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Fuzzy Control of Magnetic Bearing System Using Modified PDC Algorithm

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Abstract

A new fuzzy control algorithm for the control of active magnetic bearing (AMB) systems is proposed in this paper. It combines PDC design of Joh *et al.* [8], [9] and Mamdani-type control rules using fuzzy singletons to handle the nonlinear characteristics of AMB systems efficiently. They are named fine mode control and rough mode control, respectively. The rough mode control yields the fastest response for large deviation of the rotor and the fine mode control gives desired transient response for small deviation of the rotor. The proposed algorithm is applied a AMB system to verify the performance of the method. The comparison of the proposed method to a linear controller using a linearized model about the equilibrium point and PDC algorithm in [7] show the superiority of the proposed algorithm.

Keywords : Active Magnetic Bearing, Fuzzy Control, Modified PDC, LMI

1 Introduction

Bearing is an important mechanical element which supports rotating element of any machine. An ideal bearing should have some properties like low loss of mechanical energy due to friction, long life, high ratio of load to unit area, and excellent high speed performance. Especially, excellent high speed performance is very important since every rotary machinery is going toward high speed recently. Therefore, active magnetic bearing (AMB) has become an important issue in the field of rotary machinery since it has most of properties for ideal bearing due to its noncontacting structure. It is, however, very difficult to get good control performance for large air gap because of its high nonlinearity. It is well-known that performance of any linear controller obtained from a linearized model about the nominal equilibrium point becomes worse dramatically when the shaft deviates far from the nominal equilibrium point [1]. Hung proposed that fuzzy control, which is nonlinear control inherently, can be used to improve the sensitivity due to modeling error and control performances [2]. It is, however, too simple to be used for control of practical systems and does not have systematic design methodology.

In general, there are two types of fuzzy control which are so-called Mamdani type and Takagi-Sugeno(T-S) type. Mamdani type fuzzy controllers are usually designed empirically but T-S type fuzzy controllers are designed from several local linear models of nonlinear dynamic equation [3] [4]. Wang *et al.* [5] proposed the so-called PDC(Parallel Distributed Compensator)algorithm as a framework for design of T-S fuzzy controller which uses T-S fuzzy model. Tanaka and Sugeno [6] showed that the stability of the T-S fuzzy model can be checked by finding common symmetric positive definite matrix P satisfying n simultaneous Lyapunov inequalities. It needs, however, predetermined feedback gains to apply the stability criterion to any practical control problem [5] [7] and it becomes trial and error approach. Joh *et al.* proposed a systematic design method to overcome such drawback [8] [9]. It applies the Schur complements [10, page 7] to the previous stability criterion to treat the feedback gains as unknown and derives LMIs(Linear Matrix Inequalities) for desired regions for closed-loop poles to obtain the desired control performances [11] [12].

Nonlinearity of AMB system is very particular so Joh *et al.*'s systematic design method should be modified to handle the nonlinearity of the AMB system. It is named modified PDC algorithm in this paper and applied to the AMB system to get the improved control performances.

2 Modeling and Characteristics of AMB

2.1 AMB System

The AMB system which is used this paper for simulation is in Figure 1. It has two electromagnetic poles which is symmetric about y axis. It is assumed that the rotor is a rigid body with mass m and it moves along the x axis and does not rotate. The same numbers of coil is winded on the two electro-magnetic poles and the areas of the them are the same. The air gap between the poles and the rotor is composed of left air gap and right air gap and the summation of them is G and constant. The displacement of the rotor x is zero when the rotor is placed at the center and the left and right air gap is $\mp G/2$ at that instance respectively. The current in the coil is composed of i_b and i_p and

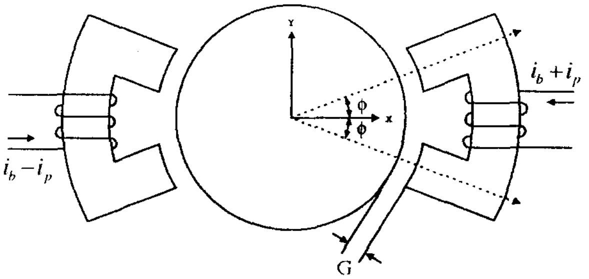

Figure 1: Simplified model of AMB

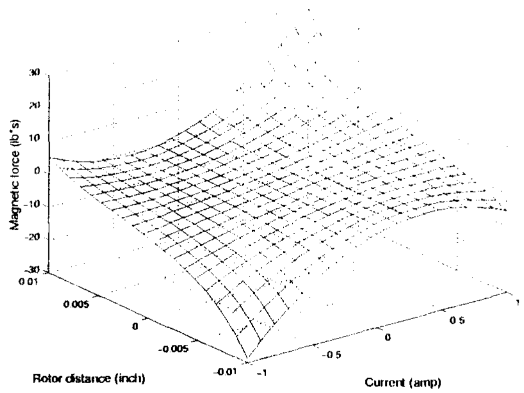

Figure 2: Characteristics of the Electromagnetic Force

they are bias current and perturbation current, respectively. Parameters for the AMB system which is used for simulation is in the Table 1 [7].

2.2 Modeling and Characteristics of AMB

The behavior of the rotor is affected by magnetic force between the rotor and two poles which is governed mainly by i_p and air gap. The Newton's second law for the rotor in this case is

$$\sum F_x = m\ddot{x} \tag{1}$$

where $\sum F_x$ is the resultant force generated by the two electro-magnetic poles. Each poles generates dragging force which is proportional to the square of i_p and inversely proportional to the displacement x. It is approximated [2] [13] as

$$\sum F_x = k\frac{(i_b + i_p)^2}{(G - \beta x)^2} - k\frac{(i_b - i_p)^2}{(G + \beta x)^2}. \tag{2}$$

Figure 2 represents the characteristics of the dragging force with respect to the current and the air gap and it shows its high nonlinearity apparently. It is investigated more thoroughly in the section 3 to compensate the nonlinearity.

3 Modified PDC Algorithm for AMB

3.1 T-S Fuzzy Model of AMB

T-S fuzzy model [3] for dynamic systems is a nonlinear input/output relation which is composed of several locally linearlized submodels. The i^{th} rule for continuous system can be represented as

$$\begin{aligned} &\text{IF } x_1 \text{ is } M_1^i \text{ and } \cdots \text{ and } x_n \text{ is } M_n^i \\ &\text{THEN } \dot{x} = A_i x + B_i u, \quad i = 1, \cdots, r \end{aligned} \tag{3}$$

where x_j : j^{th} state (or linguistic) variable,
M_j : fuzzy term set of x_j,
M_j^i : a fuzzy term of M_j selected for plant rule i,
$x = [\,x_1 \quad \cdots \quad x_n\,]^T \in R^n$,
$u = [\,u_1 \quad \cdots \quad u_m\,]^T \in R^m$,
$A_i \in R^{n\times n}$,
$B_i \in R^{n\times m}$.

T-S fuzzy model for AMB can be obtained by expanding the combined equation of (1) and (2) to Taylor's series. It is a state-space equation which has the displacement of the rotor x_1 and its velocity x_2 as states [7].

$$\begin{aligned} \dot{x} &= Ax + Bu + d \\ y &= Cx \end{aligned} \tag{4}$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & 1 \\ \frac{1}{m}\frac{\partial F_x}{\partial x}(x^*, i_p^*) & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 \\ \frac{1}{m}\frac{\partial F_x}{\partial i_p}(x^*, i_p^*) \end{bmatrix}, \\ d &= \begin{bmatrix} 0 \\ \frac{1}{m}\left(\begin{array}{c} F_x(x^*, i_p^*) - \left[\frac{\partial F_x}{\partial x}(x^*, i_p^*)\right] x^* \\ - \left[\frac{\partial F_x}{\partial i_p}(x^*, i_p^*)\right] i_p^* \end{array} \right) \end{bmatrix}, \\ C &= \begin{bmatrix} 1 & 0 \end{bmatrix}. \end{aligned}$$

The several local linearized model can be obtained from (4) for desired operating points (x^*, i_p^*). They form the r consequent parts in (3).

Seven nominal displacements for the total air gap are selected to compensate the high nonlinearity of the magnetic force as shown in Figure 2 in this paper (See Figure 3). Relationships between the magnetic force and current at two groups of nominal displacements, i.e., $(x = 0, 0.001)$ and $(x = 0.002, 0.004, 0.006, 0.008, 0.01)$, are very different. The first group shows somewhat linear relation but the other group reveals highly nonlinear relation with an inflection point. Therefore, we distinguished them as two different regions, i.e., the first and the second regions are named fine region and rough region, respectively.

Table 1: Model Parameters

Symbol	Parameter	Value	Unit
k	force constant	0.00186	$lb \cdot in^2/amp^2$
β	sensitivity of air gap to shaft disp. $(\partial g/\partial x)$	0.974	
i_b	bias current	0.3	amp
G	nominal air gap	0.02	in
m	mass of the rotor	0.0126	$lb \cdot sec^2/in$

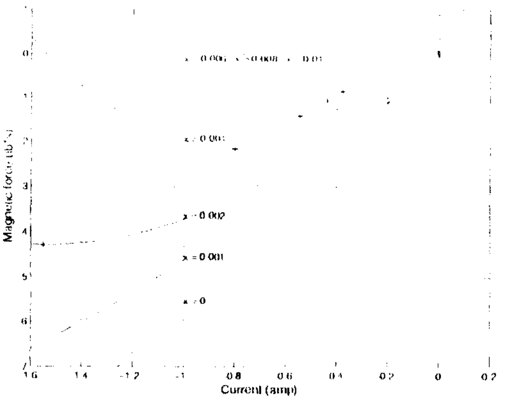

Figure 3: 2 dimensional view of the characteristic of magnetic force

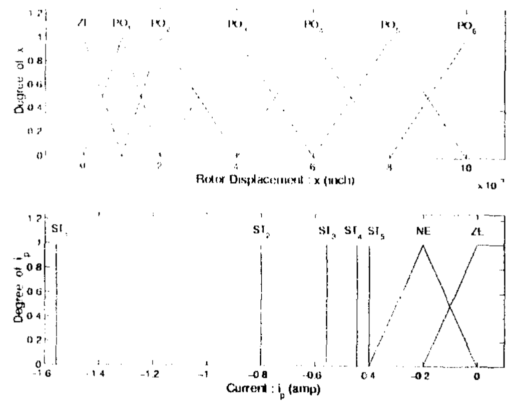

Figure 4: Membership function

The fine region is the region for fine mode control. $x = 0$ and $x = 0.001$ are named linguistically as ZE and PO_1, respectively and two linguistic terms ZE and NE are assigned to current. So, there are four operating points, i.e., (x_i, u_i), in the fine region. The T-S fuzzy model for the fine region in if-then rules is represented as

Plant Rule 1 :
IF $x_1 = ZE$ and $i_p = ZE$, THEN

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 6470.1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 442.9 \end{bmatrix} \tilde{u}$$

Plant Rule 2 :
IF $x_1 = PO_1$ and $i_p = ZE$, THEN

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 6562.8 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 446.0 \end{bmatrix} \tilde{u} + \begin{bmatrix} 0 \\ -0.06 \end{bmatrix}$$

Plant Rule 3 :
IF $x_1 = ZE$ and $i_p = NE$, THEN

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 9345.8 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 442.9 \end{bmatrix} \tilde{u}$$

Plant Rule 4 :
IF $x_1 = PO_1$ and $i_p = NE$, THEN

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 8209.1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 417.1 \end{bmatrix} \tilde{u} + \begin{bmatrix} 0 \\ -4.6 \end{bmatrix}$$

and it yields

$$\dot{x} = \frac{\sum_{i=1}^{r} w_i \left[A_i x + B_i \tilde{u} + d_i\right]}{\sum_{i=1}^{r} w_i} \qquad (5)$$

where

$$w_i = \prod (M_{ix}(x) \cdot M_{i\tilde{u}}(\tilde{u})) \qquad (6)$$

The rough region is the region for rough mode control. The purpose of the rough mode control is to move the rotor to the fine region as fast as possible. There are five nominal displacements as shown in Figure 3. The maximum possible magnetic forces for each nominal displacements are their inflection points. Therefore, the control outputs for these five nominal displacements in the rough region are assigned as fuzzy singletons $(ST_1, \cdots, ST_5)$. Fuzzy membership functions used in this paper are represented in 4.

3.2 Modified PDC Algorithm

The modified PDC algorithm for the control of AMB systems is explained in this section. It uses Joh *et al.*'s [8], [9] pole-placement design method for guaranteed stability and performance in the fine region and Mamdani-type fuzzy control using fuzzy singletons for the fastest response in the rough region.

The i^{th} fuzzy control rule according to the PDC

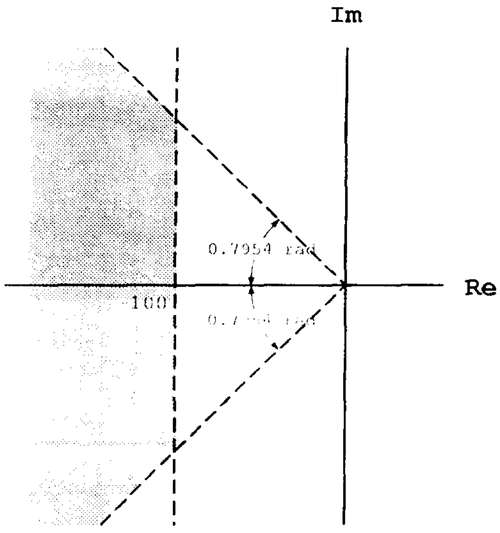

Figure 5: Desired Pole Placement Constraint Region

algorithm in the fine region can be represented as

$$\begin{array}{l}\text{IF } x_1 \text{ is } M_1^i \text{ and } i_p \text{ is } M_2^i \\ \text{THEN } u = -K_i x, \quad i = 1, \cdots, r.\end{array} \tag{7}$$

We use, however, $\tilde{u}$ as in (8) instead of u to cancel out the constant term d in (4).

$$\tilde{u} = -K_i x - k_o \tag{8}$$

where $k_o = B^{-1}d$. Therefore, the control for the fine region can be represented as follows as a result of the inference of (7) and (8).

$$\tilde{u} = \frac{\sum_{i=1}^{r} w_i K_i x}{\sum_{i=1}^{r} w_i}. \tag{9}$$

The desired region of the closed-loop poles for each local T-S model and control pair is represented as the shaded region in the Figure 5. It corresponds to the damping ratio and settling time in (10).

$$\begin{array}{l}\zeta > 0.7 \;\text{ or }\; \%OS < 4.6\ \% \\ \text{and }\; T_s < 0.04\ (sec)\end{array} \tag{10}$$

The shaded region can be represented as LMIs from [11] [12] using the following two convex regions in (11) and (12).

$$f_D(z) = \begin{pmatrix} \sin\frac{\theta}{2}(z+\bar{z}) & -\cos\frac{\theta}{2}(z-\bar{z}) \\ \cos\frac{\theta}{2}(z-\bar{z}) & \sin\frac{\theta}{2}(z+\bar{z}) \end{pmatrix} \tag{11}$$

$$f_D(z) = \begin{pmatrix} 2h_1 - (z+\bar{z}) & 0 \\ 0 & (z+\bar{z}) - 2h_2 \end{pmatrix} \tag{12}$$

where (11) is a conic sector center at the origin and with inner angle θ and (12) is a vertical strip $h_1 < x < h_2$. Matrices L and M which are used for the Joh *et al.*'s [8], [9] method are determined as

$$L = \begin{bmatrix} 200 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.714 & -0.700 \\ 0 & 0.700 & 0.714 \end{bmatrix} \tag{13}$$

Feedback gain K can be determined using Joh *et al.*'s [8], [9] method which is summarized as the following LMIs

$$\begin{array}{l} QA_i^T + A_iQ + V_i^TB_i^T + B_iV_i < 0, \quad i = 1, \cdots, r \\ QA_i^T + A_iQ + QA_j^T + A_jQ + V_j^TB_i^T + B_iV_j \\ +V_i^TB_j^T + B_jV_i < 0, \quad i < j \le r \\ \;[\lambda_{kl}Q + \mu_{kl}A_iQ + \mu_{kl}B_iV_i + \mu_{lk}QA_i^T \\ \;+\mu_{lk}V_i^TB_i^T]_{1\le k,l\le m} < 0, \quad i = 1, \cdots, r \\ Q > \alpha I, \quad \alpha = \text{positive constant} \end{array} \tag{14}$$

where $Q = P^{-1}$ and $V = KQ$.

The common symmetric positive definite matrix P and feedback gain K can be obtained as (15) and (16) by solving (14).

$$P = \begin{bmatrix} 3473.8 & 29.2 \\ 29.2 & 0.3 \end{bmatrix} \tag{15}$$

$$\begin{array}{l} K_1 = \begin{bmatrix} -808.3 & -7.0 \end{bmatrix}, K_2 = \begin{bmatrix} -803.7 & -7.0 \end{bmatrix} \\ K_3 = \begin{bmatrix} -814.7 & -7.0 \end{bmatrix}, K_4 = \begin{bmatrix} -853.2 & -7.4 \end{bmatrix} \end{array} \tag{16}$$

It can be easily seen that the P is symmetric and has eigenvalues as 3474.1 and 0.008. It means the feedback gains K for the fine mode control is stable since satisfies the Wang *et al.*'s [5] stability criterion

$$\begin{array}{r} (A_i + B_iK_i)^TP + P(A_i + B_iK_i) < 0, \quad i = 1, \cdots, r \\ G_{ij}^TP + PG_{ij} < 0, \quad i < j \le r \end{array} \tag{17}$$

where

$$G_{ij} = \frac{(A_i + B_iK_j) + (A_j + B_jK_i)}{2}, \; i < j \le r. \tag{18}$$

The fuzzy control rules for the rough region are represented as Mamdani-type using fuzzy singletons. Each fuzzy singletons in Figure 4 are equal to the maximum magnetic forces as shown in Figure 3. The fuzzy control rules for the rough region yields the result of interpolation of each rules and expressed as

$$\tilde{u} = \frac{\sum_{i=5}^{9} w_i^* c_i}{\sum_{i=5}^{9} w_i^*}. \tag{19}$$

The overall control rules for the control of AMB systems according to the proposed modified PDC algorithm are as follows.

- **Fine Region**

  Control Rule 1 :
  IF $x_1 = ZE$ and $i_p = ZE$, THEN
  $\tilde{u} = -808.3\ x_1 - 7.0\ x_2$

  Control Rule 2 :
  IF $x_1 = PO_1$ and $i_p = ZE$, THEN
  $\tilde{u} = -803.7\ x_1 - 7.0\ x_2$

  Control Rule 3 :
  IF $x_1 = ZE$ and $i_p = NE$, THEN
  $\tilde{u} = -814.7\ x_1 - 7.0\ x_2$

  Control Rule 4 :
  IF $x_1 = PO_1$ and $i_p = NE$, THEN
  $\hat{u} = -853.2\ x_1 - 7.4\ x_2 + 0.01$

- **Rough Region**

  Control Rule 5 :
  IF $x_1 = PO_2$, THEN $\tilde{u} = -1.5556$

  Control Rule 6 :
  IF $x_1 = PO_3$, THEN $\tilde{u} = -0.7988$

  Control Rule 7 :
  IF $x_1 = PO_4$, THEN $\hat{u} = -0.5566$

  Control Rule 8 :
  IF $x_1 = PO_5$, THEN $\hat{u} = -0.4444$

  Control Rule 9 :
  IF $x_1 = PO_6$, THEN $\tilde{u} = -0.4$

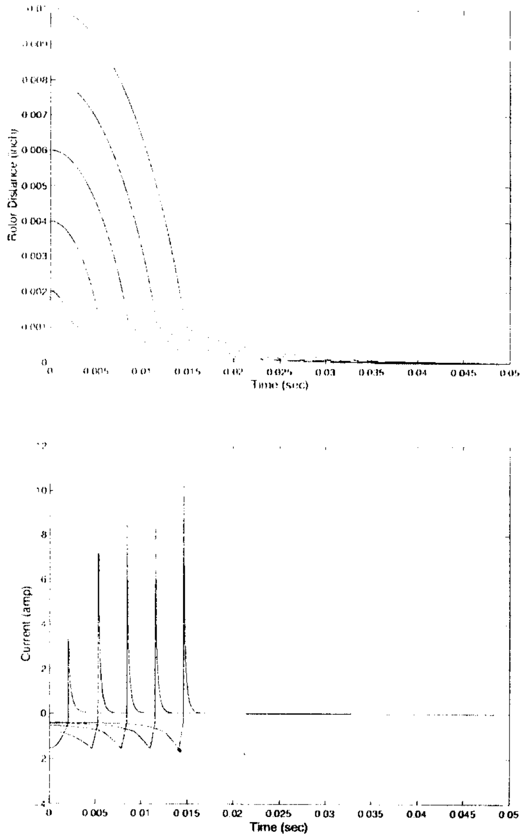

Figure 6: Response of Fuzzy Control with Several Initial Conditions

4 Simulation Results

Simulation results in Figure 6 show the performance of the proposed method for the AMB system. It represents that the AMB system is controlled well for the entire range of air gap for various initial displacements. The effect of the rough mode and fine mode control scheme can be seen clearly. The desired performance specifications for the AMB system are well satisfied.

Figure 7 shows the comparison of the proposed method to a linear controller using a linearized model about the equilibrium point and PDC algorithm in [7]. The linear controller and PDC controller are designed using pole assignment method whose desired poles in the shaded region in the Figure 5. We can see the superiority of the proposed method against the other control methods. It is the reason for this result that the later two methods can not handle the highly nonlinearity of magnetic force efficiently.

5 Concluding Remarks

We proposed the so-called modified PDC algorithm which shows superior performance for the control of AMB systems. It combines PDC design of Joh *et al.* [8], [9] and Mamdani-type control rules using fuzzy singletons to handle the nonlinear characteristics of AMB systems efficiently. They are named fine mode control and rough mode control, respectively. The rough mode control yields the fastest response for large deviation of the rotor and the fine mode control gives desired transient response for small deviation of the rotor. The simulation results show that the proposed method has superior performance against the other control method for AMB systems in the literature. Furthermore, it can be readily implemented to the real AMB systems since the required computation is very small.

The authors believe that the proposed method may be applied well to other nonlinear systems.

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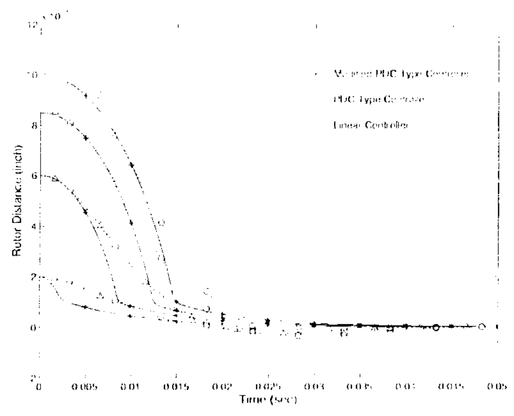

Figure 7: Control Performance Compare with Other Controller

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