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On fuzzy semi-topological properties

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Abstract : We introduce the concept of a fuzzy irresolute mapping and a fuzzy semi-homeomorphism. And we find some properties of them

1. Preliminaries

Let $I = [0, 1]$. For a set X, let I^X denote the collection of all mappings from X into I. A member A of I^X is called a *fuzzy set* of (or in) X (cf. [6]).

Definition 1.1[2]. A subfamily $\mathfrak{T}$ of I^X is called a *fuzzy topology* on X if $\mathfrak{T}$ satisfies the following conditions:

(i) $\emptyset, X \in \mathfrak{T}$. (ii) If $\{ U_\alpha : \alpha \in \Lambda \} \subset \mathfrak{T}$, then $\bigcup_{\alpha \in \Lambda} U_\alpha \in \mathfrak{T}$, where Λ is an index set.

(iii) If $A, B \in \mathfrak{T}$, then $A \cap B \in \mathfrak{T}$.

Members of $\mathfrak{T}$ are called *fuzzy open sets* in X and their complements *fuzzy closed sets* in X. The pair $(X, \mathfrak{T})$ is called a *fuzzy topological space(fts*, in short).

Notation 1.A. For a fts X, let :

(a) $FO(X)$ denote the collection of all the fuzzy open sets in X.

(b) $FC(X)$ denote the collection of all the fuzzy closed sets in X.

Definition 1.2[1]. For a fuzzy set A in a fts $(X, \mathfrak{T})$, the *closure*, $\bar{A}$ and the *interior*, $\mathring{A}$ of A are defined respectively, as

$\bar{A} = \bigcap \{ B : A \subset B, B^c \in \mathfrak{T} \}$ and $\mathring{A} = \bigcup \{ B : B \subset A, B \in \mathfrak{T} \}$.

Definition 1.3[1]. Let A be a fuzzy set in a fts X. Then :

(a) A is called a *fuzzy semi-open set(f-semi-open set*, in short) in X if there is a $B \in FO(X)$ such that $B \subset A \subset \overline{B}$.

(b) A is called a *fuzzy semi-closed set(f-semi-closed set*, in short) in X if there is a $B \in FC(X)$ such that $B \subset A \subset \overline{B}$.

Notation 1.B. For a fts X, let :

(a) $FSO(X)$ denote the collection of all f-semi-open sets in X

(b) $FSC(X)$ denote the collection of all f-semi-closed sets in X

It is clear that $A \in FSO(X)$ if and only if $A^c \in FSC(X)$.

Proposition 1.4. Let X be a fts. Then :

(a) $FO(X) \subset FSO(X)$ and $FC(X) \subset FSC(X)$.

(b) If $A \in FOS(X)$ and $A \subset B \subset \overline{A}$, then $B \in FSO(X)$.

Throughout the next sections, X, Y, $Z \cdots$etc, will denote fuzzy topological spaces.

2. Some properties of f-irresolute mappings.

The characterization of f-irresolute mapping was known in [7]. We will investigate another properties of f-irresolute mappings.

Definition 2.1[1,2,4,5]. Let $f: X \to Y$ be a mapping. Then f is said to be :

(i) *fuzzy continuous (f-continuous,* in short) if $f^{-1}(A) \in FO(X)$ for each $A \in FO(Y)$ or equivalentely $f^{-1}(B) \in FC(X)$ for each $B \in FC(Y)$.

(ii) *fuzzy open* (*f-open,* in short) if $f(A) \in FO(Y)$ for each $A \in FO(X)$.

(iii) *fuzzy closed(f-closed,* in short) if $f(B) \in FC(Y)$ for each $B \in FC(X)$.

(iv) *fuzzy semi-continuous(f-semi-continuous,* in short) if $f^{-1}(A) \in FSO(X)$ for each $A \in FO(Y)$.

(v) *fuzzy semi-open (f-semi-open,* in short) if $f(A) \in FSO(Y)$ for each $A \in FO(X)$.

(vi) *fuzzy semi-closed(f-semi-closed,* in short) if $f(B) \in FSO(Y)$ for each $B \in FC(X)$.

(vii) *fuzzy irresolute (f-irresolute,* in short) if $f^{-1}(A) \in FSO(X)$ for each $A \in FSO(Y)$.

Notation 2.A. (a) $FC_n(X, Y) = \{ f: X \to Y : f$ is f-continuous $\}$.

(b) $FO(X, Y) = \{ f: X \to Y : f$ is f-open $\}$.

(c) $FC(X, Y) = \{ f: X \to Y : f$ is f-closed $\}$.

(d) $FSC_n(X, Y) = \{ f: X \to Y : f$ is f-semi-continuous $\}$.

(e) $FSO(X, Y) = \{ f: X \to Y : f$ is f-semi-open $\}$.
(f) $FSC(X, Y) = \{ f: X \to Y : f$ is f-semi-closed $\}$.
(g) $FI(X, Y) = \{ f: X \to Y : f$ is f-irresolute $\}$.

Proposition 2.3. (a) $FC_n(X, Y) \subset FSC_r(X, Y)$ and $FI(X, Y) \subset FSC_n(X, Y)$.
(b) $FO(X, Y) \subset FSO(X, Y)$ and $FC(X, Y) \subset FSC(X, Y)$.

Remark 2.4. (a) $FC_n(X, Y) \not\supset FSC_n(X, Y)$, $FO(X, Y) \not\supset FSO(X, Y)$ and $FC(X, Y) \not\supset FSC(X, Y)$ (See Example 6.3 in [1]).
(b) $FI(X, Y) \not\supset FSC_n(X, Y)$ (See Example 2.4 in [4]).

Example 2.5. A f-continuous, f-irresolute mapping need not be f-open.

Let $X = \{ a, b, c \}$ and consider the fuzzy topologies $\mathfrak{T}^* = \{ \emptyset, O_1, O_2, X \}$ and $\mathfrak{T} = \{ \emptyset, O_1, O_2, O_3, X \}$, where

$O_1 = \{ (a, 0.3), (b, 0), (c, 0) \}$, $O_2 = \{ (a, 0.3), (b, 0.6), (c, 0) \}$,
$O_3 = \{ (a, 0.3), (b, 0), (c, 0.7) \}$.

Then clearly, $FSO(X, \mathfrak{T}) = FSO(X, \mathfrak{T}^*)$. Let $id: (X, \mathfrak{T}) \to (X, \mathfrak{T}^*)$ be the identity mapping. Then clearly id is f-continuous and f-irresolute. But id is not f-open.

Theorem 2.6. Let $f: X \to Y$ be f-continuous and f-open. If $A \in FSO(X)$, then $f(A) \in FSO(Y)$.

Definition 2.7[3]. For a fuzzy set in a fts X, the *fuzzy semi-closure(f-s-closure,* in short) $\underline{A}$ and the *fuzzy semi-interior(f-s-interior,* in short), A_o of A are defined respectively, as

$$\underline{A} = \bigcap \{ B: A \subset B, B^c \in FSC(X) \} \text{ and } A_o = \bigcup \{ B: B \subset A, B \in FSO(X) \}.$$

It is clear that $A \in FSC(X)$ if and only if $A = \underline{A}$.

Theorem 2.8. $(\bar{A})^o \subset (\underline{A})_o$ for each $A \in I^X$.

Theorem 2.9. If $f: X \to Y$ is f-continuous and f-open, then $f^{-1}(\overline{A}) = \overline{f^{-1}(A)}$.

Corollary 2.9.1. If $f: X \to Y$ is f-continuous and f-open, then f is f-irresolute.

Theorem 2.10. $f: X \to Y$ is f-irresolute if and only if for each $B \in FSC(Y)$, $f^{-1}(B) \in FSC(X)$.

Corollary 2.10.1. A mapping $f: X \to Y$ is f-irresolute if and only if for each

$A \in I^X$, $f(\underline{A}) \subset \underline{f(A)}$.

Corollary 2.10.2. A mapping $f: X \to Y$ is f-irresolute if and only if for each $B \in 2^Y$, $\underline{f^{-1}(B)} \subset f^{-1}(\underline{B})$.

Theorem 2.11. If $f: X \to Y$ and $g: Y \to Z$ are both f-irresolute, then $g \circ f: X \to Z$ is f-irresolute.

Definition 2.12. A mapping $f: X \to Y$ is said to be *fuzzy pre-semi-open* (*f-pre-semi- open*, in short) if for each $A \in FSO(X)$, $f(A) \in FSO(Y)$.

Theorem 2.13. If $f: X \to Y$ is f-continuous and f-open, then f is f-irresolute and f-pre-semi-open.

Definition 2.14. X and Y are said to be *fuzzy semi-homeomorphIc*(*f-semi-homeomorpic*, in short) if there exists mapping $f: X \to Y$ such that f is bijective, f-irresolute and f-pre-semi-open. Such an f is called a *fuzzy semi-homeomorphism*(*f-semi-homeomorphism*, in short).

Corollary 2.13.1. If $f: X \to Y$ is a f-homeomorphism, then f is a f-semi-home omorphism.

Example 2.15. A f-semi-homeomorphism need not be a f-homeomorphism. Consider $f: (X, \mathfrak{T}) \to (X, \mathfrak{T}^*)$ as in Example 2.5. Then f is a f-semi-homeomorp-hism, but f is not a f-homeomorphism.

3. Some properties of fuzzy semi-homeomorphisms

Theorem 3.1. If $f: X \to Y$ is a f-semi-homeomorphism, then $\underline{f^{-1}(B)} = f^{-1}(\underline{B})$ for each $B \in I^Y$.

Corollary 3.1.1. If $f: X \to Y$ is a f-semi-homeomorphism, then $\underline{f(B)} = f(\underline{B})$ for each $B \in 2^X$.

Corollary 3.1.2. If $f: X \to Y$ is a f-semi-homeomorphism, then $f(B_o) = (f(B))_o$ for each $B \in 2^X$.

Corollary 3.1.3. If $f: X \to Y$ is a f-semi-homeomorphism, then

$f^{-1}(B_o) = (f^{-1}(B))_o$ for each $B \in 2^X$.

Definition 3.2. Let $A \in 2^X$. Then A is said to be *nowhere dense* in X. If $(\overline{A})^o = \emptyset$.

Theorem 3.3. For each $A \in 2^X$, $(\underline{A})_o = \emptyset$ iff A is nowhere dense in X.

Corollary 3.3.1. If $f: X \to Y$ is a f-semi-homeomorphism, and A is nowhere dence in X, then $f(A)$ is nowhere dense in Y.

Theorem 3.4. Fuzzy semi-homeomorphic is an equivalent relation between fts.

Definition 3.5. A property which is preserved under f-semi-homeomorphism is called a *fuzzy semi-topological property(f-semi-topological property*, in short).

Definition 3.6. A fuzzy set A in X is said to be *of the first category* if A can be written as a countable union of fuzzy sets nowhere dense in X. The fuzzy set A is said to be *of the second cauntable* if A is not of the first countable.

Theorem 3.7. The property that a fts is of the first category is a f-semi-topological property.

Corollary 3.7.1. The property that a fts is of the second category is a f-semi-topological property.

Theorem 3.8. A f-semi-topological property is a f-topological property.

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