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On regular generalized fuzzy closed sets and generalizations of fuzzy continuous functions

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Abstract: In this paper, we define and study another various generalizations of fuzzy continuous functions by using the concept of regular generalized fuzzy closed sets. A comparative study regarding the mutual interrelations among these functions along with those functions obtained in [3] is made.Finally, we have introduced and studied the notions of *rgf*-connectedness, *rgf*-extremally disconnectedness and *rgf*-compactness.

1 Introduction

In [3], using the concept of generalized fuzzy closed sets, Balasubramanian and Sundaram have introduced certain types of near-fuzzy continuous functions between fuzzy topological spaces, i.e., generalized fuzzy continuous, fuzzy gc-irresolute, strongly gf-continuous and perfectly gf-continuous functions etc. They also introduced the notions of gf-connectedness, gf-extremally disconnectedness and gf-compactness and studied properties of those notions under above-mentioned functions.

In this paper, we study another generalizations of fuzzy continuous functions and their applications. Section 2 is devoted to regular generalized fuzzy closed sets and study their properties. In Section 3 we introduce regular generalized fuzzy continuous functions and their properties by using regular generalized fuzzy closure Cl_*. In Section 4 we introduce fuzzy rgc-irresolute functions and study their properties, whereas in Section 5 we introduce and study strongly rgf-continuous and perfectly rgf-continuous functions and investigate inter-relations among these functions and those functions defined in [3]. In Section 6 and 7, using the concept of regular generalized closed (open) set, we introduce and study the notions of rgf-connectedness, rgf-extremally disconnectedness and rgf-compactness, respectively.

2 Regular generalized fuzzy closed sets in fuzzy topology

Definition 2.1 (3). *Let X be an fts. A fuzzy set A in X is called generalized fuzzy closed (in short, gf-closed) if $Cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy open. A fuzzy set A is called generalized fuzzy open (in short, gf-open) if its complement $1 - A$ is gfc.*

Definition 2.2. *Let X be an fts. A fuzzy set A in X is called regular generalized fuzzy closed (in short, rgf-closed) if $Cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy regular open. A fuzzy set A is called regular generalized fuzzy open (in short, rgf-open) if its complement $1 - A$ is rgf-closed.*

Remark 2.3. *Every fuzzy closed (resp. fuzzy open) set is gf-closed (resp. gf-open) and every gf-closed (resp. gf-open) set is rgf-closed (resp. rgf-open), but the converses are not true.*

Example 2.4. *Let $X = \{a, b, c\}$, $\tau_1 = \{0_X, 1_X, A_1\}$ and $\tau_2 = \{0_X, 1_X, A_2\}$, where $A_1(a) = 0.5$, $A_1(b) = 0.7, A_1(c) = 0.6$; $A_2(a) = 0.4, A_2(b) = 0.7, A_2(c) = 0.3$. Define fuzzy sets A_3 and A_4 in X as follows: $A_3(a) = 0.4, A_3(b) = 0.3, A_3(c) = 0.4$; $A_4(a) = 0.3, A_3(b) = 0.3, A_3(c) = 0.2$. Then we have*

(1) In (X, τ_1), A_3 is gf-closed set but not fuzzy closed.

(2) In (X, τ_2), A_4 is rgf-closed set but not gf-closed.

Theorem 2.5. *If A and B are rgf-closed sets, then $A \vee B$ is a rgf-closed set.*

However, the intersection of two rgf-closed sets is not rgf-closed set as the following example shows.

Example 2.6. *Let $X = \{a, b, c\}$ and $\tau = \{0_X, 1_X, A_1\}$, where $A_1(a) = 0.4, A_1(b) = 0.3, A_1(c) = 0.5$. Define fuzzy sets A_2 and A_3 in X as follows: $A_2(a) = 0.3, A_2(b) = 0.9, A_2(c) = 0.3$; $A_3(a) = 0.7, A_2(b) = 0.2, A_2(c) = 0.8$. Then A_2 and A_3 are rgf-closed sets but $A_2 \wedge A_3$ is not rgf-closed set.*

Theorem 2.7. *If A is rgf-closed set and $A \leq B \leq Cl(A)$, then B is rgf-closed set.*

Theorem 2.8. *A fuzzy set A is rgf-open if and only if $F \leq Int(A)$ whenever F is fuzzy regular closed and $F \leq A$.*

Theorem 2.9. *If A and B are rgf-open sets with $A \wedge Cl(B) = B \wedge Cl(A) = 0_X$, then $A \vee B$ is rgf-open.*

Proof. Let F be a fuzzy regular closed set such that $F \leq A \vee B$. Then $F \wedge \text{Cl}(A) \leq A$ since $B \wedge \text{Cl}(A) = 0_X$, and hence by Theorem 2.5, $(F \wedge \text{Cl}(A)) \leq \text{Int}(A)$. Similarly, $(F \wedge \text{Cl}(B)) \leq \text{Int}(B)$. Now we have

$$\begin{aligned} F &= F \wedge (A \vee B) \leq (F \wedge \text{Cl}(A)) \vee (F \wedge \text{Cl}(B)) \\ &\leq \text{Int}(A) \vee \text{Int}(B) \leq \text{Int}(A \vee B). \end{aligned}$$

Hence $F \leq \text{Int}(A \vee B)$ and hence Theorem 2.5, $A \vee B$ is rgf-open. □

Theorem 2.10. *If $Int(A) \leq B \leq A$ and A is rgf-open set, then B is rgf-open.*

Definition 2.11. *A function $f : X \to Y$ is called fuzzy regular continuous (in short, fr-continuous) if the inverse image of every fuzzy closed set in Y is fuzzy regular closed in X.*

Clearly, every fr-continuous function is fuzzy continuous.

Theorem 2.12. *If A is a rgf-closed set in X and if $f : X \to Y$ is fr-continuous and fuzzy closed, then $f(A)$ is gf-closed in Y.*

However, under fuzzy closed and fr-continuous functions, the image of rgf-open set need be not gf-open.

Example 2.13. *Let $X = \{a\}$, $Y = \{a, b, c\}$, $\tau_1 = \{0_X, 1_X, A\}$ and $\tau_2 = \{0_Y, 1_Y, B_1, B_2\}$ where A, B_1 and B_2 is fuzzy sets defined by $A(a) = 0.5$; $B_1(a) = 0, B_1(b) = 0.5, B_1(c) = 0$; $B_2(a) = 0$, $B_2(b) = 1, B_2(c) = 0$. Define a function $f : (X, \tau_1) \to (Y, \tau_2)$ by $f(a) = b$. Clearly, f is fr-continuous and fuzzy closed. Now we shall show that image of rgf-open set is not gf-open. Consider a fuzzy set A_1 in X defined by $A_1(a) = 0.8$. Then A_1 is rgf-open in (X, τ_1) but $f(A_1)$ is not gf-open in (Y, τ_2).*

Definition 2.14. *A fuzzy set A in an fts X is called rgf-q-nbd of a fuzzy point x_α if there is a rgf-open set U such that $x_\alpha qU \leq A$.*

Theorem 2.15. *Let X be an fts. Then A is rgf-open if and only if for each fuzzy point x_α with $x_\alpha qA$, A is rgf-q-nbd of x_α.*

3 Regular generalized fuzzy continuous functions and their properties

Definition 3.1 (3). *A function $f : X \to Y$ is called generalized fuzzy continuous (in short, gf-continuous) if the inverse image of every fuzzy closed set in Y is gf-closed in X.*

Definition 3.2. *A function $f : X \to Y$ is called regular generalized fuzzy continuous (in short, rgf-continuous) if the inverse image of every fuzzy closed set in Y is rgf-closed in X.*

Every fuzzy continuous function is gf-continuous and every gf-continuous function is rgf-continuous. However, the converses are not true as Example 3.3 in [3] and the following example show.

Example 3.3. *Let $X = \{a, b, c\}$, $\tau_1 = \{0_X, 1_X, A_1\}$ and $\tau_2 = \{0_X, 1_X, A_2\}$, where $A_1(a) = 0.4, A_1(b) = 0.7, A_1(c) = 0.3$; $A_2(a) = 0.7, A_2(b) = 0.7, A_2(c) = 0.8$. Let $f : (X, \tau_1) \to (X, \tau_2)$ be the identity. Then f is rgf-continuous but not gf-continuous since $f^{-1}(1 - A_2)$ is not gf-closed in (X, τ) for fuzzy closed set $1 - A_2$ in (X, τ_2).*

Balasubramanian and Sundaram [3] defined the generalized fuzzy closure operator Cl^* to obtain some properties of gf-continuity. So, in similar way, we define the regular generalized fuzzy closure operator Cl_* for any fuzzy set A in (X, τ) as follows:

$$\text{Cl}_*(A) = \bigwedge \{B \mid A \leq B \text{ and } B \text{ is } rgf\text{-closed}\}.$$

Theorem 3.4. *Let A be a fuzzy set in X and x_α be a fuzzy point in X. Then $x_\alpha \in Cl_*(A)$ is and only if for each rgf-q-nbd U of x_α, UqA.*

The following are the properties of rgf-continuous functions.

Theorem 3.5. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function.*

(i) The following statements are equivalent:

(a) f is rgf-continuous.

(b) The inverse image of each fuzzy open set in Y is rgf-open in X.

(ii) If $f : (X, \tau) \to (Y, \sigma)$ is rgf-continuous, then $f(Cl_*(A)) \leq Cl(f(A))$ for any fuzzy set A in X.

(iii) The following statement are equivalent:

(a) For each fuzzy point x_α in X and each fuzzy open q-nbd of $f(x_\alpha)$, there exists a rgf-open q-nbd U of x_α such that $f(U) \leq V$.

(b) For each fuzzy set A in X, $f(Cl_*(A)) \leq Cl(f(A))$.

(c) For each fuzzy set B in Y, $Cl_*(f^{-1}(B)) \leq f^{-1}(Cl(B))$.

(d) The function $f : (X, \tau_*) \to (Y, \sigma)$ is fuzzy continuous.

The converse of Theorem 3.4 (ii) need not be true as seen from the following example.

Example 3.6. *Let $X = \{a, b, c\}$, $\tau_1 = \{0_X, 1_X, A_1\}$ and $\tau_2 = \{0_X, 1_X, A_2\}$ where A_1, A_2 are fuzzy sets in X defined by $A_1(a) = 0.6, A_1(b) = 0.7, A_1(c) = 0.6$; $A_2(a) = 0.3, A_2(b) = 0.7, A_2(c) = 0.3$. Consider a function $f : (X, \tau_1) \to (X, \tau_2)$ defined by $f(a) = f(b) = f(c) = b$. Then for any fuzzy set A, $f(Cl_*(A)) \leq Cl(f(A))$, but f is not rgf-continuous. (Since $1 - A_2$ is a fuzzy closed in (X, τ_2) but $f^{-1}(1-A_2)$ is not rgf-closed in (X, τ_1).)*

Definition 3.7 (3). *An fts X is said to be fuzzy $T_{1/2}$ if every gf-closed set in X is fuzzy closed in X.*

Definition 3.8. *An fts X is said to be fuzzy regular-$T_{1/2}$ if every rgf-closed set in X is fuzzy regular closed in X.*

Every fuzzy regular-$T_{1/2}$ space is fuzzy $T_{1/2}$ but the converse need not be true as seen from the following example.

Example 3.9. *Let $X = \{a, b\}$ and $\tau = \{0_X, 1_X, A\}$ where A is fuzzy set in X defined by $0 \leq A(a) \leq 1, 0 \leq A(b) < \frac{1}{2}$. Then (X, τ) is fuzzy $T_{1/2}$ but not fuzzy regular-$T_{1/2}$.*

Theorem 3.10. *Let $f : X \to Y$ and $g : Y \to Z$ be functions.*

(i) If f and g are rgf-continuous and Y is fuzzy regular-$T_{1/2}$, then the composition $g \circ f : X \to Z$ is also rgf-continuous.

(ii) If f is rgf-continuous and g are fuzzy continuous, then the composition $g \circ f$ is rgf-continuous.

The following Example shows that the composition of any two rgf-continuous functions need not be rgf-continuous.

Example 3.11. *Let $X = \{a, b, c\}$, $\tau_1 = \{0_X, 1_X, A_1\}$, $\tau_2 = \{0_X, 1_X, A_2\}$ and $\tau_3 = \{0_X, 1_X, A_3\}$ where A_1, A_2 and A_3 are fuzzy sets in X defined as follows:*

$$\begin{aligned} A_1(a) &= 0.4, A_1(b) = 0.3, A_1(c) = 0.4; \\ A_2(a) &= A_2(b) = A_2(c) = 0.5; \\ A_3(a) &= 0.7, A_3(b) = 0.8, A_3(c) = 0.7. \end{aligned}$$

Let $f : (X, \tau) \to (X, \tau_2)$ be a function defined by $f(a) = f(b) = f(c) = b$ and $g : (X, \tau_2) \to (X, \tau_3)$ be the identity. Then f and g are rgf-continuous but $g \circ f$ is not rgf-continuous; for $1 - A_3$ is fuzzy closed in (X, τ_3), $f^{-1}(g^{-1}(1-A_3))$ is not rgf-closed in (X, τ_1). Hence $g \circ f$ is not rgf-continuous.

Remark 3.12. *For $f : X \to Y$, when X is fuzzy regular-$T_{1/2}$, then rgf-continuity, gf-continuity, fuzzy continuity and fr-continuity are equivalent.*

4 Fuzzy rgc-irresolute functions and their properties

Definition 4.1 (3). *A function $f : X \to Y$ is called fuzzy gc-irresolute if the inverse image of every gf-closed set in Y is gf-closed in X.*

Definition 4.2. *A function $f : X \to Y$ is called fuzzy rgc-irresolute if the inverse image of every rgf-closed set in Y is rgf-closed in X.*

Every fuzzy rgc-irresolute function is rgf-continuous but the converse is not true (see Example 4.3). And the following Examples 4.4 and 4.5 show that fuzzy gc-irresolute function and fuzzy gc-irresolute function are, in general, independent.

Example 4.3. *Let $X = \{a, b, c\}$, $\tau_1 = \{0_X, 1_X, A_1\}$ and $\tau_2 = \{0_X, 1_X, A_2\}$, where A_1 and A_2 are fuzzy sets in X defined by $A_1(a) = 0.7, A_1(b) = 0.8, A_1(c) = 0.7$; $A_2(a) = A_2(b) = A_2(c) = 0.5$. Let $f : (X, \tau_1) \to (X, \tau_2)$ be the identity. Then f is rgf-continuous but not fuzzy rgc-irresolute; for a fuzzy set A_3 in X defined by $A_3(a) = 0.3, A_3(b) = 0.1, A_3(c) = 0.3$ is rgf-closed in (X, τ_2), $f^{-1}(A_3)$ is not rgf-closed in (X, τ_1).*

Example 4.4. *Let $X = \{a, b, c\}$, $\tau_1 = \{0_X, 1_X, A_1\}$ and $\tau_2 = \{0_X, 1_X, A_2\}$ where A_1 and A_2 are fuzzy sets in X defined by $A_1(a) = A_1(c) = 0, A_1(b) = 0.5$; $A_2(a) = A_2(c) = 0, A_2(b) = 1$. Let $f : (X, \tau_1) \to (X, \tau_2)$ be the identity. Then f is fuzzy gc-irresolute but not fuzzy rgc-irresolute; for A_1 is rgf-closed in (X, τ_2), $f^{-1}(A_1)$ is not rgf-closed in (X, τ_1). Hence f is not fuzzy rgc-irresolute.*

Example 4.5. *Let $X = \{a, b, c\}$, $\tau_1 = \{0_X, 1_X, A_1\}$ and $\tau_2 = \{0_X, 1_X, A_2\}$ where A_1 and A_2 are fuzzy sets defined by $A_1(a) = A_1(c) = 0, A_1(b) = 1$; $A_2(a) = A_2c) = 0.7, A_2(b) = 0.5$. Let $f : (X, \tau_1) \to (X, \tau_2)$ be the identity. Then f is fuzzy rgc-irresolute but not fuzzy gc-irresolute; for a fuzzy set A_3 in X defined by $A_3(a) = A_3(c) = 0, A_3(b) = 0.5$ is gf-closed in (X, τ_2), $f^{-1}(A_3)$ is not gf-closed in (X, τ_1).*

The following are the properties of fuzzy rgc-irresolute functions.

Theorem 4.6. *Let $f : (X, \tau) \to (Y, \sigma)$ be a function.*

(i) The following statements are equivalent:

(a) f is fuzzy rgc-irresolute.

(b) The inverse image of every rgf-open set in Y is rgf-open in X.

(ii) If $f : (X, \tau) \to (Y, \sigma)$ is fuzzy rgc-irresolute, then $f(Cl_*(A)) \leq Cl_*(f(A))$ for any fuzzy set A in X.

(iii) The following statements are equivalent:

(a) For each fuzzy point x_α in X and each rgf-open q-nbd of $f(x_\alpha)$, there exists a rgf-open q-nbd U of x_α such that $f(U) \leq V$.

(b) For each fuzzy set A in X, $f(Cl_*(A)) \leq Cl_*(f(A))$.

(c) For each fuzzy set B in Y, $Cl_*(f^{-1}(B)) \leq f^{-1}(Cl_*(B))$.

(d) The function $f : (X, \tau_*) \to (Y, \sigma_*)$ is fuzzy continuous.

Theorem 4.7. *Let $f : X \to Y$ and $g : Y \to Z$ be functions.*

(i) If f and g are fuzzy rgc-irresolute, then the composition $g \circ f$ is fuzzy rgc-irresolute.

(ii) If f is fuzzy rgc-irresolute and g are rgf-continuous, then the composition $g \circ f$ is rgf-continuous.

5 Strongly rgf-continuous and perfectly rgf-contnuous functions

Definition 5.1 (3). *A function $f : X \to Y$ is called perfectly fuzzy continuous if the inverse image of every fuzzy open set in Y is both fuzzy open and fuzzy closed in X.*

Definition 5.2 (3). *A function $f : X \to Y$ is called strongly gf-continuous if the inverse image of every gf-open set in Y is fuzzy open in X.*

Definition 5.3 (3). *A function $f : X \to Y$ is called perfectly gf-continuous if the inverse image of every gf-open set in Y is both fuzzy open and fuzzy closed in X.*

Definition 5.4. *A function $f : X \to Y$ is called strongly rgf-continuous if the inverse image of every rgf-open set in Y is fuzzy open in X.*

Definition 5.5. *A function $f : X \to Y$ is called perfectly rgf-continuous if the inverse image of every rgf-open set in Y is both fuzzy open and fuzzy closed in X.*

Remark 5.6. *When Y is fuzzy regular-$T_{1/2}$, strongly rgf-continuity, strongly gf-continuity and fuzzy continuity are equivalent concepts, and also perfectly rgf-continuity, perfectly gf-continuity and perfectly fuzzy continuity are equivalent.*

Theorem 5.7. *Strong rgf-continuity $\Rightarrow$ strong gf-continuity $\Rightarrow$ fuzzy continuity.*

The converses of Theorem 5.7 are not true as Example 5.7 in [3] and the following example show.

Example 5.8. *Let $X = \{a, b\}$ and $\tau_1 = \{0_X, 1_X, A_1\}$ where A_1 is fuzzy set in X defined by $A_1(a) = 1, 0.5 \leq A(b) \leq 1$. Let $f : (X, \tau_1) \to (X, \tau_1)$ be the identity. Then f is strongly gf-continuous but not strongly rgf-continuous.*

Theorem 5.9. *A function $f : X \to Y$ is strongly rgf-continuous if and only if the inverse image of every rgf-closed set in Y is fuzzy closed in X.*

Theorem 5.10. *Let $f : X \to Y$, $g : Y \to Z$ be functions. If f is strongly rgf-continuous and g is rgf-continuous, then $g \circ f$ is fuzzy continuous.*

Theorem 5.11. *Perfect rgf-continuity $\Rightarrow$ perfect gf-continuity, and perfect rgf-continuity $\Rightarrow$ strong rgf-continuity.*

The converses of Theorem 5.11 are not true.

Example 5.12. *Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A_1, A_2\}$ and $\tau_2 = \{0_X, 1_X, A_1\}$ where A_1 and A_2 are fuzzy sets in X defined by $A_1(a) = 1$, $0.5 \leq A_1(b) \leq 1$; $A_2(a) = 0$, $0 \leq A_2(b) \leq 0.5$. Let $f : (X, \tau_1) \to (X, \tau_2)$ be the identity. Then f is perfectly gf-continuous but not perfectly rgf-continuous.*

Example 5.13. *Let $X = \{a, b\}$, $\tau_1 = \{0_X, 1_X, A_1\}$ and $\tau_2 = \{0_X, 1_X, A_2\}$ where A_1 and A_2 are fuzzy sets in X defined by $0 \leq A_1(a) \leq 1$, $0 \leq A_1(b) \leq \frac{1}{2}$; $0 \leq A_2(a) \leq \frac{1}{2}$, $0 \leq A_2(b) \leq 1$. Define $f : (X, \tau_1) \to (X, \tau_2)$ by $f(a) = b$ and $f(b) = a$. Then f is strongly rgf-continuous but not perfectly rgf-continuous.*

Theorem 5.14. *A function $f :\to Y$ is perfectly rgf-continuous if and only if the inverse image of rgf-closed set in Y is both fuzzy open and fuzzy closed in X.*

Regarding the results above-mentioned so far, we have the table of implications as shown in follwing table.

$\Rightarrow$	a	b	c	d	e	f	g	h	i	j	k
a	1	1	1	1	0	0	0	0	0	0	0
b	0	1	1	1	0	0	0	0	0	0	0
c	0	0	1	1	0	0	0	0	0	0	0
d	0	0	0	1	0	0	0	0	0	0	0
e	0	0	1	1	1	0	0	0	0	0	0
f	0	0	0	1	0	1	0	0	0	0	0
g	1	1	1	1	0	0	1	0	0	0	0
h	0	1	1	1	1	0	0	1	0	0	0
i	1	1	1	1	1	0	1	1	1	0	0
j	0	1	1	1	1	1	0	1	0	1	0
k	1	1	1	1	1	1	1	1	1	1	1

In above table, a, b, c, d, e, f, g, h, i, j and k denote fuzzy regular continuity, fuzzy continuity, gf-continuity, rgf-continuity, fuzzy gc-irresolute, fuzzy rgc-irresolute, perfect fuzzy continuity, strong gf-continuity, perfect gf-continuity, strong rgf-continuity and perfect rgf-continuity, respectively. Also 1 denotes 'implies' and 0 denotes 'does not imply'.

6 rgf-connectedness and their properties

Definition 6.1 (3). *An fts X is said to be gf-connected if the only fuzzy sets which are both gf-open and gf-closed are 0_X and 1_X.*

Definition 6.2. *An fts X is said to be rgf-connected if the only fuzzy sets which are both rgf-open and rgf-closed are 0_X and 1_X.*

Theorem 6.3. *Every rgf-connected space is gf-connected and every gf-connected space is fuzzy connected [6].*

However, the converses are not true as Example 7.3 in [3] and the following example show.

Example 6.4. *Let $X = \{a, b\}$ and $\tau = \{0_X, 1_X, A\}$ where A is a fuzzy set in X defined by $\frac{1}{2} \leq A \leq 1$, $A(b) = 1$. Then (X, τ) is gf-connected but not rgf-connected; For any fuzzy set B in X, B is rgf-open and rgf-closed in (X, τ). Hence (X, τ) is not rgf-connected.*

Theorem 6.5. *For fuzzy regular-$T_{1/2}$ space X, the following are equivalent:*

(i) X is rgf-connected.

(ii) X is gf-connected.

(iii) X is fuzzy connected.

Theorem 6.6. *If $f : X \to Y$ is rgf-continuous surjection and X is rgf-connected, then Y is fuzzy connected.*

Theorem 6.7. *If $f : X \to Y$ is fuzzy rgc-irresolute surjection and X is rgf-connected, then Y is rgf-connected.*

Theorem 6.8. *If $f : X \to Y$ is more strongly gf-continuous surjection and X is fuzzy connected, then Y is rgf-connected.*

Theorem 6.9. *An fts X is rgf-connected if and only if it has no non-zero rgf-open sets A and B such that $A + B = 1$.*

Corollary 6.10. *An fts X is rgf-connected if and only if it has no non-zero rgf-open sets A and B such that $A + B = 1$, $Cl(A) + B = A + Cl(B) = 1$.*

Now, we define the regular generalized fuzzy interior operator Int_* for any fuzzy set A in a fts (X, τ) as follows: $\mathrm{Int}_*(A) = \bigvee\{B \mid B \leq A$ and B is rgf-open$\}$. It is easy to see that for any fuzzy set A in X, $1 - \mathrm{Cl}_*(A) = \mathrm{Int}_*(1 - A)$.

Definition 6.11. *A rgf-open set A is called regular rgf-open if $A = Int_*(Cl_*(A))$. The fuzzy complement of regular rgf-open set is called regular rgf-closed.*

Definition 6.12. *An fts X is called rgf-super connected if there is no proper regular rgf-open set in X.*

Theorem 6.13. *In an fts X, the following are equivalent:*

(i) X is rgf-super connected.

(ii) For every non-zero rgf-open set A, $Cl_*(A) = 1$.

(iii) For every rgf-closed set A with $A \neq 1$, $Int_*(A) = 0$.

(iv) X does not have non-zero rgf-open sets A and B such that $A + B \leq 1$.

(v) X does not have non-zero fuzzy setsA and B such that $Cl_*(A) + B = A + Cl_*(B) = 1$.

Definition 6.14. *An fts X is said to be rgf-strongly connected if it has no non-zero rgf-closed sets A and B such that $A + B \leq 1$.*

Theorem 6.15. *An fts X is rgf-strongly connected if and only if it has no non-zero rgf-open sets A and B such that $A \neq 1$, $B \neq 1$ and $A + B \geq 1$.*

Remark 6.16. *Every rgf-super connected space is gf-super connected, and every rgf-strong connected space gf-strong connected.*

The converses of Remark 6.16 are not true. Also the following examples show that gf-super connectedness and gf-strong connectedness are independent.

Example 6.17. *Let $X = \{a, b\}$ and $\tau = \{0_X, 1_X, A\}$ where A is fuzzy set defined by $\frac{2}{3} \leq A(a) \leq 1$ and $A(b) = 0$. Then (X, τ) is gf-super connected but it is neither gf-strongly connected nor rgf-super connected.*

Example 6.18. *Let $X = \{a, b\}$ and $\tau = \{0_X, 1_X, A_1, A_2\}$ where A_1 and A_2 are fuzzy sets defined by $A_1(a) = \frac{2}{3}$, $A_1(b) = 1$; $0 \leq A_2(a) < \frac{1}{3}$, $A_2(b) = 0$. Then (X, τ) is gf-strongly connected but it is neither gf-super connected nor rgf-strongly connected.*

7 rgf-extremally disconnectedness and rgf-compactness

Definition 7.1. *An fts X is said to be regular generalized fuzzy extremally disconnected (in short, rgf-extremally disconnected) if $Cl_*(A)$ is rgf-open, whenever A is rgf-open.*

Theorem 7.2. *For any fts X, then following are equivalent:*

(i) X is rgf-extremally disconnected.

(ii) For each rgf-closed set A, $Int_*(A)$ is rgf-closed.

(iii) For each rgf-open set A, $Cl_*(A) + Cl_*(1 - Cl_*(A)) = 1$.

(iv) For each pair of rgf-open set A, B with $Cl_*(A) + B = 1$, $Cl_*(A) + Cl_*(B) = 1$.

Definition 7.3. *A collection $\{A_\lambda\}_{\lambda \in \Lambda}$ of rgf-open sets in X is called rgf-open cover of a fuzzy set B in X if $B \leq \bigvee_{\lambda \in \Lambda} A_\lambda$.*

Definition 7.4. *An fts X is called rgf-compact if every rgf-open cover of X has a finite subcover.*

Definition 7.5. *A fuzzy set B in X is said to be rgf-compact relative to X (which we shall call a rgf-compact set) if for every collection $\{A_\lambda\}_{\lambda \in \Lambda}$ of rgf-open sets of X such that $B \leq \bigvee_{\lambda \in \Lambda} A_\lambda$, there exists a finite subset Λ_0 of Λ such that $B \leq \bigvee_{\lambda \in \Lambda_0} A_\lambda$.*

Theorem 7.6. *Let X be a rgf-compact fts and A be a rgf-closed set in X. Then A is rgf-compact set.*

Theorem 7.7. *(i) If $f : X \to Y$ is rgf-continuous and X is rgf-compact, then $f(X)$ is a fuzzy compact set.*

(ii) If $f : X \to Y$ is fuzzy rgc-irresolute and A is rgf-compact set of X, then $f(A)$ is rgf-compact set of Y.

(iii) If $f : X \to Y$ is strongly rgf-continuous and X is fuzzy compact, then $f(X)$ is a rgf-compact set of Y.

References

[1] G. Balasubramanian, *On extensions of fuzzy topologies*, Kybernetika 28 (1992), 239–244.

[2] G. Balasubramanian, *Fuzzy disconnectedness and its stronger forms*, Indian J. Pure Appl. Math. 23 (1993), 27–30.

[3] G. Balasubramanian and P. Sundaram, *On some generalizations of fuzzy continuous functions*, Fuzzy Sets and Systems 86 (1997), 93–100.

[4] C. L. Chang, *Fuzzy topologcal spaces*, J. Math. Anal. Appl. 24 (1968), 182–190.

[5] W. Dunham, *A new closure operator for non-T_1 topologies*, Kyungpook Math. J. 22 (1982), 55–60.

[6] U. V. Fatteh and D. S. Bassan, *Fuzzy connectedness and its stronger forms*, J. Math. Anal. Appl. 111 (1985) 449–464.

[7] N. Levine, *Generalized closed sets in topology*, Rend. Cir. Mat. Palermo 19 (1970), 89–96.

[8] P. Sundaram, *Studies on generalizations of continuous maps in topological spaces*, Ph.D. Thesis (Bharathiar University, July 1991).