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ON FUZZY VIETORIES TOPOLOGY

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Abstract:We introduce the concept of a fuzzy vietories topology and we obtain its fundamental properties.

1. Preliminaries.

Let $I = [0,1]$. For a set X, Let I^X be the collection of all the mappings from X into I. Each member of I^X is called a fuzzy set in X $(cf.(6))$. Let $F_p(X)$ denote the collection of all the fuzzy points in a set X $(cf.(1))$

Definition 1.1[4]. A fuzzy point x_λ in a set X is said to be *quasi-coincident*(*q-coincident*, in shorts) with a fuzzy set A in X, denoted by $x_\lambda qA$, if $\Lambda + A^c(x) > 1$ or $\lambda + A(x) > 1$. A fuzzy set A is said to be *q-coincident* with a fuzzy set B, denoted by AqB, if there exists an $x \in X$ such that $A(x) > B^c(x)$ or $A(x) + B(x) > 1$. In this case, we say that A and B are *q-coincident.*

Definition 1.2[1]. A subfamily $\mathcal{T}$ of I^X is called a *fuzzy topology* on X if $\mathcal{T}$ satisfies the following conditions:

(i) $\emptyset, X \in \mathcal{T}$,

(ii) If $\{U_\alpha : \alpha \in \Lambda\} \subset \mathcal{T}$, the $\bigcup_{\alpha\in\Lambda} U_\alpha \in \mathcal{T}$, where Λ is an index set,

(iii) If $A, B \in \mathcal{T}$, then $A \cap B \in \mathcal{T}$.

Each member of $\mathcal{T}$ is called a *fuzzy open set* in X and its complement a *fuzzy closed set* in X. The pair $(X, \mathcal{T})$ is called a *fuzzy topological space*(*fts*, in short)

Theorem 1,A[4]. *$A \subset B$ if and only if A and B^c are not q-coincident(denoted by $A\overline{q}B^c$). In particular, $x_\lambda \in A$ if and only if $x_\lambda \overline{q}A$.*

Definition 1.3[4]. For a fuzzy set A in a fts $(X, \mathcal{T})$, the *closure* $\overline{A}$ and the *interior*, $\overset{\circ}{A}$ of A are defined respectively ,as

$$\overline{A} = \bigcap\{B : A \subset B, B^c \in \mathcal{T}\} \qquad \text{and} \qquad \overset{\circ}{A} = \bigcup\{B : B \subset A, B \in \mathcal{T}\}.$$

Theorem 1.B[4]. *Let X be a fts and let $A \in I^X$. Then:*
(a) $X_\lambda \in \overset{\circ}{A}$ if and only if X_λ has a neighborhood contained in A.
(b) $x_\lambda \in \overline{A}$ if and only if for each q-neighborhood V of x_λ, VqA.

Theorem 1.C[4]. *Let X be a fts and let $A \in I^X$. Then,*

$$\overset{\circ}{A} = (\overline{A^c})^c \qquad \textit{and} \qquad \overline{A} = ((\overset{\circ}{A^c}))^c.$$

Definition 1.4[4]. A fts X is said to be:
(a) $\mathcal{T}_0$, if for any two distinct fuzzy points x_λ and y_μ:
(Case 1) When $x \neq y$, either x_λ has an open nbd which is not q-conincident with y_μ or y_μ has an open nbd which is not q-coincident with x_λ.
(Case 2) When $x = y$ and $\lambda < \mu$(say), then there exists a q-nbd V of y_μ which is not q-coincident with x_λ.
(b) $\mathcal{T}_1$, if for any two distinct fuzzy points x_λ and y_μ:
(Case 1) When $x \neq y$, x_λ has an open nbd which is not q-coincident with y_μ and y_μ has an open nbd which is not q-coincident with x_λ.
(Case 2) When $x = y$, and $\lambda < \mu$(say), then there exists a q-nbd V of y_μ such that $x_\lambda qV$.
(c) $\mathcal{T}_2$(*Hausdorff*) if for any two distinct fuzzy points x_λ and y_μ:
(Case 1) When $x \neq y$, then x_λ and y_μ have open nbds which are not q-coincident.
(Case 2) When $x = y$ and $\lambda < \mu$(say), then y_μ has an open q-nbd V and x_λ has an open nbd U such that $V\overline{q}U$.

Theorem 1.D[2]. *A fts X is $\mathcal{T}_1$ if and only if every singlton set is closed in X.*

2. Definition of a fuzzy Vietories topology

Notations. Let X be a fts. then:
(a) 2^X={E : E is nonempty and closed in X}.
(b) $2^A=\{E \in 2^X : E \subset A\}$, where $A \in I^X$.

Proposition 2.1. *Let X be a fts and let $A \in I^X$. Then*

$$2^X - 2^{A^c} = \{E \in 2^X : EqA\}.$$

Lemma 2.2. *Let X be a fts and let $\mathfrak{T}$ be a collection of all sets 2^G and of all sets $2^X - 2^{G^c}$, where G is a fuzzy open set in X. Let $\mathfrak{B}_e$ be the collection of all finite intersections of membrs of $\mathfrak{T}$. Then for each $\mathbb{B} \in \mathfrak{B}_e$,*

$$\mathbb{B} = \{E \in 2^X : E \subset A_0, EqA_i \text{ for each } i = 1, \cdots, n\},$$

where A_i is open in X for each $i = 0, 1, \cdots, n$. In this case, $\mathbb{B}$ will be denoted as $< A_0, A_1, \cdots, A_n > e$.

Theorem 2.3. *Let $(X, \mathcal{T})$ be a fts and let $\mathfrak{T}$ be a collection of all sets 2^G and of all set $2^X - 2^{G^c}$, where G is a fuzzy open set in X. Then there is a unique fuzzy topology $\mathcal{T}_e$ on 2^X such that $\mathfrak{T}$ is a subbase for $\mathcal{T}_e$. In fact, $\mathfrak{B}_e$ is a base for $\mathcal{T}_e$, and $\mathcal{T}_e$ an ordinary topology on 2^X.*

Definition 2.4. Let $(X, \mathcal{T})$ be a fts. Then the **fuzzy Vietories**(or *finite*)topology $\mathcal{T}_v$ on 2^X is the generated by the collection of the forms $< U_1, \cdots, U_n >_v$ with $U_1, \cdots, U_n$ fuzzy open sets in X, where $< U_1, \cdots, U_n >_v = \{E \in 2^X : E \subset \bigcup_{i=1}^n U_i$ and EqU_i for each $i = 1, \cdots, n\}$.

In fact, $\mathcal{T}_v$ is an ordinary topology on 2^X.

Theorem 2.5. *The collection $\mathfrak{B}_v$ of the forms $< U_1, \cdots, U_n >_v$ with $U_1, \cdots, U_n$ fuzzy open sets in X, forms a base for $\mathcal{T}_v$.*

Theorem 2.6. *$\mathfrak{B}_e$ and $\mathfrak{B}_v$ are equivalent. Hence $\mathcal{T}_v = \mathcal{T}_e$.*

3. Fundamental properties.

Proposition 3.1. *Let X be afts. then:*

(a) $2^{A_0 \cap A_1} = 2^{A_0} \cap 2^{A_1}$ and generally $2^{\bigcap_\alpha A_\alpha} = \bigcap_\alpha 2^{A_\alpha}$, where $A_0, A_1, A_\alpha \in I^X$.

(b) $A \subset B$ if and only if $2^A \subset 2^B$.
Hence $A = B$ if and only if $2^A = 2^B$.

Lemma 3.2. *Let $(X, \mathcal{T})$ be a fuzzy $\mathcal{T}_1$-space and let $A \in I^X$. Then:*

(a) If $A(x) < 1$ for each $x \in X$, then $\overline{2^A} = 2^{\overline{A}}$.

(b) $\overset{\circ}{\widehat{2^A}} = 2^{\overset{\circ}{A}}$

Theorem 3.4. *Let $(X, \mathcal{T})$ be fuzzy $\mathcal{T}_3$-space, and $A \in I^X$. Then the set $\{E \in 2^X : A \subset E\}$ is closed in $(2^X, \mathcal{T}_v)$.*

Theorem 3.5. *Let $(X, \mathcal{T})$ be a fts. Then:*
(a) $(2^X, \mathcal{T}_v)$ is always $\mathcal{T}_0$.
(b) If X is $\mathcal{T}_1$, then 2^X is $\mathcal{T}_1$. But the converse is false.

Example 3.6. Let X be a finite set containing more than two points. Let the topology $\mathcal{T}$ on X be the fuzzy trivial topology. Then $2^X = \{X\}$. So 2^X is $\mathcal{T}_1$. But $(X, \mathcal{T})$ is not $\mathcal{T}_1$.

Definition 3.7. A fuzzy set A in a fts X is said to be *dense* in X if $\overline{A} = X$. In particular, A is said to be *countably dense* in X if A is dense in X and $S(A)$ is countable. If X has a fuzzy countable dense set, we say that X is *separable.*

Theorem 3.8. *Let $\mathfrak{F}(X)$ be the family of all the fuzzy finite sets in a fuzzy $\mathcal{T}_1$-space X. Then $\mathfrak{F}(X)$ is dense in $(2^X, \mathcal{T}_v)$.*

Theorem 3.9. *X is separable if and only if 2^X is separable .*

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