left[\sum_{j=1}^{n} \tilde{A}_{kjq}\tilde{X}_j\right] \gtrsim \lambda_q, \quad k \in I \qquad (15)$$

$$\tilde{X}_j \geq 0, \quad j = 1, ..., n \qquad (16)$$

Because of $\mu_{z_{kq}}(\tilde{\mathbf{X}}) = (\sum_{j=1}^{n} A_{kjq}\tilde{X}_j - g_{kq}^L)/(g_{kq}^M - g_{kq}^L)$, Eq. (15) can be expanded as follow.

$$\sum_{j=1}^{n} \tilde{A}_{kjq}\tilde{X}_j - g_{kq}^L - \lambda_q(g_{kq}^M - g_{kq}^L) \gtrsim 0 \qquad (17)$$

If we want to solve [P5], there are some difficult problems such as how to integrate the scenarios whose probability is not equal or how interpret the fuzzy parameters $\tilde{A}_{kjq}$. So, we can not judge whether the fuzzy inequality (17) is satisfied or not for a λ_q. The fuzzy inequality (17) should be judged ambiguous, e.g. almost satisfy or little satisfy.

3.4. Degree of satisfied the inequality relation

It is not clearly whether the fuzzy inequality (17) is satisfied or not for a λ_q. So, we consider the degree of satisfied the relationship. Generally, if the value of λ_q is larger. then the degree of satisfied the inequality relationship is smaller. The converse is almost true. The decision maker want to obtain a solution that both λ_q and the degree of satisfied the inequality relationship are as large as possible. We express that $\mu_{Ekq}(\tilde{\mathbf{X}})$ is the membership functions of the degree of the inequality relationship for the qth scenario of the kth goal function. Formulation under considering these conditions is [P6].

[P6] $(q = 1, ..., Q)$

$$\max \quad \lambda_q \qquad (18)$$

$$\max \quad \mu_{Ekq}(\mathbf{X}), \quad k \in I \qquad (19)$$

$$\text{s.t.} \quad \mu_{z_{kq}}(\tilde{\mathbf{X}}) \gtrsim \lambda_q, \quad k \in I \qquad (20)$$

$$\tilde{X}_j \geq 0, \quad j = 1, ..., n \qquad (21)$$

Then, in the same manner of λ_q, we integrate the I membership functions of the degree of the inequality relationship.

[P7]

$$\max \quad \lambda_q \tag{22}$$
$$\max \quad \tau_q \tag{23}$$
$$\text{s.t.} \quad \mu_{z_{qk}}(\tilde{\mathbf{X}}) \gtrsim \lambda_q, \quad k \in I \tag{24}$$
$$\mu_{Ekq}(\tilde{\mathbf{X}}) \geq \tau_q, \quad k \in I \tag{25}$$
$$\tilde{X}_j \geq 0, \quad j = 1, ..., n \tag{26}$$

where $\tau_q = \min(\mu_{E1q}(\tilde{\mathbf{X}}), \mu_{E2q}(\tilde{\mathbf{X}}), ...\mu_{E,\#Iq}(\tilde{\mathbf{X}}))$.

Next, we must integrate λ_q and τ_q. If the decision maker has some prior knowledge or information about λ_q and τ_q, we can use these information. But in this study, we suggest a method in the case of unable to get them.

3.5. Formulation for α level set

As we pointed out above, [P5] has fuzzy parameters and fuzzy relation and it is constructed by plural scenarios. Moreover, when both the coefficients and variables are fuzzy number, then it can not calculate these them without the reference function determined. So it is too difficult to solve [P5] directly. Then, we focus on the α level set of fuzzy set. α level set of arbitrary fuzzy number becomes interval. Divided into some α level sets, fuzzy programming problem is decomposed some interval programming problems. Then, we obtain the optimal fuzzy solution through reconstructed all the solution for each α level using the decomposition theorem of fuzzy set. The α level set of fuzzy set $\tilde{A}$ is represented in the following form.

$$A^\alpha = [\underline{a}^\alpha, \ \bar{a}^\alpha] \tag{27}$$

Using the α level set, [P5] becomes as next formulations.

[P8] $(q = 1, ..., Q)$

$$\max \quad \lambda_q \tag{28}$$
$$\text{s.t.} \quad \sum_{j=1}^{n} A_{kjq}^\alpha X_j^\alpha - g_{kq}^L - \lambda_q(g_{kq}^M - g_{kq}^L), \quad k \in I \tag{29}$$
$$X_j^\alpha \geq 0, \quad j = 1, ..., n \tag{30}$$

where A_{kjq} and X_j^α are the interval coefficients and variables for an α level, respectively. So, whole Eq. (29) is interval. Eq. (29) is represented in following manner.

$$\sum_{j=1}^{n} A_{kjq}^\alpha X_j^\alpha - g_{kq}^L - \lambda_q(g_{kq}^M - g_{kq}^L) = [\underline{I}_{kq}^\alpha, \ \bar{I}_{kq}^\alpha] \tag{31}$$

Each element of interval $[\underline{I}_{kq}^\alpha, \ \bar{I}_{kq}^\alpha]$ can be expanded as next.

$$\underline{I}_{kq}^\alpha = \sum_{j=1}^{n} \underline{C}_{kjq}^\alpha \underline{X}_j^\alpha - g_{kq}^\alpha - \lambda_q(g_{kq}^M - g_{kq}^L), \quad k \in I_1 \tag{32}$$
$$= \sum_{j=1}^{n} \underline{C}_{kjq}^\alpha \bar{X}_j^\alpha - g_{kq}^\alpha - \lambda_q(g_{kq}^M - g_{kq}^L), \quad k \in I_2 \tag{33}$$
$$\bar{I}_{kq}^\alpha = \sum_{j=1}^{n} \bar{C}_{kjq}^\alpha \bar{X}_j^\alpha - g_{kq}^\alpha - \lambda_q(g_{kq}^M - g_{kq}^L), \quad k \in I_1 \tag{34}$$
$$= \sum_{j=1}^{n} \bar{C}_{kjq}^\alpha \underline{X}_j^\alpha - g_{kq}^\alpha - \lambda_q(g_{kq}^M - g_{kq}^L), \quad k \in I_2 \tag{35}$$

To measure the degree of the inequality relationship, Kono et al. [3] defined the membership function for the inequality sign. This is near sense of Ishibuchi et al. [2].

Definition 1 *For the inequality relation* $[\underline{I}_{kq}^\alpha, \ \bar{I}_{kq}^\alpha] \gtrsim 0$, *the membership function for the inequality sign is identified as follow,*

$$\mu_{Eqk}^\alpha(\mathbf{X}^\alpha) = \begin{cases} 0, & \bar{I}_{kq}^\alpha < 0 \\ \dfrac{\bar{I}_{kq}^\alpha}{\bar{I}_{kq}^\alpha - \underline{I}_{kq}^\alpha}, & \underline{I}_{kq}^\alpha \leq 0 < \bar{I}_{kq}^\alpha \\ 1, & \bar{I}_{kq}^\alpha \geq 0 \end{cases} \tag{36}$$

Using above definition, [P8] can be transformed into [P9] in the same sense from [P5] to [P7].

[P9]

$$\max \quad \lambda_q \tag{37}$$
$$\max \quad \tau_q \tag{38}$$
$$\text{s.t.} \quad \mu_{z_{qk}}^\alpha(\mathbf{X}^\alpha) \gtrsim \ \lambda_q, \quad k \in I \tag{39}$$
$$\mu_{Ekq}^\alpha(\mathbf{X}^\alpha) \geq \tau_q, \quad k \in I \tag{40}$$
$$\bar{x}_j^\alpha - \underline{x}_j^\alpha \leq \theta_j, \quad j = 1, ..., n \tag{41}$$
$$\bar{x}_j^\alpha \geq \underline{x}_j^\alpha \geq 0, \quad j = 1, ..., n \tag{42}$$

where θ_j is a positive threshold value for the jth variable to prevent a meaninglessly wide range solution. Eq. (40) is equivalence as next inequality form.

$$(1 - \tau_q)\bar{I}_{kq}^\alpha - \tau_{kq}\underline{I}_{kq}^\alpha \geq 0 \tag{43}$$

As pointing out above section, the decision maker wants to obtain a solution where both λ_q and τ_q are as large as possible. But These operators have the trade off relationship, generally. So in this study, we normalize λ_q about τ_q. To put it concretely, using [P9]. The optimal value of λ_q when τ_q equals 1 and 0 is $\lambda_q^{\rm pe}$ and $\lambda_q^{\rm pe}$, respectively. Recall that there has some trade off relationship between λ_q and τ_q, so when it is the largest $\tau_q, (\tau_q = 1)$, we consider the most pessimistic condition about λ_q. Conversely, when $\tau_q = 0$,then the most optimistic condition about λ_q. Using $\lambda_q^{\rm pe}$ and $\lambda_q^{\rm op}$, we normalize λ_q as follow.

$$\mu_{\lambda_q}(\lambda_q) = \frac{\lambda_q - \lambda_q^{\rm pe}}{\lambda_q^{\rm op} - \lambda_q^{\rm pe}} \quad (44)$$

Hence, [P9] can be written in [P10].

[P10] $(q = 1, ..., Q)$

$$\begin{array}{lll} \max & \eta_q & (45) \\ \text{s.t.} & \mu_{\lambda_q}(\lambda_q) \geq \eta_q & (46) \\ & \mu_{Ekq}^{\alpha}(\mathbf{X}^{\alpha}) \geq \tau_q \geq \eta_q, \quad k \in I & (47) \\ & \bar{x}_j^{\alpha} - \underline{x}_j^{\alpha} \leq \theta_j, \quad j = 1, ..., n & (48) \\ & \bar{x}_j^{\alpha} \geq \underline{x}_j^{\alpha} \geq 0, \quad j = 1, ..., n & (49) \end{array}$$

[P10] is the single objective programming, but there remains nonintegrated scenarios that set up in first. We integrate all the scenarios which considering the probability of each scenario (p_q) as [P11].

[P11] $(q = 1, ..., Q)$

$$\begin{array}{lll} \max & \omega & (50) \\ \text{s.t.} & (1 - \eta_q) p_q \leq \omega, \quad q = 1, ..., Q & (51) \\ & \mu_{\lambda_q}(\lambda_q) \geq \eta_q, \quad q = 1, ..., Q & (52) \\ & \mu_{Ekq}^{\alpha}(\mathbf{X}) \geq \tau_q \geq \eta_q, k \in I; q = 1, ..., Q & (53) \\ & \bar{x}_j^{\alpha} - \underline{x}_j^{\alpha} \leq \theta_j, \quad j = 1, ..., n & (54) \\ & \bar{x}_j^{\alpha} \geq \underline{x}_j^{\alpha} \geq 0, \quad j = 1, ..., n & (55) \end{array}$$

In [P11], Eq. (51) is the form that multiplied the index integrated λ_q and τ_q for each scenario by its probability. So, we can obtain a solution considering the index being relatively large value and large probability.

[P11] is a nonlinear programming problem, but the optimal solution is in the interval [0, 1]. So, we can obtain it using the method of bisection and the first stage of simplex method [6].

3.6. Extention to other α level sets

When we solve [P12], there remain some problems. I do not know how to select α level to solve and even if we solved individually about some α level, these solutions may not consistent. In these problems, Kono et al. [3] proposed a method that is based on the solution about $\alpha = 1$ (they named it the *reference solution*), then expanded the other $\alpha (< 1)$ level sets. Then, they defined the satisfiable condition for fuzzy inequality sign.

Definition 2 *If the value of $\mu_{Ekq}^{\alpha}(\mathbf{X}^{\alpha})$ in anbitrarily α level sets is not smaller than $\mu_{Ekq}^{1}(\mathbf{X}^{*})$, then the fuzzy inequality "$\gtrsim$" or "$\lesssim$" is satisfied.*

In others words, it is satisfied following relations.

$$\begin{array}{rcll} \lambda_q & \geq & \mu_{\lambda_q}^{-1}(\eta_q^*) & (56) \\ \mu_{Ekq}^{\alpha}(\mathbf{X}^{\alpha}) & \geq & \mu_{Ekq}^{1}(\mathbf{X}^{*}) & (57) \end{array}$$

where $\mathbf{X}^*$ is the reference solution and η_q^* is the optimal solution for the qth scenario.

Based on this reference solution, we obtain the solution about the other α levels using the next algorithm.

Step 1. Set $\tau_q = \min\{\mu_{Ekq}^{1}(\mathbf{X})\}$ for $k,\ \alpha = 0$.

Step 2. Change the threshold θ_j.

Step 3. Solve [P12].

Step 4. Substitute α' for α.

Step5. Identify a concerned $\alpha (\alpha' < \alpha < 1)$, go to Step 2. If no existence such α, then end.

[P12]

$$\begin{array}{lll} \min & d^- - \varepsilon \delta & (58) \\ \text{s.t.} & \lambda_q + d^- \geq \mu_{\lambda_q}^{-1}(\eta_q^*), \quad q = 1, ..., Q & (59) \\ & \mu_{Ekq}^{\alpha}(\mathbf{X}^{\alpha}) + d^- \geq \tau_q, \quad q = 1, .., Q & (60) \\ & \bar{x}_j^{\alpha} - \underline{x}_j^{\alpha} \geq \delta, \quad k \in I & (61) \\ & \bar{x}_j^{\alpha} - \underline{x}_j^{\alpha} \leq \theta_j, \quad k \in I & (62) \\ & \bar{x}_j^{\alpha'} \geq \bar{x}_j^{\alpha} \geq \bar{x}_j^{1}, \quad j = 1, ..., n & (63) \\ & \underline{x}_j^{\alpha'} \leq \underline{x}_j^{\alpha} \leq \underline{x}_j^{1}, \quad j = 1, ..., n & (64) \\ & \bar{x}_j^{\alpha} \geq \underline{x}_j^{\alpha} \geq 0, \quad j = 1, ..., n & (65) \end{array}$$

where ε is the non-archimedean infinitesimal and η_q^* is the optimal value of the qth scenario in [P11] in the case of $\alpha = 1$. Moreover, we assume that $\underline{x}_j^{\alpha'} = 0$ and $\bar{x}_j^{\alpha'} = \infty$ in the case of $\alpha = 0$.

Often, the stochastic programming is that the objective function is the expected value. In this case, the objective function is formulated that weighted by the probability. Hence, its optimal solution may be biased on the specific scenario which the expected value is very large. If there is sufficiently alternative relationship among the scenarios, it is reasonable to use the expected value to the objective function. But in many case, we think that these relationships is not sufficiently, so it has potential peril to use the expected value to the objective function.

Then, we obtain the optimal fuzzy solution $\tilde{\mathbf{X}}^* = [\tilde{X}_j^*]$ using the decomposition theorem of fuzzy set.

$$\tilde{X}_j^* = \bigcup_{\alpha \in \Phi} \alpha \wedge X_j^{\alpha *} \tag{66}$$

where $X_j^{\alpha *}$ is the optimal interval solution about α level, i.e. $[\underline{x}_j^{\alpha *},\ \bar{x}_j^{\alpha *}]$, and Φ is the set of solved α levels.

4. Numerical Example

A numerical example will be reported in the presentation.

5. Summary

In this study, we considered the fuzzy solution and the relationship between the parameters in fuzzy mathematical programming. To represent the relationship among the fuzzy parameters, we applied the scenario approach in the stochastic problem. This approach is relatively easy to introduce the mathematical programming. Any other approach to be represented the relationship of fuzzy numbers, it is difficult to apply to the mathematical programming. So, they are not reasonable to solve realistic problems. Moreover, we proposed a method to obtain the fuzzy solution. Such a long term planning problem, we may have some managerial risks such as changing the business condition. If we can obtain only a crisp solution, we may puss up the revisable chance of the planning. So, the fuzzy solution can be useful information for the decision maker.

We remain some problems such as how to estimate the fuzzy parameters and these relationships.

Furthermore, if the decision maker can not accept the optimal solution, we need an interactive process.

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