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# Fuzzy Logic-based Satisfactory Control of Multiple Objective Systems : Theory and Applications

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## I. Introduction

Many real control systems are designed and realized to achieve multiple objectives. However, it is usually a very difficult task to find an optimal control strategy that optimizes all the multiple objectives simultaneously. The problem can often be mathematically intractable or the objectives may be conflicting with each other not to allow any feasible solutions. In addition, if the system is complex or uncertain, the solution process may be time consuming and the literal meaning of optimality may be lost or at best much degraded.

If we carefully examine the design process of typical control systems, the performance requirements for the objectives are given in advance in the form of specifications and/or desired satisfactory levels. Thus, instead of finding optimal control and measuring the performance in terms of objective functions, we may directly try to find control that are satisfactory with respect to the multiple performance criteria. In fact, the concept of satisfactory control has emerged as an alternative to the optimization-based control.(Goodrich [10]). Such an approach may render the solutions which are easily understood by the designer.

It is noted that, in engineering design, the level of one's satisfaction degree may not be easily expressed by a crisp numerical number. Instead, such a concept of perception is better represented by a fuzzy set. As such, it is attempted in this paper to resolve the control problem of a complex system with multiple objectives by means of fuzzy logic and the concept of satisfaction degree. This approach is a modification of conventional fuzzy logic control.

The fuzzy logic control has been successful as one of the most fruitful tools for automation applications. Earlier well-known fuzzy logic control can be found in many different subjects such as control of heat engine(Mamdani[1], 1974), cement kiln(Holmblad[2], 1982), water purification process(Yagishita[3], 1985), automatic train operation systems(Yasubobu[4], 1985), and automatic crane operation systems(Yamada[6] and [5][7][8][9]). Those applications of fuzzy logic control confirm that fuzzy control can be successfully utilized for complex and uncertain processes that are controlled by a skilled human operator without deep knowledge of underlying dynamics. Fuzzy logic controller can use the experts' heuristic or qualitative knowledge in order to make the fuzzy if-then rules.

In the paper are reviewed various methodologies on multi-objective control and are presented our recent results on satisfactory control of complex and uncertain multi-objective systems. Also as an application, it will be shown that the fuzzy model based satisfactory controller is applied to the design of high level control of an automatic train operation system for magnetically levitated train.

## II. Control of Multi-Objective Systems : Methodologies Reviewed

Consider the multiple objective systems that have multiple objectives to be satisfied. In general, the control problem of multiple objective system can be mathematically formulated as follows :

$$\begin{array}{lll} \textit{Optimize} & Q_1(x(\cdot),u(\cdot)),\cdots,Q_N(x(\cdot),u(\cdot)) & : \textit{objectives} \\ \textit{subject to} & \lambda_i(x(\cdot),u(\cdot))=0,\quad i=1,\cdots,L & : \textit{equality constraints} \\ & \tilde{\lambda}_j(x(\cdot),u(\cdot))\leq 0,\quad j=1,\cdots,M & : \textit{inequality constraints} \end{array}$$

We are interested in dealing with complex/uncertain dynamic systems with multiple objectives to satisfy. Real example plants in that category include refuse incineration plants, high performance induction motor drivers, overhead cranes and magnetically levitated train.

In general, conventional approaches such as pareto optimality, sequential optimization, linear quadratic method and vector inequality approach, are applicable only to the systems whose dynamic characteristics are completely known. Therefore, there should be some other means to deal with the complex and uncertain multi-objective systems. Recently, there have been proposed various kinds of intelligent system approaches including fuzzy logic controller-based system. In fact, FLC-based approaches are known very effective in many applications.

Yasunobu and Miyamoto([4], 1985) studied the automatic train operation system for Sendai rail system based on the model-based fuzzy predictive controller. For the system, the dynamic characteristic of the train system is not precisely identified because of complexity in its own dynamics variation and uncertainty in the environment. Moreover, there were many objectives to be satisfied such as safety, comfortability, traceability, stop gap accuracy and running time accuracy. To control this complex and uncertain multi-objective system, they proposed a design scheme to predict the result via a prediction model and select the most likely control rule among finite fuzzy IF-THEN rules derived from skilled human operators. This scheme is called the model-based fuzzy predictive control, and was successful for the Sendai rail system. This method is, however, difficult to generalize due to the limitation in the number of control alternatives, and the stability of the overall system is not guaranteed.

K.Kim and J.Kim([11], 1994) proposed a design method to assign certainty factors to the obtained rules and applied them to calculate control inputs. When applied to a large scale systems, this method may have difficulty in determining the certainty factors because the relationships between the control objectives and rules are not clear and the method requires too many certainty factors to be determined heuristically.

Since the fuzzy if-then rules are usually derived from many different sources in designing a fuzzy logic based controller for complex and uncertain multi-objective systems, inconsistency between the rules may be unavoidable, and has to be solved. Ginsberg([12], 1988) proposed to sort out less confident rules defined as rules which have smaller number of antecedent conditions based on traditional AI approach. This method is known to be effective for single objective control but inappropriate for multi-objective control systems. Pedrycz([13], 1994) used a design scheme to sort out less likely rules via measure of inconsistency. He defined index of inconsistency and level of inconsistency, and eliminated the rules which have high level of inconsistency. In the controller based on this approach, its performance is highly affected by the definition of measure of inconsistency and the information loss can be substantial dut to elimination process.

A new measure of inconsistency between rules is defined in [9] and [14], and a control method was proposed based on the definition. According to Yu[9][14], two rules are called inconsistent rules in case they have identical or similar antecedents (if-parts) but have different or dissimilar consequents (then-part). Based on the definition, index of inconsistency, level of inconsistency and certainty factor of a rule were determined. The certainty factors are used as weights during defuzzification process. Lim and Bien[9] also proposed a rule modification scheme via pre-determined satisfaction degree function.

It is also possible to apply the concept of satisfactory control to design a controller for the complex and uncertain multi-objective systems. The satisfactory control method renders more meaningful solutions for the designers and it takes shorter time to calculate the solutions in comparison with conventional optimal solutions. In fact, it is very complicated, time-consuming, and sometimes meaningless to find an optimal solution with respect to the given multiple performance indices with the conventional optimal control method. A satisficing control approach via the epistemic utility theory is proposed by M.A.Goodrich, W.C.Stirling, and K.L. Frost([10], 1996). Since the conventional superlative decision making method often confronts with a great barrier caused by computational limitation and lack of information on the dynamic characteristics, a comparative decision making method was proposed as a new paradigm of decision making. The comparative decision making method guarantees the minimum performance in spite of the complexity and uncertainty.

## III. Approaches within Multi-objecitve Decision Making Framework

Multi-objective decision making(MODM) problems have been studied within traditional mathematical optimization methodologies, such as LP, integer programming and NLP. The typical MODM problems is as follows :

$$\begin{aligned} &\text{min} \quad f_1(x), f_2(x), \cdots, f_K(x) \\ &\text{s.t.} \quad x \in \mathbf{X} \equiv \{x \in \mathbf{R}^n \mid g_i(x) \leq 0, i \in [1, I], h_j(x) = 0, j \in [1, J]\} \end{aligned}$$

The above type of problem may be handled by (i) weighting method (ii) trade-off method (iii) goal-programming method (iv) hierarchical (sequential) method (v) min-max method and (vi) vector optimality method.

In this paper, the last two methodologies are discussed in more detail.

### III.1 Vector Optimality via Evolutionary Programming

As a non-gradient-based, probabilistic search technique, it is recently well-known that EA(Evolutionary Algorithm) may solve real-world optimization problems. The approach by Park[16] is to use the Pareto-optimal set frontier(see Fig. 1) and locate a Pareto solution vector via EP(see Fig. 2).

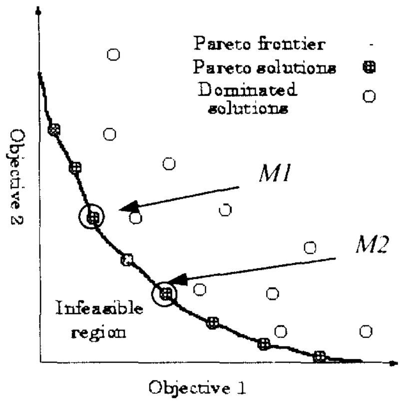

Fig 1. Pareto optimal set(frontier) when minimizing each objective.

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function: selection
Select two candidates, $C_1$ and $C_2$ randomly from the current population.
Select $t_{dom}$ individuals as a comparison set, $CP$, from the population.
    for i =1 to $t_{dom}$ do
        begin
            Evaluate all objective values of $C_1$ and $C_2$ and $CP_i$.
            Check out the dominance of each candidate with
            Respect to the comparison individual.
        End
function: reproduction I
    if $C_1$ is non-dominated and $C_2$ is dominated,
        then return $C_1$ and reproduce it.
    else if $C_1$ is dominated and $C_2$ is non-dominated,
        then return $C_2$ and reproduce it.
    else Apply sharing schemes, and calculate the niche counts
        ($m_1$ and $m_2$) of $C_1$ and $C_2$, respectively.
        If $m_1 \le m_2$
            then return $C_2$ and reproduce it.
        else return $C_1$ and reproduce it.
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Fig 2. Pseudocode for Pareto domination tournament and niche sharing. [16]

By simulation for non-minimum phase plant with dynamics

$$\frac{s-1}{(s+2)(s-0.128)}$$

it was shown that the multi-objective system designed by Pareto-based optimal vector cases (M1,M2) shows better performance than PID-type control and controllers obtained by the weighting method.(Fig.1)

### III.2 Max-min Solution by Math Programming Method

Consider the TS fuzzy model for the plant:

$R_i$ : If $y(k)$ is $A_{i1}$ and $y(k-1)$ is $A_{i2}$ and $\cdots$ and $y(k-n+1)$ is $A_{in}$,
then $\tilde{y}^i(k+1) = \mathbf{a}_i\mathbf{y}(k) + \mathbf{b}_i\mathbf{u}(k)$

or

$R_i$ : If $y(k)$ is $A_{i1}$ and $y(k-1)$ is $A_{i2}$ and $\cdots$ and $y(k-n+1)$ is $A_{in}$,
then $\tilde{y}^i(k+1) = \mathbf{a}_i\mathbf{y}(k) + \mathbf{b}_i\mathbf{u}(k) + c_i$

where $\mathbf{a}_i = (a_{i1},\cdots,a_{in})^T$, $\mathbf{b}_i = (b_{i1},\cdots,b_{im})^T$, $\mathbf{y}(k) = (\mathrm{y(k)},\cdots,\mathrm{y(k-n+1)})^T \in \mathbf{R}^n$, $\mathbf{u}(k) = (u(k),\cdots,u(k-m+1))^T \in \mathbf{R}^m$

The output of the TS fuzzy model is given by

$$\begin{aligned}
\tilde{y}(k+1) &= \frac{\sum_{i=1}^{N} w_i(k)\tilde{y}^i(k+1)}{\sum_{i=1}^{N} w_i(k)} \\
&= \frac{\sum_{i=1}^{N} w_i(k)(\mathbf{a}_i\mathbf{y}(k) + \mathbf{b}_i\mathbf{u}(k))}{\sum_{i=1}^{N} w_i(k)} \\
&\text{or} \\
&= \frac{\sum_{i=1}^{N} w_i(k)(\mathbf{a}_i\mathbf{y}(k) + \mathbf{b}_i\mathbf{u}(k) + c)}{\sum_{i=1}^{N} w_i(k)} \\
&= \tilde{f}(k) + \tilde{g}(k)u(k)
\end{aligned}$$

It is also defined the satisfaction degree functions, $P_i$ as the following examples:

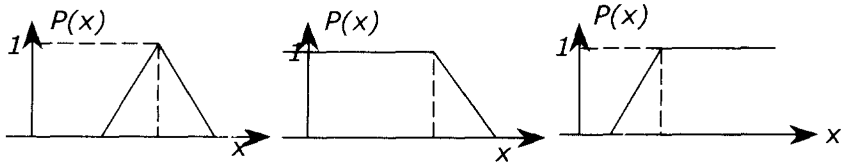

Fig 3. Three types of satisfaction degree fuctnions

The multi-objective control problem under these settings are defined as follows:

$$\begin{aligned}
&\max_{u(k)} \min_{1 \le i \le l} \quad P_i(\tilde{y}(k+1)) \\
&\text{subject to} \quad \tilde{y}(k+1) = \tilde{f}(k) + \tilde{g}(k)u(k) \\
&\qquad\qquad P_i(\tilde{y}(k+1)) > 0, 1 \le \forall i \le l \\
&\qquad\qquad u(k) \in \mathbf{U}
\end{aligned}$$

This problem has to be solved at every moment k. The solution of this problem is effectively obtained, since it is easily converted to the linear programming formulation[17]. In addition, if we consider the uncertainty factors in consequence parts of the TS fuzzy model given by the following equations,

$$\mathbf{a}_i = \bar{\mathbf{a}}_i + \mathbf{F}_a\mathbf{p}, \quad \mathbf{b}_i = \bar{\mathbf{b}}_i + \mathbf{F}_b\mathbf{p}, \quad \mathbf{F}_a \in R^{n \times q}, \mathbf{F}_b \in R^{m \times q}, p \in R^q \text{ and } \|p\| \le 1$$

and formulate it with the worst case optimization scheme, the problem is newly defined as follows:

$$\begin{array}{ll}\max\limits_{u(k)}\min\limits_{\|\mathbf{p}\|\le 1}\min\limits_{1\le i\le l} & P_i(y(k+1)) \\ \text{subject to} & y(k+1)=\tilde{f}(k)+\tilde{g}(k)u(k)+\mathbf{p}^T\mathbf{q}+\mathbf{p}^T\mathbf{f}_{h1}^T u(k) \\ & P_i(y(k+1))>0, 1\le\forall i\le l \\ & u(\mathrm{k})\in U\end{array}$$

This problem can be converted to the second-order cone programming problem(SOCP) using simple mathematics, and therefore, effectively solved under that formulation.[17]

### III.3 Application : MAGLEV ATO system

An automatic train operation(ATO) system for magnetically levitated train(MAGEV) is the system that has the following objectives :

1) Track a given speed pattern.
2) Do not exceed the pre-determined maximum speed.
3) Comfortable ride quality.

This system can be viewed as a high level control system for the propulsion of the MAGLEV train, whose dynamic characteristics are given by a set of, at least, 5 coupled nonlinear differential equations, and have undesirable and unpredictable uncertainty caused by many environmental elements such as levitation gap, weather condition, passenger status, etc.

We have applied the max-min approach mentioned above to the ATO system. We have firstly defined three satisfaction degree functions related to the three objectives, respectively. And then, using input-output data of the MAGLEV train obtained from the previous operation of human operators or conventional PID controllers, we have trained a TS fuzzy model of the form given in section III.2.

The experiment has been undertaken in the rotary test bed system, which is not a real MAGLEV train, but a test model for the propulsion part, using the speed pattern for the real MAGLEV train. We are now performing an experiment in the real MAGLEV train system. The test bed experiment shows that the ATO system based on the proposed algorithm operates MAGLEV with the half of speed tracking error and 2-times better comfortability than the conventional PID algorithm. One of the result is shown in Fig 5.

## IV. Conclusions

In the paper, the past and recent research results are discussed on the satisfactory control methods for the complex and uncertain multi-objective systems. The fuzzy model based satisfactory controller was proposed as a better alternative and applied to the ATO system for the MAGLEV train. The result shows the effectiveness of the control method over the conventional control methods. Further study should be addressed to the issues of convergence, error bound, and stabilization as well as computationally efficient solutions for the complex and uncertain multi-objective systems.

(a) Maglev Train in Korea (b) Rotary Test Bed System

Fig 4. Real Maglev Train and Rotary Test Bed System

(a) Conventional PID controller (b) Proposed Controller

Fig 5. Experiment Result (Rotary Test Bed)

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