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Design of a PID type Fuzzy Controller

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Abstract

A PID type fuzzy Controller is proposed based on a crisp type model in which the consequent parts of the fuzzy control rules are functional representation or real numbers. Using the conventional PID control theory, a new PID type fuzzy controller is developed , which retains the characteristics of the conventional PID controller. An advantage of this approach, is that it simplifies the complicated defuzzification algorithm which could be time consuming. Computer simulation results have shown that the proposed PID fuzzy controller has satisfactory tracking performance.

Key Words : Control theory; PID control; Fuzzy controller

1.0 Introduction

It is a well known fact that conventional PID controllers are widely used in practical industrial processes, even though it has some draw backs. This is due to their effectiveness for linear systems, ease of design and inexpensive cost. Despite their effectiveness for the control of linear systems, conventional PID controllers are not suitable for nonlinear systems, high - order and time delayed systems, not to talk about complex and vague systems that require expert knowledge.

Furthermore, different versions of PID type fuzzy controller have been proposed. However, the algorithm (Min - Max compositional rule and Product - Sum method) of the fuzzy controller for the defuzzification procedure, is always a complicated and time consuming process.

Takagi and Sugeno proposed a crisp type model [6] to simplify the complicated and time consuming defuzzification algorithm. In this model, the consequent parts of the fuzzy control rules are crisp functional representation or crisp real numbers rather than fuzzy sets in the simplest form.

In this paper, it is proposed to utilize the properties of the crisp type fuzzy model which simplifies the complicated defuzzification algorithm to design a PID type fuzzy controller by relating to the conventional PID control theory.

2.0 Crisp type Fuzzy Controller

Consider a two input and one output fuzzy controller with e (error) and $\dot{e}$ (derivative of error) being the inputs to the controller and u be the output of the controller. Let the linguistic values of e and $\dot{e}$ be A_i $(i \in I = [-m, \cdots, -1, 0, 1, \cdots, m])$ and B_j $(j \in J = [-n, \cdots, -1, 0, 1, \cdots n])$ respectively. Also suppose the membership functions of A_i and B_j are $A_i(e)$ and $B_j(\dot{e})$ respectively. The fuzzy control rules of the controller is of the form

$$\textit{If } e \textit{ is } A_i \text{ and } \textit{if } \dot{e} \textit{ is } B_j \text{ then } u \textit{ is } u_{ij} \qquad (1)$$

where u_{ij} is a crisp value.

Using product - sum inference methods, the truth value of the antecedent part of (1) is

$$f_{ij} = A_i(e)B_j(\dot{e}) \quad (i \in I, j \in J) \tag{2}$$

Using gravity-center method [3] as used for defuzzifying sets, the output of the controller is of the form

$$u = \frac{\sum_{i,j} f_{ij} u_{ij}}{\sum_{i,j} f_{ij}} \tag{3}$$

put(2) in (3), we have

$$u = \frac{\sum_{i,j} A_i(e)B_j(\dot{e})u_{ij}}{\sum_{i,j} A_i(e)B_j(\dot{e})} \tag{4}$$

Selecting triangular membership functions for A_i and B_j such that

$$\sum_{i,j} A_i(e)B_j(\dot{e}) = 1 \tag{5}$$

Therefore, (4) becomes

$$u = \sum_{i,j} A_i(e)B_j(\dot{e})u_{ij} \tag{6}$$

Linearizing (4) around an operating point, (4) becomes

$$u = u_{ij} - \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i} e_i - \frac{u_{i(j+1)} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j} \dot{e}_j + \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i} e_i - \frac{u_{i(j+1)} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j} e_j$$

$$\Rightarrow u = A + Pe + D\dot{e} \tag{7}$$

where $A = u_{ij} - \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i} e_i - \frac{u_{i(j+1)} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j} \dot{e}_j$,

$$P = \frac{u_{(i+1)j} - u_{ij}}{e_{i+1} - e_i}, \text{and } D = \frac{u_{i(j+1)} - u_{ij}}{\dot{e}_{j+1} - \dot{e}_j}$$

Thus, the crisp type fuzzy controller represented by (7) is a PD controller.

3.0 PID type Fuzzy Controller

Since the crisp type fuzzy controller is a PD fuzzy controller, it is proposed to formulate a PI fuzzy controller. Thereafter, combining these controllers in parallel to form a fuzzy PID type controller as is obtained in conventional PID controller.

From (7), we have

$$u_1 = u - A = Pe + D\dot{e} \tag{8}$$

Integrating (8), we obtain

$$\begin{aligned} u_2 &= \int u_1 dt \\ &= P\int edt + De \end{aligned} \tag{9}$$

(9) is a PI fuzzy controller.

Combining (7) and (9), we have a PID fuzzy control law as

$$\begin{aligned} u_c &= k_4 u_2 + k_3 u \\ &= k_3 A + (k_3 P + k_4 D)e \\ &\quad + k_4 P\int edt + k_3 D\dot{e} \end{aligned} \tag{10}$$

where k_3 and k_4 are scaling factors.

The proposed structure of the PID type fuzzy controller is shown in figure 1.

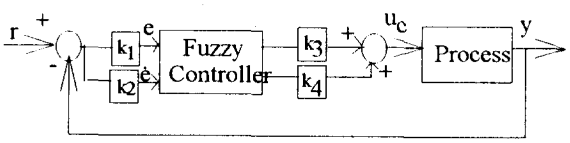

Figure 2: Proposed Fuzzy PID Type Contr

k_1 and k_2 are scaling factors for e and $\dot{e}$ respectively

4.0 Fuzzy Control rules

The standard procedure of fuzzy controller design which consists of fuzzification, control rule base formulation and defuzzification is adopted. A triangular membership functions for each fuzzy linguistic value is employed for the error e and the rate of change of error $\dot{e}$ as shown in figure 2.

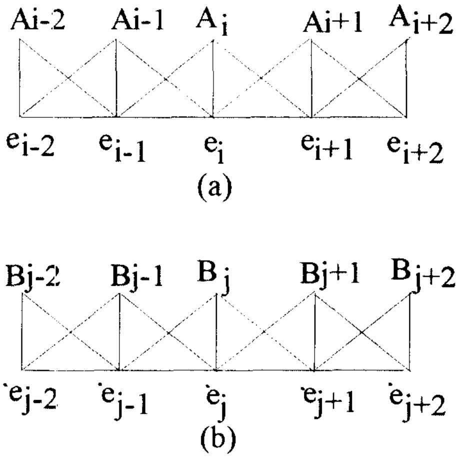

Figure 2 : The Membership function of $\tilde{A}$ and $\tilde{B}$

The core of the fuzzy subsets e and $\dot{e}$ defined in figure 2 are

$$\{e_i\} = \{e_{-2}, e_{-1}, e_0, e_1, e_2\} = \{-1, -0.4, 0, 0.4, 1\}$$

$$\{\dot{e}_j\} = \{\dot{e}_{-2}, \dot{e}_{-1}, \dot{e}_0, \dot{e}_1, \dot{e}_2\} = \{-1, -0.4, 0, 0.4, 1\}$$

The error e and change of error $\dot{e}$ are fuzzified, taking into consideration the overall PID control law (10). Using the aforementioned membership functions, the fuzzy control rules are established for the fuzzy controller. This is as represented in Table 1.

Table 1 : The Fuzzy Control Rules

u_c \ e / $\dot{e}$	e-2	e-1	e0	e1	e2
ė-2	-1	-0.75	-0.5	-0.25	0
ė-1	-0.75	-0.5	-0.25	0	0.25
ė0	-0.5	-0.25	0	0.25	0.5
ė1	-0.25	0	0.25	0.5	0.75
ė2	0	0.25	0.5	0.75	1

5.0 Simulation

Some simulation studies were carried out to prove the effectiveness of the PID type fuzzy controller. First we consider fuzzy PID controller for lower - order linear processes, to show how well it performs for these simple cases. This is because, it is well established that the conventional PID controller is designed for linear processes, for which it works very well. We show that the fuzzy PID controller is as good as, if not better than , the conventional PID controller for such lower - order linear processes. In our simulation studies, we use the discrete simulation method.

Example 1: A first order linear process whose Transfer function is given as

$$y(t+1) = 0.9048y(t) + 0.095u(t)$$

is considered. The parameters of the fuzzy type controller are T = 0.1, k_1 = 0.8, k_2 = 0.3, k_3 = 0.9, k_4 = 0.1 and the set - point r = 1. The tracking response of the fuzzy PID controller for a step input is shown in figure 3, which is remarkable.

Example 2: A second - order linear process whose Transfer function is given as

$$y(t+1) = 1.7236y(t) - 0.7408y(t-1) + 0.1449u(t) + 0.1311u(t-1)$$

where the fuzzy PID parameters are : T = 0.1 , k_1 = 0.4, k_2 = 0.3, k_3 = 0.2, k_4 = 0.1 and the set - point r = 1. The tracking response of the fuzzy PID controller for a step input is shown in figure 4. The tracking response is perfect and has no steady - state error. However a little overshoot exist.

Example 3 : A nonlinear process whose dynamic is given as

$$\dot{y}(t) = 0.0001|y(t)| + u(t)$$

In this case, the fuzzy PID parameters are : T = 0.1 , k_1 = 0.1, k_2 = 2.9, k_3 = 6.9, k_4 = 0.1 and the set - point r = 1. The tracking response of the fuzzy PID controller for this nonlinear process to a step input is shown in figure 5, which depicts a very good tracking result.

Example 4 : A linearized Combustion model with significant time delay whose dynamic is given as

$$y(t+1) = 0.978y(t) + 0.0157u(t-td)$$

where $td = 18\,seconds$. The fuzzy PID parameters are : T = 0.1 , k_1 = 1.0, k_2 = 0.1, k_3 = 0.5, k_4 = 1.85 and the set - point r = 1. The tracking response of the fuzzy PID controller for a step input is shown in figure 6.

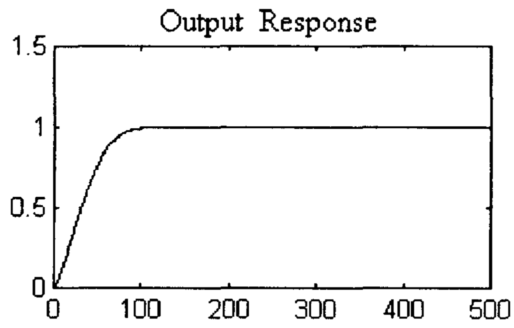

Figure 3 :Result of Example 1

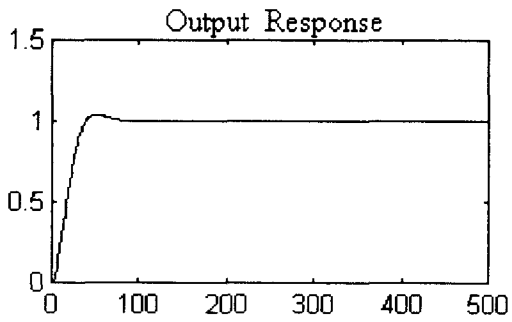

Figure 4 : Result of Example 2

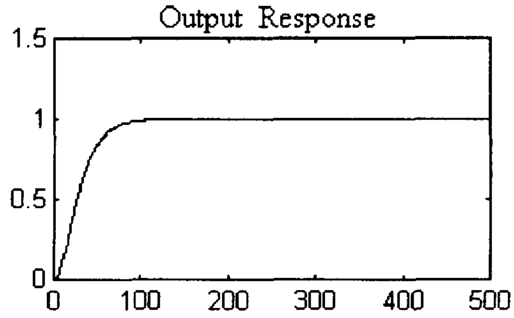

Figure 5 : Result of Example 3

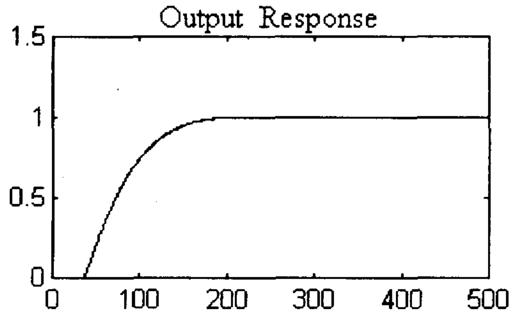

Figure 6 : Result of Example 4

These simulation results revealed that the fuzzy PID controller has better transient and steady state tracking responses, especially when the system under control is nonlinear or time delay.

6.0 Conclusion

In this paper, we have described the design principle of a fuzzy PID type controller based on crisp type model and its tracking performance. The crisp type model greatly simplifies the complicated defuzzification algorithm which could be time consuming. The fuzzy PID type controller preserves the simple linear structure of the conventional PID controller, in addition to enhancing self - tuning control capability. We have shown convincing computer simulation results that demonstrate the effectiveness of the fuzzy PID controller, particularly when the process under control is nonlinear or time delay.

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