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On fuzzy c^*-continuous mappings

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Abstract : We introduce the concept of a fuzzy c^*-continuity. And we obtain some properties.

1. Preliminaries.

Let $I=[0,1]$. For a set X, let I^X be the collection of all mappings of X into I. Each member of I^X is called a *fuzzy set* in X(cf, [9]) For each $A \in I^X$, let $S(A)=\{x \in X: A(x)>0\}$ (called the *support* of A).

Definition 1.1[2]. A subfamily $\Im$ of I^X is called a *fuzzy topology* on X if $\Im$ satisfies the following conditions:

(i) $\emptyset, X \in \Im$.

(ii) If $\{U_\alpha : \alpha \in \Lambda\} \subset \Im$, then $\bigcup_{\alpha \in \Lambda} U_\alpha \in \Im$, where Λ is an index set.

(iii) If $A, B \in \Im$, then $A \cap B \in \Im$.

Members of $\Im$ are called *fuzzy open sets* in X and their complements *fuzzy closed sets* in X. The pair $(X, \Im)$ is called a *fuzzy topological space(fts*, in short).

Notations. For a fts X, let :

(a) $FO(X)$ denote the collection of all the fuzzy open sets in X.

(b) $FC(X)$ denote the collection of all the fuzzy closed sets in X.

Definition 1.2[6]. Let $(X, \Im)$ be a fts and let A be subset of X. Then the family $\Im_A = \{U\mid_A : U \in \Im\}$ is called the *relative fuzzy topology* of $\Im$ to A. Such a fuzzy topological space $(A, \Im_A)$ is called a *subspaces* of $(X, \Im)$. A $\Im_A$-open(resp. $\Im_A$-closed) fuzzy set is also called a *relative open* (resp. *closed*) fuzzy set in A.

It is clear that $\Im_A$ is a fuzzy topology on A.

Definition 1.3[4]. Let $\mathcal{B}$ be a collection of fuzzy sets in a fts X. Then $\mathcal{B}$ is called a *filter* *base* if for any finite subset $\{ U_i : i = 1, \cdots, n \}$ of $\mathcal{B}$, $\bigcap_{i=1}^{n} U_i \neq \emptyset$.

Definition 1.4[4]. A subset A of a fts $(X, \Im)$ is said to be *compact* if for each filter base $\mathcal{B}$ such that every finite intersection of members of $\mathcal{B}$ is q-coincident with A, $(\bigcap_{B \in B} clB) \cap A \neq \emptyset$.

Theorem 1.A[4]. Every closed subset of a compact space is compact.

Definition 1.5[3]. $(X, \Im)$ said to be T_2 iff for two distinct points x_λ and y_μ in X : (i) $x \neq y$ implies that x_λ and y_μ have open nbds which are not q-coincident ; (ii) $x = y$ and $\lambda < \mu$(say) imply that x_λ has an open nbd and y_μ has an open q-nbd which are not q-coincident.

Theorem 1.B[4]. A compact subset of a T_2-space is closed.

Theorem 1.C[4]. Every f-continuous image of a compact space is compact.

Definition 1.6[5]. Let X and Y be fts's, let $f: X \to Y$ be a mapping and let $x_\lambda \in F_p(X)$. Then f is said to be *fuzzy c-continuous*(or simple, *f-c-continuous*) *at* x_λ if for each open nbd V of $f(x_\lambda)$ and V^c is fuzzy compact in Y, there exists an open nbd U of x_λ such that $f(U) \subset V$. The mapping f is said to be *fuzzy c-continuous*(or simple, *f-c-continuous*)(on X) if f is f-c-continuous at each $x_\lambda \in F_p(X)$.

It is clear that each f-continuous mapping is f-c-continuous.

Definition 1.7[8] A fts is *countably compact* iff every countable open cover of the space has a finite subcover.

It is clear that each fuzzy compact space is countably compact.

2. Basic properties of fuzzy c^*-continuous mapping.

Definition 2.1. Let X and Y be fts's, let $f: X \to Y$ be a mapping and let $x_\lambda \in F_p(X)$. Then f is said to be *fuzzy* c^**-continuous*(or simple *f-* c^**-continuous*) *at* x_λ if for each open nbd V of $f(x_\lambda)$ and V^c is fuzzy countably compact in Y, there exists an open nbd U of x_λ such that $f(U) \subset V$. And the mapping f is said to be *fuzzy* c^**-continuous*(or simple, *f- c^*-continuous*)(on X) if f is f-c-continuous at each $x_\lambda \in F_p(X)$.

It is clear that every f-continuous mapping is f- c^*-continuous and every f- c^*-continuous mapping is f-c-continuous. But the inverse is not necessarily true.

Theorem 2.2. Let X and Y is fts's, and let $f: X \to Y$ be a mapping. Then the following statement are equivalent:

(a) f is f- c^*-continuous.

(b) If V is fuzzy open set in Y with V^c is a fuzzy countably compact in Y, then $f^{-1}(V) \in FO(X)$.

(c) If C is a fuzzy countably compact and closed subset of Y, then $f^{-1}(C) \in FC(X)$.

Theorem 2.3. If $f: X \to Y$ is f- c^*-continuous and A is a subset of X, then $f|_A: A \to Y$ is f- c^*-continuous.

Theorem 2.4. If $f: X \to Y$ is f-continuous and $g: Y \to Z$ is f- c^*-continuous . Then $g \circ f: X \to Z$ is f- c^*-continuous .

Theorem 2.5. Let X and Y be fts's and let $X = A \cup B$, where $A, B \in FO(X)$ (resp, $A, B \in FC(X)$) such that $A = S(A)$ and $B = S(B)$. Suppose $f: X \to Y$ is mapping such that $f|_A$ and $f|_B$ are f- c^*-continuous. then f is f- c^*-continuous.

Corollary 2.5.1. If X and Y are fts's, and either (1) $X = \bigcup_{\alpha \in \Gamma} A_\alpha$, where each A_α is a fuzzy open subset of X with $S(A_\alpha) = A_\alpha$ for each α or (2) $X = \bigcup_{i=1}^{n} B_i$, where each B_i is a fuzzy closed subset of X with $S(B_i) = B_i$ for each i and $f: X - Y$ is a mapping such that either each $f|_{A_\alpha}$ or $f|_{B_i}$ is f- c^*-continuous, then f is f- c^*-continuous.

3. Further results

Definition 3.1[7]. A fts $(X, \mathfrak{T})$ is said to be C_I if every fuzzy point in X has a countable local base.

Theorem 3.2. Let $f: X \to Y$ be a f-c-continuous and Y a fuzzy T_2-compact space. Then f is f-continuous.

Corollary 3.2.1. Let $f: X \to Y$ be a f- c^*-continuous and Y be fuzzy T_2-compact space. Then f is f-continuous.

Theorem 3.3. Let $f: X \to Y$ be bijection, f-continuous mapping, and Y C_I . Then $f^{-1}: Y \to X$ is f- c^*-continuous.

Corollary 3.3.1. Let $f: X \to Y$ be bijection, f-continuous function, and Y C_I . Then $f^{-1}: Y \to X$ is f-c-continuous.

Corollary 3.3.2. Let $f: X \to Y$ be bijection and f-continuous. If Y is C_I and X T_2, then f is homeomorphism.

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