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Acquirement and Linguistic Expressions of Fuzzy Rules

Satoru Maebashi
Master's Program
in Science and Engineering,

Takehisa Onisawa*
Institute of Engineering Mechanics,
Email: onisawa@esys.tsukuba.ac.jp,

University of Tsukuba 1-1-1, Tennodai, Tsukuba, 305-8573 Japan
*Corresponding author

Abstract *Fuzzy rules are often obtained by experts who know an objective system well. Fuzzy rules acquired by experts, however, do not express all input-output relations of the system. This paper proposes a method of fuzzy rules acquisition by genetic algorithm, which uses an integer coding. Obtained fuzzy rules are expressed in plain language so that the fuzzy rules are understood easily. The proposed method is applied to the control of the distance between cars and running through a crank-typed road, and the validity of the method is confirmed.*

1. Introduction

Fuzzy Logic Controller (FLC) is usually designed by experts who know a controlled system well. However, it is difficult for experts to have every knowledge of system input-output relations in order to control the system and fuzzy rules of the FLC obtained by experts do not necessarily express control rules well. In recent years, neural network and Genetic Algorithm(GA) are applied to fuzzy rules acquisition in order to solve above problems[1-4]. However expressions of acquired knowledge in plain language[5] have not been considered.

This paper proposes a method which applies GA to the acquirement of fuzzy rules and expresses obtained fuzzy rules in plain language. The method is consisted of two parts; Fuzzy Rules Generation part by GA and Linguistic Expressions part of obtained fuzzy rules. The Fuzzy Rules Generation part codes parameter values of membership functions of fuzzy rules not in a binary type but in an integer type, and generates fuzzy rules by applying integer coded chromosomes to GA. In the Linguistic Expressions part, membership functions of the acquired rules are labeled by plain language. The proposed method is applied to the control of the distance between cars and running through a crank-typed road.

2. Fuzzy rules generation with GA

Coding a fuzzy rule to a chromosome is necessary to acquire fuzzy rules with GA. However binary coding, the ordinary method for simple GA, is not effective when a chromosome is long.

In this paper an integer coding of fuzzy rules is used, and GA operators are redefined for the integer coding.

2.1 An individual as a set of chromosomes with integer coding

In this paper, the trapezoidal membership function is used for the representation of a fuzzy set since it is more flexible in its expression than the triangular one. The trapezoidal membership function has a set of 4 parameter values (as, az, bs, bz), where the as is the beginning point and the az is the end point of which membership values are equal to 1.0, and the bs and the bz express the left and the right widths of the membership function respectively. The as and the az take a value in [0,100], on the other hand the bs and the bz take the one in [0,50]. These parameter values are coded by integers as shown in Fig.1, which is called an integer coding in this paper.

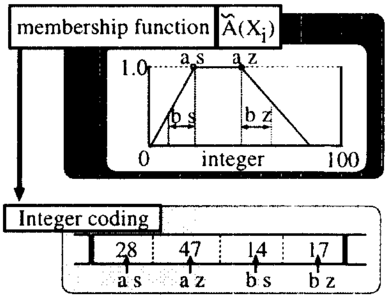


Fig.1. Integer coding

This paper considers that a set of fuzzy rules composed of antecedent(if part) statements and consequent(then part) statements represents the knowledge of an objective system. Therefore, an individual has a set of fuzzy rules as shown in Fig.2, that is, an individual has a set of some chromosomes.

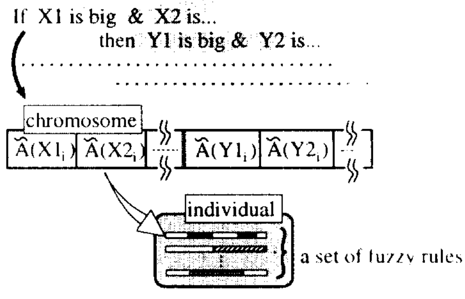


Fig.2. Individual

2.2 GA operators for individuals

Fuzzy rules in an individual are improved by GA operators, a mutation and a crossover. Conventional GA operators defined for binary coded chromosomes, which change bits on a chromosome, are not useful for the individual as mentioned in 2.1. In this paper GA operators for integer coded chromosomes are defined as follows.

The crossover is defined as a change of chromosomes which are selected at random from two individuals as shown in Fig.3. The two individuals are also selected at random. This implies that a part of the character of each parent is changed and inherited.

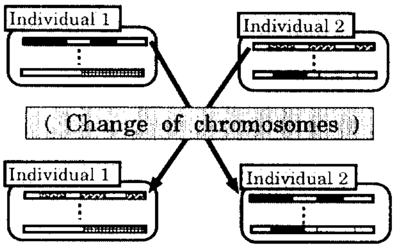


Fig.3. Crossover

A mutation is usually considered as generating a new character which parents do not have. The definition of a mutation in this paper is divided into three types as shown in Fig.4. The first type mutaion means the shape transformation of a membership function. This type mutation is performed by the change of integer coded four parameter values (as, az, bs, bz) in a chromosome. The second type mutation is the parallel shift of a membership function. This is done by the change of integer coded two parameter values (as, az) in a chromosome. The third type mutation is done by the mirror image change of each membership function. This means that the shape of a mutated membership function and that of a pre-mutated one are symmetry with respect to the center of the domain of a membership function.

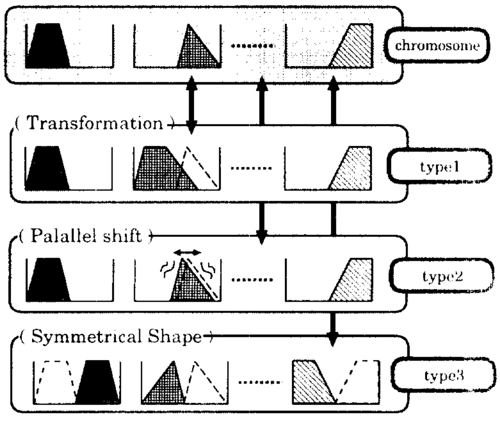


Fig.4. 3 types of mutation

2.3 Evaluation of an individual

Evaluation of each chromosome in an individual has the advantage of the evaluation of a partially meaningful chromosome for an objective system. This evaluation method, however, has a difficulty to evaluate whether chromosomes in fuzzy rules are appropriate for the representation of a system behavior as a whole or not. This paper considers an individual, i.e., a set of some chromosomes, as a set of fuzzy rules and obtained fuzzy rules are applied to a fuzzy control system. Therefore, this paper employs a Pittsburgh approach [6], and the error between a system response and a reference value is considered as the evaluation. Eq.(1) and Eq.(2) are employed as evaluation functions.

$$H(i) = \sum_{t=0}^{T} |u(t) - f_i(t)| \cdot t \quad (i = 1, \cdots, M) \quad (1)$$

$$N = min(H(i)) \quad (2)$$

where t is time, $u(t)$ is the reference value, $f_i(t)(i = 1, \cdots, M)$ is a system response when the i-th individual is used as a set of fuzzy rules, and T is the time within which a system state becomes steady.

The individual, which makes the error a minimum, is selected as a parent in a next generation. That is, the strategy of *elitist selection*[1][6] is used. However other individuals, of which evaluations are not so good, are also selected at random as parents in a next generation because of efficient search of an optimal solution.

All initial outputs for whole inputs spaces are assumed to be zero. The output zero means the maintenance of a system state as a control strategy.

2.4 Restrictions for individuals in each generation

The number of fuzzy rules is unknown beforehand. The acquisition of fuzzy rules is equivalent to the determination of the number of fuzzy rules. And the following five restrictions are considered for the simplification of the fuzzy rules acquisition algorithm.

(1)The initial number of fuzzy rules in an individual is 2. (2)The number of individuals for each genaration is 50. (3)If the evaluation is not improved for 30 generations, another one chromosome, i.e., another one fuzzy rule, is added into each individual in order to escape a local solution of GA. (4)The maximum number of chromosomes in each individual is assumed to be 15 because of the limited simulating time. (5)If more than two similar chromosomes, i.e., fuzzy rules, in an individual are obtained, they are unified as mentioned later.

2.5 Unification of similar rules

If similar rules are obtained in an individual , they are unified. The similarity measure $Sim(X_{ij})$ is defined by Eq.(3) in order to estimate the similarity of two fuzzy sets.

$$Sim(X_{ij}) = \max\left(\frac{W_{area}(\mu_i(x) \wedge \mu_j(x))}{W_{area}(\mu_i(x))}, \frac{W_{area}(\mu_i(x) \wedge \mu_j(x))}{W_{area}(\mu_j(x))}\right) \quad (3)$$

where $\mu_i(x)$ and $\mu_j(x)$ are membership functions representing fuzzy sets of an antecedent statement about an element X in the i-th and in the j-th fuzzy rules, $\wedge$ stands for a minimum operation, and $W_{area}(\mu_i(x))$ is the weighted value of $\mu_i(x)$ defined by

$$W_{area}(\mu_i(x)) = \int_0^{1.0} \alpha \cdot (\mu_i(x_{max}) - \mu_i(x_{min}))d\alpha \quad (4)$$

where α-cut of $\mu_i(x)$ is $[\mu_i(x_{min}), \mu_i(x_{max})]$.

Now if Eq.(5) is satisfied with the i-th and the j-th fuzzy rules, they are estimated to be similar and they are unified.

$$I < \min(Sim(X1_{ij}) \cdots Sim(Xp_{ij}), Sim(Y1_{ij}), \cdot\cdot, Sim(Yq_{ij})) \quad (5)$$

where $Sim(X1_{ij}), Sim(X2_{ij}), \cdots, Sim(Xp_{ij})$(p is the number of elements of antecedent statements) are similarities of antecedent statements about $X1, X2, \cdots, Xp$ elements between the i-th and the j-th fuzzy rules. $Sim(Y1_{ij}), Sim(Y2_{ij}), \cdots, Sim(Yq_{ij})$(q is the number of elements of the consequent statements) are similarities of consequent statements about $Y1, Y2, \cdots, Yq$ elements between the i-th and j-th fuzzy rules. And I is a threshold. In this paper I is set as 0.8.

If the right side of Eq.(5) is larger than I, the i-th and the j-th rules are estimated to be similar. If two fuzzy rules are estimated to be similar, fuzzy sets in antecedent and in consequent statements are combined by the union of them. The union of fuzzy sets, however, is not usually a convex fuzzy set. A non-convex fuzzy set is changed to a convex fuzzy set as shown in Fig.5 in order to express fuzzy rules in plain language. The convexity has an influence on inference results before the convexity since shapes of membership functions are changed a little by the convexity. Therefore, parameters of membership functions of convexed fuzzy sets are readjusted by GA, where initial parameters values of membership functions are those of convexed fuzzy sets.

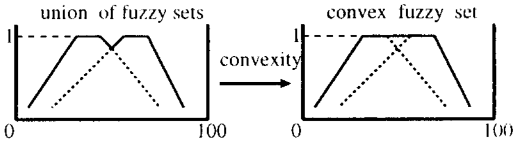


Fig.5. Convexity

3. Linguistic expressions of obtained fuzzy rules

Linguistic expressions of acquired fuzzy rules help to make them more comprehensible for the people who use them. The obtained fuzzy rules are expressed in plain language according to the following procedures.

(1)Fuzzy sets $B_i (i = 1, \cdots, 15)$ with linguistic labels are prepared beforehand.

(2)The similality between $\mu_{B_i}(x)$ and the membership function of a fuzzy set A, which is one of antecedent or consequent statements in obtained fuzzy rules $\mu_A(x)$ is estimated by

$$DS1_i = \frac{W_{area}(\mu_A(x) \wedge \mu_{B_i}(x))}{W_{area}(\mu_A(x) \vee \mu_{B_i}(x))} \quad (i = 1, \cdot\cdot, 15) \quad (6)$$

where $\vee$ stands for a maximum operation.

(3)The fuzzy set B_i^*, which makes the $DS1_i$ the largest, is selected, and the fuzzy set A is expressed by the linguistic label of B_i^*.

If B_i^* and B_j^* $(i \neq j)$ are selected by Eq.(6), the second similarity measure $DS2_k$ is used in order to select only one.

$$DS2_k = \frac{\max(W_{area}(\mu_A(x)), W_{area}(\mu_{B_k^*}(x)))}{W_{area}(\mu_A(x) \vee \mu_{B_k^*}(x))} \quad (k = i, j) \quad (7)$$

where $DS2_k$ is the ratio of the weighted value of the union of A and B_k^* to the weighted value of A or B_k^*.

4. Experiment 1: Control distance between cars

4.1 Situations

The proposed method is applied to a simple fuzzy control system. In this application the distance between two cars is controlled with 2 inputs and 1 output fuzzy rules.

A following car runs after a preceding car keeping the distance between two cars constant. The initial conditions are that a preceding car runs at a constant speed and that a following car starts running when a preceding car passes by a following car. The fuzzy control system controls the acceleration of the following car in order to keep the distance constant.

Inputs of the fuzzy control system are a relative distance ($-100m \leq x1 \leq 100m$) and a relative velocity ($-10m/s \leq x2 \leq 10m/s$) between two cars, and an output of the fuzzy control system is a relative acceleration($-5.56m/s^2 \leq y \leq 5.56m/s^2$).

The negative relative distance means that a following car is ahead of a preceding car, and the positive relative distance means that a following car is behind a preceding car. The initial number of fuzzy rules is 2 and the reference relative distance is settled x=0(m).

4.2 Simulation results

Table 1 shows the evalutions for acquired fuzzy rules with their number of fuzzy rules from 2 through 6. Table 1 shows that evaluation does not change even when the number of rules increases, since the fuzzy control system does not have situations which correspond to the added rules. Fig.6 shows the acquired fuzzy rules when their number is 2.

Table 1 :Evaluations for each fuzzy rule

Number	Generation	Evaluation
2	60	2548
3	120	2548
4	160	2548
5	220	2548
6	260	2548

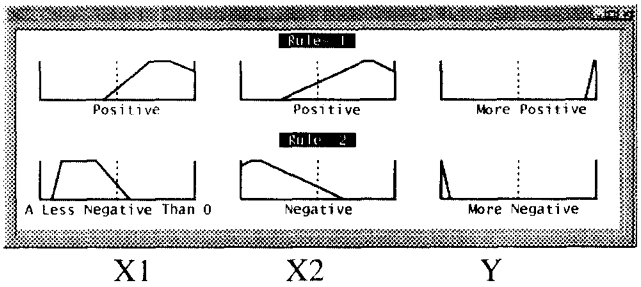


Fig.6. Acquired fuzzy rules

Linguistic expressions of obtained fuzzy rules are

Rule 1 :

If a following car is behind a preceding car and a speed of a following car is slower than that of preceding car, then accelerate a following car more.

Rule 2 :

If a following car is a little ahead of a preceding car and a speed of a following car is faster than that of a preceding car, decelerate a following car more.

Fig.7 shows control results with three different reference distances $-50(m)$, $0(m)$, and $50(m)$ by the use of fuzzy rules shown in Fig.6. It is found that although the fuzzy rules in Fig.6 are obtained under the condition that the reference distance is $0(m)$, and overshoots in reference distances $-50(m)$ and $50(m)$ are rather big, control results of the reference distance $-50(m)$ and $50(m)$ are also comparative good. And it is found that linguistic expressions of acquired fuzzy rules are easy to understand and reasonable.

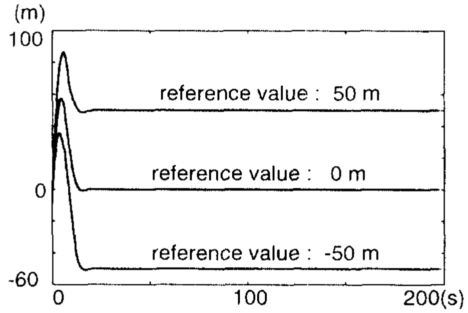


Fig.7. Simmulation results

5. Experiment 2: Running through crank-typed road

5.1 Situations

The proposed method is applied to the control of the running through a crank-typed road[7] with 4 inputs and 2 outputs fuzzy rules.

Inputs of the fuzzy control system are a distance between a forward wall and a car, $0(m) \leq x1 \leq 10(m)$, a distance between a left side wall and a car, $0(m) \leq x2 \leq 10(m)$, a distance between a right side wall and a car, $0(m) \leq x3 \leq 10(m)$, and a direction angle of a car, $-90(deg) \leq x4 \leq 90(deg)$. If each vaule of $x1$, $x2$, and $x3$ are smaller than $0(m)$, its value is assumed to be $0(m)$, and if $x1$, $x2$, and $x3$ are larger than $10m$, each is assumed to be $10(m)$. If the value of $x4$ is smaller than $-90(deg)$, $x4$ is assumed to be $-90(deg)$, and if larger than $90(deg)$, $x4$ is $90(deg)$.

Outputs of the fuzzy control system are the degree of the angle of the wheel, $-30(deg) \leq y1 \leq 30(deg)$, and the value of acceleration of a car, $-10(m/s^2) \leq y2 \leq 10(m/s^2)$, with the condition that the velocity of a car is not over the upper limit $10m/s$ and the lower limit $-10m/s$. The lenght of the axle is $L = 2(m)$.

5.2 Learned concepts

People who drive a car along a crank-typed road usually have concepts, "go straight", "turn to the left" , "turn to the rigth". They control a car keeping their balance. These concepts are considered in the acquisition of fuzzy control rules.

The number of chromosomes in an initial individual is assumed to be three. Individulals, which can control a car along a crank-typed road well for 12 initial conditions as shown in Fig.8, are selected as parents in a next generation.

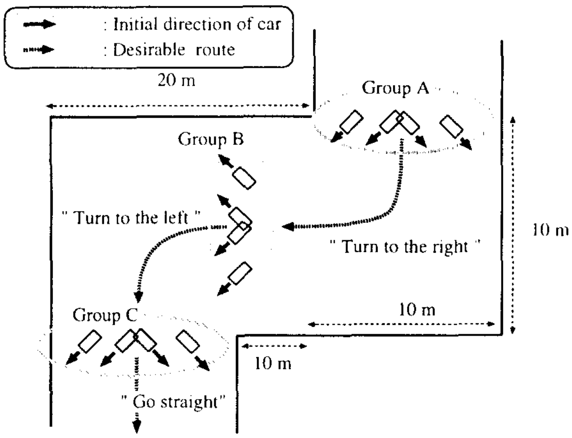


Fig.8. 12 initial states of a car

The group A of initial states is considered as the concept "turn to the right". The group B of initial states is considered as the concept "turn to the left", and the group C is as the concept "go straight"

5.3 Evaluation for an individual

The evaluation space as shown in Fig.9 is defined, where the desirable area as shown in Fig.9 corresponds to Eq.(1). Individuals, which make a car pass along the route with better evaluations, are selected by GA.

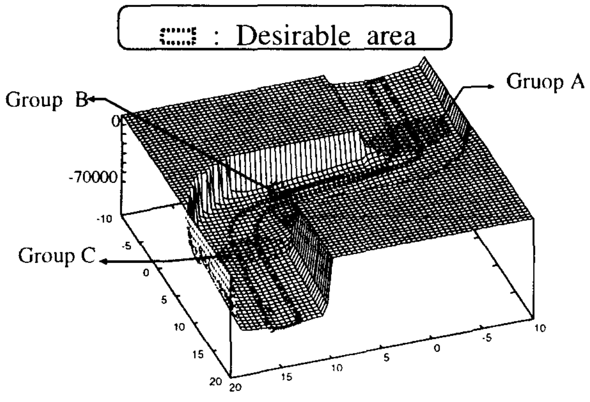


Fig.9. Evaluation space

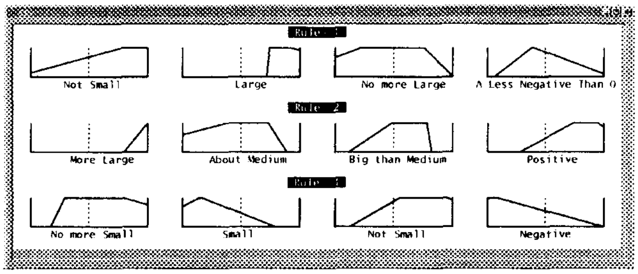
5.4 Simulation results

Table 2 shows evaluations for acquired fuzzy rules with their number from 3 through 8, where the smaller value, the better of the control result.

It is found that evaluations are almost similar among 3 rules through 8 rules. Obtained fuzzy rules with the number 3 are shown in Fig.10 and Fig.11. Fig.10 shows the antecedent part and Fig.11 shows the consequent part.

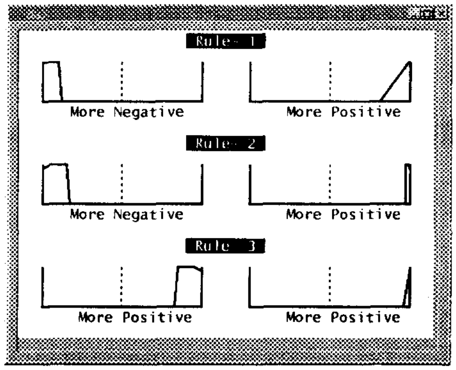
Table 2 : Evaluations for each fuzzy rule

Number	Generation	Evaluation
3	360	-39276685
4	400	-39272349
5	700	-39301380
6	740	-39300534
7	800	-39304863
8	1020	-39308282



(x1) (x2) (x3) (x4)

Fig.10. Antecedent part



(y1) (y2)

Fig.11. Consequent part

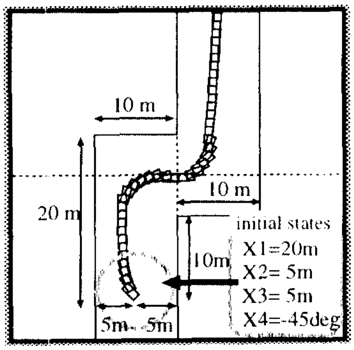
From Fig.11 it is found that the degree of the angle of the wheel ($y1$) is apt to take an extreme value, and that the value of the acceleration ($y2$) is always positive, i.e., accelerative. These reflect the following considerations. (1)The fuzzy control system must control a car along a crank-typed road with only a few rules. The control system controls a car by the interpolation of the inference results by the use of a few extreme rules with respect to $y1$. (2)Owing to the character of the evaluation space, only individuals which make a car move forward are selected.

Table 3 shows linguistic expressions of obtained rules. For example, the rule1 implies that if right wall is not farther, turn a handle more left.

Table 3 : Linguistic expressions of obtained rules

Rule 1 (IF Part)	Rule 1 (THEN Part)
Forward Wall is not near, Left Wall is far Right Wall is not farther, Direction is a little left ,	Turn a handle more left, Accelerate more
Rule 2 (IF Part)	**Rule 2 (THEN Part)**
Forward Wall is far, Left Wall is middle, Right Wall is a little far, Direction is right,	Turn a handle more left, Accelerate more
Rule 3 (IF Part)	**Rule 3 (THEN Part)**
Forward Wall is not near, Left Wall is near, Right Wall is not near, Direction is left	Turn a handle more right Accelerate more

Fig.12 shows results of computer simulations by the use of the acquired fuzzy rules where road types in Fig.12 are different from the one in Fig.8. It is found that a car is controlled well.



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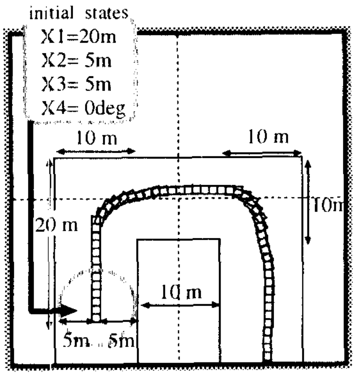

Type -2

Fig.12. Results of computer simulations.

6. Conclusion

This paper proposed a method to acquire fuzzy rules by using the integer coding and GA, and a method to express aquired fuzzy rules in plain language. The presented method is applied to the control of the distance between cars and running through a crank-typed road. Simulation results show the validity of the presented method.

In the experiment 2 the expressions of obtained fuzzy rules are not necessarily comprehensible to us, since this paper does not consider the comprehensibility of obtained rules. As the open problem, the comprehensibility is necessary to be considered in the expression of knowledge acquisition.

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