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Nonlinear Flexural Modelling of Composite Prestressed Concrete Beams Reinforced with Advanced Composite Materials

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ABSTRACT

The analytical model is developed in order to predict the nonliear flexural responses of bonded and unbonded prestressed concrete beam which contains advanced composite materials. The block concept is used, which can be regarded as an intermediate modeling method between the couple method with one block and the layered method with multiple sliced blocks in a section. The model can successfully predict the flexural behavior of variously reinforced prestressed concrete beams.

1. INTRODUCTION

FRP rebar or tendons usually made of glass, aramid, or carbon have beneficial effects compared to the traditional steel rebars or tendons. The main advantage of reinforcing structural components by FRP can be summarized as follows[1]: 1) FRP reinforcements are corrosion-free; 2) they are non-magnetic, non-conductive to electricity, and transparent to radio waves; 3) they have lighter unit weight; and FRP tendons have almost the same or higher tensile strength than steel tendons. The main objective of this research is to develop an analytical model which can comprise various characteristics of composite materials, different placement and casting of composite materials in combination with conventional cementitious matrices. A beam section is divided by different rectangular blocks characterized by their material property and location in a beam section. The analytical expressions are formulated for these individual blocks. The model takes into account the prepeak and postpeak tensile resistance of the matrix. Complete flexural load-deflections can be generated by the model, which may serve as an useful tool for analyzing and understanding structural behvaior of a composite prestressed concrete beam and thus improving their structural performance.

2. DEVELOPMENT OF THE MODEL

2.1 Assumptions of the Model

The following assumptions are made in developing the current model: 1) plane section remains plane after and before bending(Bernoulli's principle); 2) symmetrical loading type and tendon profile geometry; 3) the internal moment resistance at midspan is assumed equal to the externally applied midspan moment; 4) constitutive relationships for the steel. composite reinforcing bar(i.e., FRP tendon), concrete or composite matrices known for compression or tension; post-peak tensile behavior of ductile composite matrix, the crack opening is assumed to be and 6) the eccentricities of the bonded and/or unbonded prestressing tendons and the external moment varying along the span in accordance with
the pre-seleted tendon profile and loading type geometries.

2.2 Development of Model : Block Modeling

A composite beam is composed of different blocks where each block is identified by its own

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material property and its geometry(Fig.1). For an incremental increase of bottom strain at midspan, top strain leading to a sectional equilibrium is sought through the numerically accelerated method(Modified Regula-Falsi).

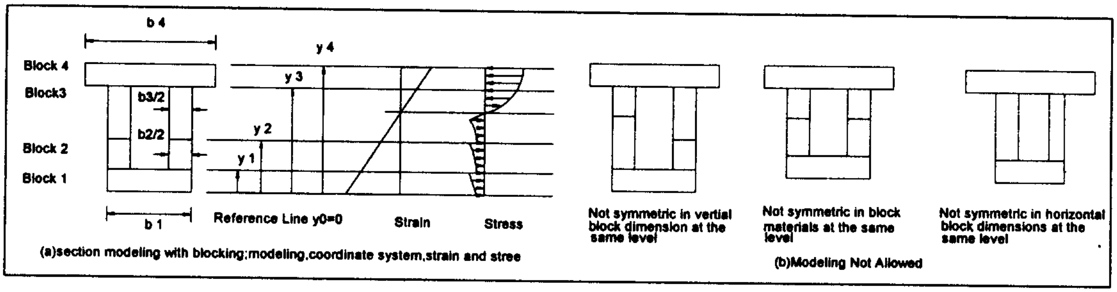

Fig.1 Concept of Block Modeling

For a given bottom strain at j-th section and a numerically chosen trial top strain, sum on axial forces and moments from each block i at section j can be calculated as follows for each block i (see Fig.2):

$$F_{i,j}{}^{k} = b_i \cdot \left(\frac{h_i}{\varepsilon^{k}{}_{i,j} - \varepsilon^{k}{}_{i-1,j}} \right) \cdot \int_{\varepsilon^{k}{}_{i-1,j}}^{\varepsilon^{k}{}_{i,j}} \sigma_{m(i)} d\varepsilon \qquad (1)$$

where :

$F_{i,j}{}^{k}$ = I-th block axial force at section j for loading step k;

$\varepsilon^{k}{}_{i,j}$ = matrix strain at top of the I-th block at section j for loading step k;

$\varepsilon^{k}{}_{i-1,j}$ = matrix strain at bottom of the I-th block at section j for loading step k;

$\sigma_{m(i)}$ = matrix stress for I-th block material, m(I).

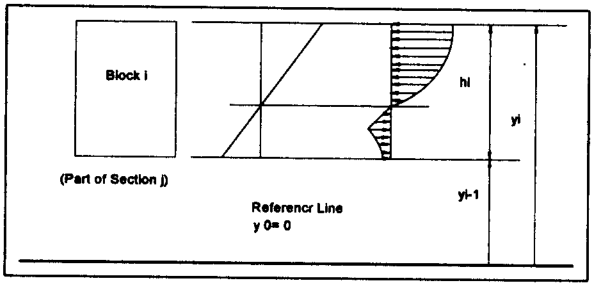

Fig.2 Typical I-th Block: Possible Strain and Stress Distribution

For all sections(j=0,1,2, . . . , N) along the beam, the total sum on the sectional forces from each block must vanish for each loading step k:

$$F_{i,j}{}^{k} = \sum_{i=1}^{NDB} F_{i,j}{}^{k} = 0 \quad (j=0,1,2, \ldots, N) \qquad (2)$$

Where:

$F_{i}{}^{k}$ = sum of the axial forces from each block I at section j for loading step k; and

$F_{i,j}{}^{k}$ = I-th block axial force at section j for loading step k.

Once the sectional equilibrium is reached at section j, the sectional moment resistance at this section is obtained by adding all the moment resistance from each block :

$$M_{j}^{k} = \sum_{i=1}^{NDB} M_{i,j}^{k} \qquad (3.a)$$

where:

M_{j}^{k} = moment resistance at section j for loading step k; (3.b)

$$M_{i,j}^{k} = -\alpha_{i,j}^{k} \cdot \left[\int_{\varepsilon^{k}_{i-1,j}}^{\varepsilon^{k}_{i,j}} \sigma_{m(i)} \cdot \varepsilon \cdot d\varepsilon + \beta_{i,j}^{k} \int_{\varepsilon^{k}_{i-1,j}}^{\varepsilon^{k}_{i,j}} \sigma_{m(i)} d\varepsilon \right] \quad (3.c)$$

= I-th block moment at section j;

$$\alpha_{i,j}^{k} = \frac{b_i \cdot h^2_i}{\left(\varepsilon_{i,j}^{k} - \varepsilon_{i-1,j}^{k} \right)^2} ; \quad (3.d)$$

$\varepsilon_{i-1,j}^{k}$ =bottom strain of block I at section j for loading step k;

$\varepsilon_{i,j}^{k}$ = top strain of block I at section j for loading step k;

y_i , y_{i-1} = y coordinate of top and bottom of I-th block, respectively;

$h_i = y_i - y_{i-1}$ = height of i-th block; and

$\sigma_{m(i)}$ = stress function corresponding to I-th block matrix.

Intergration shown above is perfmed using 5-point Gauss-Legeadre gradrature. In order to estimate curvature distributions, a beam is divided into a number of sections along the beam. At each of these sectioins, extreme bottom strain and extreme top strain are numerically found so that internal moment obtained from this strain distribution is close enough to the externally applied moment. This procedure is used in [2] and called as "Moment Equilibrium" and is expland in the following. With two given conditions that sum on axial forces equal to zero and sum on internal moment equal to a given external moment at a section, two unknowns - top extreme strain and bottom extreme strain- can be evaluated.
Basic assumptions are made on the selection of bottom strain at extreme bottom fiber at section j: 1). icreasing bottom strain increases the internal moment resistance at any loading; and 2). the extreme bottom fiber strain at loading step k ($\varepsilon_{i,j}^{b,k}$) is greater than or equal to the one at previous loading step k-1 ($\varepsilon_{i,j}^{b,k-1}$) and less than or equal to the extreme bottom fiber strain at the midspan at the loading step k ($\varepsilon_{i,j}^{b,k}$) :

$$M_{j}^{k-1} \leq M_{j}^{k} \leq M_{N}^{k} , \quad j=0,1,2, \ldots , N-1 \quad (4)$$

Let, $\Delta M_{j}^{k,L} = M_{j}^{k-1} - M_{j}^{k}$ and $\Delta M_{j}^{k,U} = M_{N}^{k} - M_{j}^{k}$. By the assumptions made above, it can be seen that $\Delta M_{j}^{k,L}$ is less than or equal to zero and $\Delta M_{j}^{k,U}$ is greater or equal to zero. The extreme bottom fiber strains corresponding to the lower bound($\varepsilon_{1,j}^{b,k,L}$) and the upper bound($\varepsilon_{1,j}^{b,k,u}$) for the given external moment at section j (M_{j}^{k}) can be assigned with the previously obtained strains at section j for loading step k-1, ($\varepsilon_{1,j}^{b,k-1}$) , and at midspan for loading step k,($\varepsilon_{1,N}^{b,k,}$) :

$$\varepsilon_{1,j}^{b,k,L} = \varepsilon_{1,j}^{b,k-1} \quad \text{and} \quad \varepsilon_{1,j}^{b,k,u} = \varepsilon_{1,N}^{b,k,} . \quad (5)$$

The correct bottom strain at extreme bottom fiber corresponding to the given external moment of is then enclosed in the interval $[\varepsilon_{1,j}^{b,k,L}, \varepsilon_{1,j}^{b,k,u}]$. Let $\triangle M_j^k = \overline{M}_j^k - M_j^k$, where M_j^k is the moment obtained by the assumed extreme bottom fiber strain at section j and loading stage k . Note that at the correct extreme bottom fiber strain ($\varepsilon_{1,j}^{b,k}$), the difference between internal moment resistance and external moment becomes zero (i.e., $\triangle M_j^k = 0$).

Midspan deflection is calculated using Simpson's Rule and the Moment Area theorems.

3. PRESTRESSED CONCRETE BEAM

3.1 Prestrains of Tendons at Reference Stage

The prestrains at the initiation stage of iteration can be obtained by.

$$\varepsilon_{psb,l} = \varepsilon_{peb,l} + \varepsilon_{ceb,l} \text{ ; and } \quad \varepsilon_{psub,l} = \varepsilon_{peub,l} + \Omega\, \varepsilon_{ceub,l} \tag{6}$$

where:

$\varepsilon_{psb,l}$, $\varepsilon_{psub,l}$ = l-th bonded and i-th unbonded tendon prestrains at the reference stage, respectively;

$\varepsilon_{peb,l}$, $\varepsilon_{pceub,l}$ = l-th bonded tendon effective strain and l-th unbonded tendon effective strain, respectively; and

$\varepsilon_{ceb,l}$, $\varepsilon_{ceub,l}$ = strain increase in the matrix at the level of l-th bonded and l-th unbonded tendon for the load beyond reference stage, respectively.

In calculating unbonded prestrains($\varepsilon_{pceub,l}$) above, the bond reduction coefficient(Ω) is used to take into account their member depedent (not section dependent) quantities.

Using the Hooke's law and classical elastic theory, the expression for Ω can be formulated as[3]:

$$\Omega = \frac{2}{L \cdot M_{\max} \cdot (e_{oub})} \cdot \int_0^{l/2} M(x) \cdot e_{ub}(x) \cdot dx \tag{7}$$

where:

L = beam span;

$M_{\max}$ = changes in bending moment at the critical section (or midspan in our study);

e_{oub} = eccentricity of unbonded tendon at the critical section; and

$e_{ub}(x)$ = eccentricity of unbonded tendon at the distance x along the beam from the support.

In the current study, the concept of bond reduction coefficient (Ω) is adopted numerically rather than analytically to find strains in unbonded tendons. Concrete strains at the level of these tendons are found for each section and then these strain values along the beam are integrated numerically in order to evaluate the corresponding elongation of the unbonded tendon. The additional strain is obtained by dividing this elongation of the tendon by corresponding tendon length. This strain is then added to the effective prestrain given at the reference stage.

4. CONSTITUTIVE MODELS

The model incorporates different type of constitutive models of for matrix, reber and prestressed steel tendons or FRP tendons. The mathematical expression for models can be found in[4-6]

5. COMPARISON WITH TEST RESULT

5.1 Bonded Prestressed Concrete Beam with Steel Tendons.

Fully or partially prestressed concrete beams are used for the comparison(Fig.3). Loading conditions and typical cross sections of the beam as well as their properties are given in[7].

Table.1 Summary of reinforcing paramenters

Beam	Tensile Steel Area	Comp. Steel Area	f'c (ksi)	fy (ksi)	ds (in)	Tendon Area	dps (in)	Pe/bar (kips)
PS3	-	2#D1 (0.017)	5.73	70.0	-	0.256	4.75 6.25 7.75	11.56
PP2S2	3#2 (0.15)	2#D1 (0.017)	5.29	70.0	8.0	0.085	6.25	11.31
PP2S3	2#3 (0.22)	2#D1 (0.017)	6.20	70.0	8.0	0.170	6.25	11.31

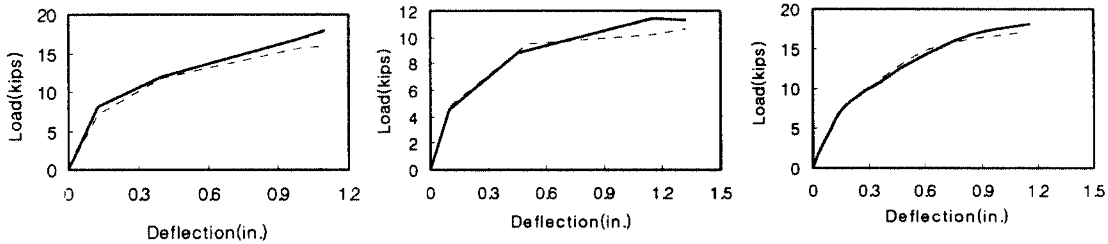

Fig.3(a). Beam PS3 Fig.3(b). Beam PP2S2 Fig.3(c). Beam PP2S3

Fig.3 Comparison between the Model and the Test Results for Bonded Prestressed Concrete Beam[7]

5.2 Bonded Prestressed Concrete Beam with CFRP Tendon

Test results of bonded prestressed concrete beam with CFRP tendons (Fig. 5.10)and SIFCON [8] are compared with model predictions.

Table.2 Summary of reinforcing parameters

Beam	Tensile Steel Area	f'c (ksi)	fy (ksi)	ds (in)	Tendon Area	Dps (in)	Pe/bar (kips)
TC6	2#4(0.4)	6.10	60.0	11.0	2CFCC(0.094) 2CFCC(0.094)	6.75 9.25	8.93 8.93
RFC	2G270(O.306)	6.10 (plain) 6.2 (SIFCON)	243.5	11.0	1CFCC(0.047) 2CFCC(0.094) 1CFCC(0.047)	7.5 9.25 10.75	8.93 8.93 8.93
RFCa-1	2G160(1.57)	6.2 (SIFCON)	142	11.0	1CFCC(0.047) 2CFCC(0.094) 1CFCC(0.047)	7.5 9.25 10.75	8.93 8.93 8.93

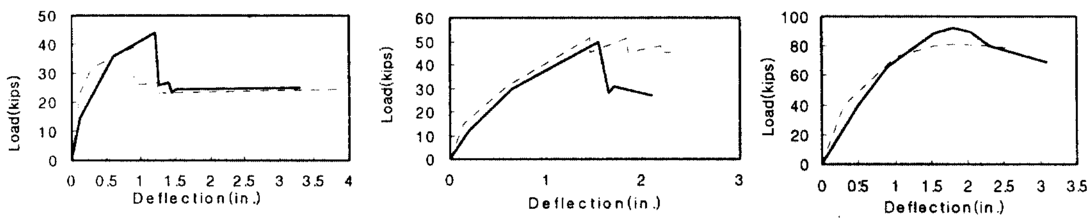

Fig.4(a). Beam TC6 Fig. 4.(b). Beam RFC Fig. 4(c). Beam RFCa-1

Fig.4 Comparison between the Model and the Test Results for Prestressed Beam with Bonded CFRP Tendons(CFCC) [8].

5.3 Unbonded Prestressed Concrete Beam

Experimental results from Tao et.al.[9] are used to compare with analytical model .

Table.3 Reinforceing paramenters

Beam	Tensile Steel (As)	fy (ksi)	f'c (ksi)	ds (in)	Tendon Area (As)	Dps (in)	Pe/bar (kips)
A-2	0.243	62.4	4.44	9.84	0.152	8.66	19.95
A-3	0.366	62.4	4.44	9.84	0.247	8.66	28.91

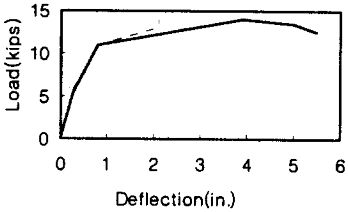

Fig.5(a). Beam A-2

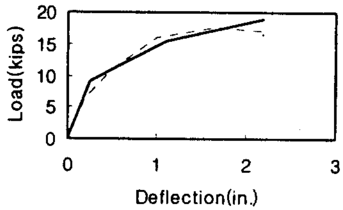

Fig.5(b). Beam A-3

Fig. 5 Comparison between the Model and the Test Results for Prestressed Concrete Beam with Unbonded Tendons[9]

6. CONCLUSION

The main features of the model can be summarized as follows : 1) the model can predict the flexural behavior of composite bonded prestressed beam and unbonded prestressed concrete beam; 2) a beam having various combinations of reinforcing materials - i.e., linear elastic material like FRP rebar or tendon and strain hardening material like conventional rebar or steel tendons - can be modeled; and 3) the model can trace the residual flexural response of the beam after one (or more) of the main reinforcements is (or are) fractured. This feature is useful when an overall flexural behavior of the beam reinforced with both brittle FRP bars (prestressed or nonprestressed) and conventional ductile reinforcements are desired.

ACKNOWLEDGDMENTS

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Reference

1. Fiber-Reinforced-Plastic(FRP) Reinforcement for Concrete Structure: Properties and Application, Edited by Nanni, A.,Elsevier,1993.
2. Alkhairi,F.D.,On the Flexural Behavior of Concrete Beams Prestressed with Unbonded Internal and External Tendons, A dissertation submitted in partial fulfillment of the requirement for the degree of Doctor of Philosophy(Civil Engineering) in the University of Michigan, 1991
3.Naaman,A.E.,A New Methodology for the Analysis of Beams Prestressed with External or Unbonded Tendons, External Prestressing in Bridges,Naaman, A.E. and Breen,J.E., Editors, SP120-16, the American Concrete Institute, Detroit, 1990,pp.339-354.
4. Absi,E. and Naaman,A.E.,Modele Rheologique Pour Les Betons De Fibres, Proceedings of 3rd International Symposium on Fiber Reinforced Cement and Concrete, Sheffield, UK, July 1986, edited by Swamy, R.N., Wagstaffe, R.L. and Oakley, D.R.
5. Virlogeux, M., Non-Linear Analysis of Externally Prestressed Structures, FIP Symposium in Israel
6. Menegotto,M. and Pinto,P.E.,Method of Analysis for Cyclically Loaded R.C. Plane Frames, IABSE Preliminary Report for Symposium on Resistance and Ultimate Deformability of Structures Acted on by Well-Defined Repeated Loads, Libson, Portugal, 1973,pp.15-22.
7. Harajli, M.H., Deformation and Cracking of Partially Prestressed Concrete Beams under Static and Cyclic Fatigue Loading, A dissertation submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy (Civil Engineering) in the University of Michigan, 1985.
8. Jeong,S.M.,Evaluation of Ductility in Prestressed Concrete Beams Using Fiber Reinforced Plastic Tendons, A Dissertation submitted in partial fulfillment of the requirements for the degreee of Doctor of Philosophy(Civil Engineering) in the University of Michigan, 1994.
9. Tao.x and Du.G., Uitimate Stress of Unbonded Tendons in Partially Prestressed Concrete Beam, PCI Journal,Vol.30, No.6, pp.72-91, 1985