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A Fuzzy Model Based Controller for the Control of Inverted Pendulum

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Abstract

In this paper, we propose a stable fuzzy logic controller architecture for inverted pendulum. In the design procedure, we represent the fuzzy system as a Takagi-Sugeno fuzzy model and construct a global fuzzy logic controller by considering each local state feedback controller and a supervisory controller. Unlike usual parallel distributed controller, one can design a global stable fuzzy controller without finding a common Lyapunov function by the proposed method. A simulation is performed to control the inverted pendulum to show the effectiveness and feasibility of the proposed fuzzy controller.

1. Introduction

Fuzzy logic control is one of the most useful approaches for utilizing the qualitative knowledge of a system to design a controller. Fuzzy logic control is generally applicable to plants that are mathematically poorly modeled and where the qualitative knowledge of experienced operators is available for providing qualitative control. The design of fuzzy logic controllers, however, has difficulties in the acquisition of expert's knowledge and relies to a great extent on empirical and heuristic knowledge which, in many cases, cannot be justified. Therefore, the performance of the controllers can be degraded in the case of plant parameter variations or unpredictable incident which a designer may have ignored, and the parameters of fuzzy logic controllers obtained by expert's control action may not be optimal. And it has not been viewed as a rigorous science due to a lack of formal synthesis techniques that guarantee the very basic requirements of global stability and acceptable performance.

With the developments of neural networks and fuzzy systems, it is known that the qualitative knowledge of a system can also be represented in a nonlinear functional form [1-3], [5]. On the basis of this idea some fuzzy model based fuzzy control system design methods [4-9], [11-13], [15] have appeared in the fuzzy control field. In these methods, sets of fuzzy rules are used to construct the suitable local linear state models from which local controllers can be determined. The stability of the overall system is then determined by a Lyapunov stability analysis. These kinds of design approaches suffer mainly from two limitations. Firstly, a common Lyapunov function is difficult to find, especially when the number of fuzzy rules required to give a good plant model is large. Secondly, the performance of the closed loop system is difficult to predict.

To resolve these problems we propose a stable fuzzy logic controller architecture. In the design procedure, we represent the fuzzy system as a family of local state space models, which is often called Takagi-Sugeno (TS) fuzzy model and construct a global fuzzy logic controller by considering each local state feedback controller and a supervisory controller. By this simple modification, one can design a global stable fuzzy controller without finding a common Lyapunov function. Finally, A simulation is performed to control the inverted pendulum to show the effectiveness and feasibility of the proposed fuzzy controller.

The proposed approach in this paper has the same advantages stated in [9]. But the structure of the controller used is different from that of [9].

The TS fuzzy model will be reviewed in Section 2. In Section 3, the proposed stability design approach will be detailed. In Section 4, an application to the inverted pendulum will be given to show the advantages of the proposed approach. Finally, conclusions will be drawn in Section 5.

2. The TS fuzzy model and the PDC controller

The fuzzy model used in this paper is the TS fuzzy model proposed in [1-3]. It combines the fuzzy inference rule and the local linear model. The ith rule of the TS fuzzy model is shown in (1).

Rule i: If x_1 is F_{i1} and ... and x_n is F_{in}

$$\text{THEN } \dot{x} = A_i x + B_i u \quad (i = 1, 2, ..., r) \tag{1}$$

where Rule i denotes the ith fuzzy inference rule, (A_i, B_i) is th ith local model of the fuzzy system (1), r is the number of fuzzy inference rules, $x \in R^n$ are the state variables of the system, $u \in R^p$ are input variables of the system, and F_{ij} are fuzzy sets. The final output of the continuous time fuzzy system is inferred as follows:

$$\dot{x} = \frac{\sum_{i=1}^{r} w_i (A_i x + B_i u)}{\sum_{i=1}^{r} w_i} = Ax + Bu \tag{2}$$

where r is the number of fuzzy rules in the TS model, $w_i = \prod_{j=1}^{n} F_{ij}(x_j)$ is the degree of fulfillment of antecedent part of the ith fuzzy rule satisfying $\sum_{i=1}^{r} w_i = 1$ and $w_i \geq 0$ (i = 1, 2, ..., r), and

$$A = \frac{\sum_{i=1}^{r} w_i A_i}{\sum_{i=1}^{r} w_i}, \qquad B = \frac{\sum_{i=1}^{r} w_i B_i}{\sum_{i=1}^{r} w_i} \tag{3}$$

We adopt the parallel distributed compensation (PDC) technique [8] to control the fuzzy model in (2). The main idea of the parallel distributed compensation is to derive each control rule so as to compensate each rule of a fuzzy system. The concept of PDC design is shown in Fig. 1.

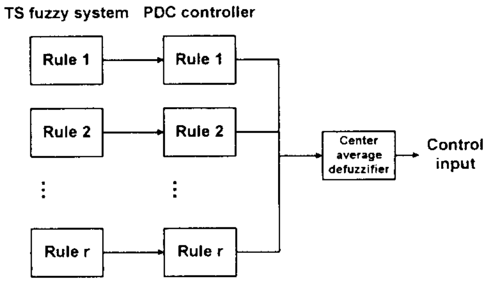

Fig. 1 The concept of PDC design

Using the same premise as (1), The ith rule of the FLC can be as follows:

Rule i: If x_1 is F_{i1} and ... and x_n is F_{in}

$$\text{THEN } u = -K_i x \quad (i = 1, 2, ..., r) \tag{4}$$

where $K_i = [K_{i1}, ..., K_{in}]$ is the feedback gain vector. The fuzzy controller (4) is represented as

$$u_f = -\frac{\sum_{i=1}^{r} w_i K_i x}{\sum_{i=1}^{r} w_i} \tag{5}$$

The overall closed loop fuzzy system combining (2) and (5) becomes (6).

$$\dot{x} = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i w_j (A_i - B_i K_j) x}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i w_j} \tag{6}$$

The stability condition of fuzzy system (6) is given as follows:

Theorem 1. ([8]) The equilibrium of a fuzzy control system (6) is asymptotically stable in the large if there exists a common positive definite matrix P such that

$$\{A_i - B_i K_j\}^T P \{A_i - B_i K_j\} - P < 0 \quad \text{for } w_i w_j \neq 0, \; i, j = 1, 2, ..., r \tag{7}$$

If the common positive definite matrix P can be found, one can determine the stability of the closed fuzzy system (6). However, in many cases, it is difficult to find the common positive definite matrix P and therefore the closed-loop fuzzy system cannot be guaranteed to be stable. In such cases, an additional control input can be added to the system to ensure system stability. In the next section, we will show the detailed derivation of the proposed scheme.

3. Stable fuzzy controller design

We solve this problem by appending another control term u_s to the u_f. Then the final control law is defined as

$$u = u_f + u_s \tag{8}$$

where u_f is defined in (5) and u_s will be derived later. For arbitrary ith rule ($1 \le i \le r$),

$$\begin{aligned}\dot{x} &= \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i w_j (A_i - B_i K_j)x}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i w_j} + Bu_s \\ &= \frac{w_i^2 (A_i - B_i K_i)x + \sum_{i=1}^{r}\sum_{j\ne i}^{r} w_i w_j (A_i - B_i K_j)x}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i w_j} + Bu_s \\ &= v_i^2 F_i x + H_i x + Bu_s\end{aligned} \tag{9}$$

where

$$\begin{aligned} v_i &= \frac{w_i}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i w_j} \\ F_i &= A_i - B_i K_i \\ H_i &= \sum_{i=1}^{r}\sum_{j\ne i}^{r} v_i v_j (A_i - B_i K_j)\end{aligned} \tag{10}$$

If the TS fuzzy model is locally controllable then the feedback control gains K_i ($i = 1, 2, \ldots, r$) can be selected so that the eigenvalues of ($A_i - B_i K_i$) are in the specified places. And there exists a symmetric positive-definite matrix P_i and Q_i which are the solutions of the Lyapunov equation

$$F_i^T P_i + P_i F_i = -Q_i \tag{11}$$

Define a Lyapunov function candidate

$$V = x^T P x \tag{12}$$

where

$$P = \sum_{j=1}^{r} P_j$$

Before going further, we define following for convenience.

$$\overline{P}_i = \sum_{j\ne i} P_j, \qquad \overline{Q}_i = F_i^T \overline{P}_i + \overline{P}_i F_i^T$$

Then we have

$$\begin{aligned}\dot{V} &= \dot{x}^T P x + x^T P \dot{x} \\ &= (v_i^2 F_i x + H_i x + Bu_s)^T Px \\ &\quad + x^T p(v_i^2 F_i x + H_i x + Bu_s) \\ &= (v_i^2 F_i x + H_i x + Bu_s)^T (P_i + \overline{P}_i)x \\ &\quad + x^T (P_i + \overline{P}_i)(v_i^2 F_i x + H_i x + Bu_s) \\ &= v_i^2 x^T (F_i^T P_i + P_i F_i)x + v_i^2 x^T (F_i^T \overline{P}_i + \overline{P}_i F_i)x \\ &\quad + 2x^T P(H_i x + Bu_s) \\ &= -v_i^2 x^T Q_i x - v_i^2 x^T \overline{Q}_i x + 2x^T P(H_i x + Bu_s) \\ &= -v_i^2 x^T Q_i x - v_i^2 x^T \overline{Q}_i x + 2x^T PH_i x + 2x^T PBu_s \\ &\le -v_i^2 x^T Q_i x + |x|^2 |v_i^2 \overline{Q}_i| + |x|^2 |2PH_i| + 2x^T PBu_s\end{aligned} \tag{13}$$

We choose the supervisory control u_f as

$$u_s = \begin{cases} -\dfrac{\operatorname{sgn}(x^T PB)}{|2x^T PB|} |x|^2 (|2PH_i| + |v_i^2 Q_i|) & \text{if } |2x^T PB| \ne 0 \\ 0 & \text{otherwise}\end{cases} \tag{14}$$

Then we have

$$\dot{V} \le -v_i^2 x^T Q_i x \tag{15}$$

Alternatively, if $u \in R^1$, we can use the following supervisory control (16) and the derivative of the Lyapunov function as (17).

$$u_s = \begin{cases} -(2x^T PB)^{-1} x^T (2PH_i - v_i^2 \overline{Q}_i)x & \text{if } 2x^T PB \ne 0 \\ 0 & \text{otherwise}\end{cases} \tag{16}$$

$$\dot{V} = -v_i^2 x^T Q_i x \tag{17}$$

Therefore the close loop fuzzy system (9) is asymptotically stable for both (14) and (16).

The design procedure of the proposed stable fuzzy controller is as follows:

1. Construct a TS fuzzy model shown in (1) by a proper identification technique.
2. Design gains K_i (i = 1, 2, ..., r) for state feedback compensators for each rule of the obtained TS fuzzy model. Then, we can construct the u_f in (8)
3. Choose ith rule in order to specify remaining design parameters. One possible choice would be the rule at the origin.
4. Calculate P_i and Q_i by solving the Lyapunov equation (11). P and $\bar{Q}_i$ can be obtained by these parameters and those of Step 1 and 2.
5. H_i and v_i can be obtained in the operation phase. Then we can construct the u_s in (8).
6. The proposed controller can be constructed by the combination of u_f and u_s.

Note 1: The selection of one rule is inevitable for the specification of design parameters. Thus, we refer selected rule as the central rule.

Note 2: The proposed fuzzy controller guarantees the stability only for the fuzzy model (1). It means that if the identification of a fuzzy model is poorly performed the obtained controller (8) may not stabilize the original nonlinear system. Thus, this kind of design technique requires an integrated procedure of identification and control.

4. Application to inverted pendulum

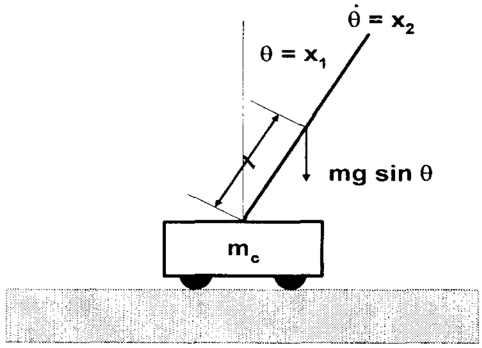

Fig. 2 The inverted pendulum system

To illustrate the proposed approach, let us consider the problem of balancing of an inverted pendulum on a cart. The dynamic equations of the pendulum are [7].

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)u}{4l/3 - aml\cos^2(x_1)} \tag{18}$$

where x_1 is the angle in radians of the pendulum from vertical axis, x_2 is the angular velocity in rad s^{-1}, g = 9.8 m/s^2 is the acceleration due to gravity, m = 2.0 kg the mass of the pendulum, $a = (m + M)^{-1}$ and M = 8.0 kg the mass of the cart, $2l$ = 1.0m is the length of pendulum, and u is the force applied to the cart. A TS fuzzy model to approximate the above system is as follows [7]:

Rule 1: IF x_1 is about 0 THEN $\dot{x} = A_1x + B_1u$

Rule 2: IF x_1 is about $\pi/2$,

THEN $\dot{x} = A_2x + B_2u$

Where,

$$A_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{4l/3 - aml} & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ -\frac{a}{4l/3 - aml} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ \frac{2g}{\pi(4l/3 - aml\beta^2)} & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 \\ -\frac{a\beta}{4l/3 - aml\beta^2} \end{bmatrix}$$

and $\beta = \cos(88^\circ)$. The membership functions for Rule 1 and Rule 2 are shown in Fig. 3.

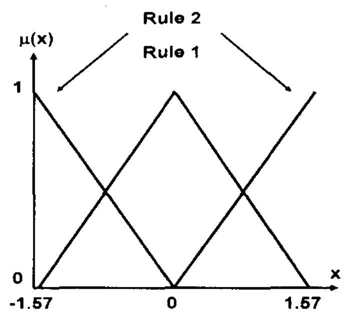

Fig. 3 Membership functions

We simply choose the closed-loop eigenvalues [-2, 2] for $A_1 - B_1K_1$, $A_2 - B_2K_2$, and $Q_1 = diag(10, 10)$, $Q_2 = diag(10, 10)$. Then, we can obtain

$$K_1 = [-120.6667 \quad -22.6667]$$
$$K_2 = [-2551.6 \quad -764.0]$$
$$P_1 = P_2 = \begin{bmatrix} 11.25 & 1.25 \\ 1.25 & 1.5625 \end{bmatrix}$$
$$P = \begin{bmatrix} 22.5 & 2.5 \\ 2.5 & 3.125 \end{bmatrix}$$

If we choose first rule as the central rule, then we have

$$\overline{Q}_1 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

Using these parameters we can construct the proposed stable fuzzy logic controller. We use the supervisory controller in the form of (16) since the inverted pendulum system is the single input system. Figure 4 shows the angle response of the pendulum system (15) for initial conditions x_1 = 45° (0.7854 rad), $x_2 = 0$ by the proposed controller. In order to assess the performance of the proposed controller and compare its performance to that of the conventional linear controller, we apply the controller to the original system (18). Figure 5 shows the response of the pendulum system using linear control $u = -K_1x$ and proposed controller for initial conditions 15° (0.2618 rad), 30° (0.5236 rad), 45°. The solid lines indicate responses with the proposed controller. The dotted lines show those of the linear controller. The proposed method can stabilize an inverted pendulum system for all of the given initial conditions while the linear one fails to balance the pendulum for x_1 = 45°. The simulation results show that the proposed controller can stabilize the nonlinear system although the proposed design procedure is based on the TS fuzzy model instead of the original nonlinear plant.

5. Conclusion

In this paper, we propose a stable fuzzy logic controller architecture for an inverted pendulum. In the design procedure, we represent the fuzzy system as a family of local state space models, which is often called Takagi Sugeno fuzzy model and construct a global fuzzy logic controller by considering each local state feedback controller and a supervisory controller. By this simple modification, one can design a global stable fuzzy controller without finding a common Lyapunov function. Finally, Simulation is performed to control the inverted pendulum to show the effectiveness and feasibility of the proposed fuzzy controller.

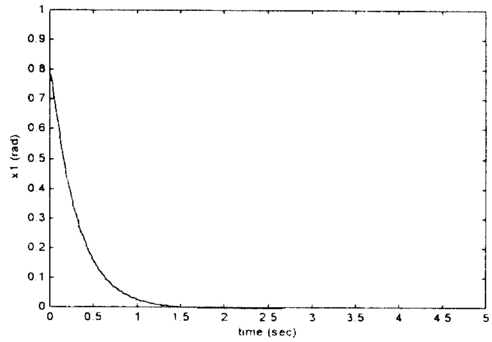

Fig. 4 Output response

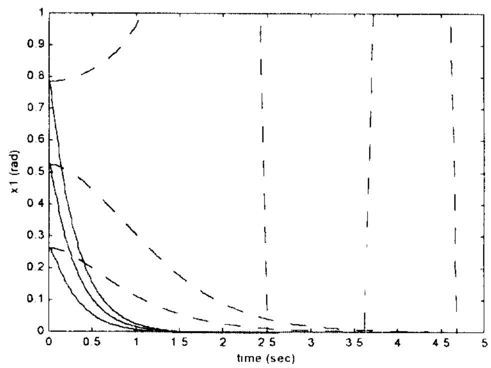

Fig. 5 Output responses by linear and proposed control

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