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Earth Pressure-Deflection Curves for Beam-column Modeling of Tieback Walls
앵커토류벽의 탄소성 보해석을 위한 토압-변위 곡선에 관한 연구

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개 요 : 탄소성 보해석 모델은 유한요소 해석에비해 상대적으로 간단하며 흙-구조물 사이의 상호작용을 적절하게 고려할 수 있는 장점으로 연성 토류벽, 앵커 토류벽, 버팀보 토류벽에 널리 이용되고 있다. 이 모델은 토류벽을 강성을 가진 일차원 선형 요소로, 지반을 비선형의 스프링으로 치환하며 벽체 변위와 토압과의 관계를 이용하여 토류벽을 해석하는 방법이다. 그러므로 앵커 토류벽의 탄소성 보해석의 결과는 지반 스프링을 표시하는 토압-변위 곡선에 의해 좌우된다. 본 연구에서는 실물크기의 앵커 토류벽을 시공하여 실측된 변위와 모멘트로부터 Cubic Spline Function을 이용 토압 분포를 산정 함으로서 앵커 토류벽의 탄소성 보해석에 필수적인 토압-변위 곡선을 구성, 제안하였다.

Key words : Earth Pressure, Anchored Wall, Tieback Wall, Retaining Wall, p-y curve

1. Introduction

A tieback wall or ground anchor wall is an innovative earth retaining system which uses tiebacks or ground anchors. A tieback functions as a load carrying element, consisting essentially of a steel tendon inserted into a suitable ground formation (Cheney, 1988). Retaining structures for transportation facilities, bridge abutments, deep excavation in urban area, underpinning of structures and stabilization of sliding soil or rock slopes are some applications (Weatherby, 1982).

The analysis technique for tieback walls has been developed for decades (Haliburton, 1968, Clough and Tsui, 1974, Briaud와 김낙경, 1998). The beam on elasto-plastic foundation model is popular due to its simplicity and wide applicability. The model represents the foundation as a series of non-linear soil springs. The success of tieback wall modeling depends primarily on the earth pressure-deflection (p-y) relationship representing non-linear soil springs.

In this study, full scale anchored walls in sand were instrumented and constructed. Earth pressure distribution was obtained from the bending moment measurements by using cubic spline function. Earth pressures obtained from the measurement were incorporated into the earth pressure-deflection curves (p-y curves) and tested for the measured wall deflection.

2. Beam-Column Modeling of Tieback Wall

Governing equation for the horizontal beam modeling of the tieback wall is as follows;

$$EI\frac{d^4y}{dz^4}+Q\frac{d^2y}{dz^2}-P(y,z)=0 \qquad (1)$$

where, EI is the wall lateral stiffness, y the wall horizontal deflection at depth of z, Q the axial load in the wall at depth of z, P horizontal soil reaction or tieback load.

The beam-column modeling of the tieback wall is shown on Figure 1 and the width of the wall to be considered is either unit width of the wall with equivalent anchor load per unit width or the horizontal spacing of the anchor. The conceptual p-y curves for the slurry wall type tieback wall are shown on Figure 2.

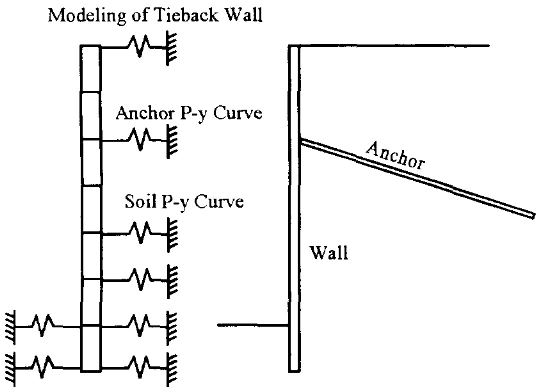


Figure 1. Beam-column Modeling

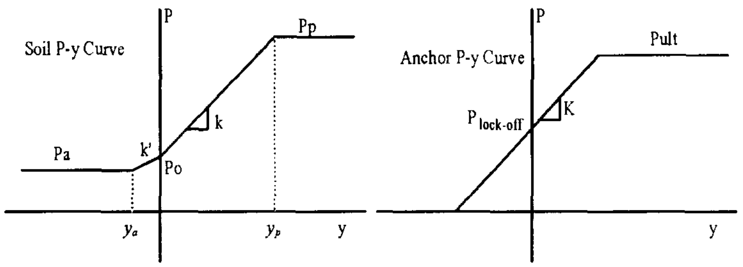


Figure 2. P-y Curves for Slurry Wall Type Wall

3. Full Scale Test Wall

A full-scale soldier pile and woodlagging tieback wall with two different instrumented sections has been built and monitored at the Texas A&M University National Geotechnical Experimentation Site. The soil condition at the site and the section of the one row tieback wall are shown on Figure 3 and 4, respectively. The measured deflection and bending moment for the one row tieback wall at three construction stages are shown on Figure 5.

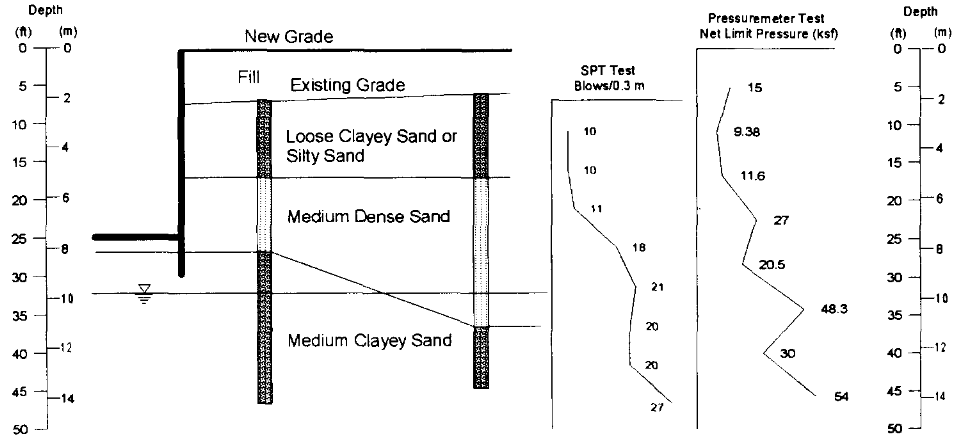


Figure 3. Soil Condition at the Site

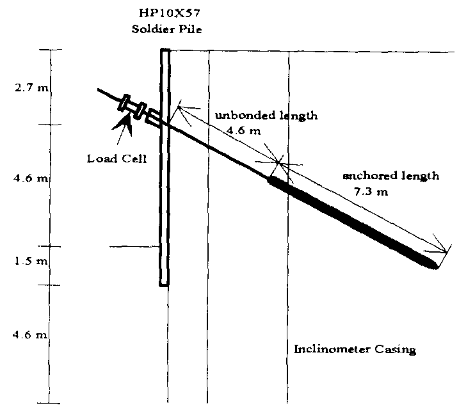
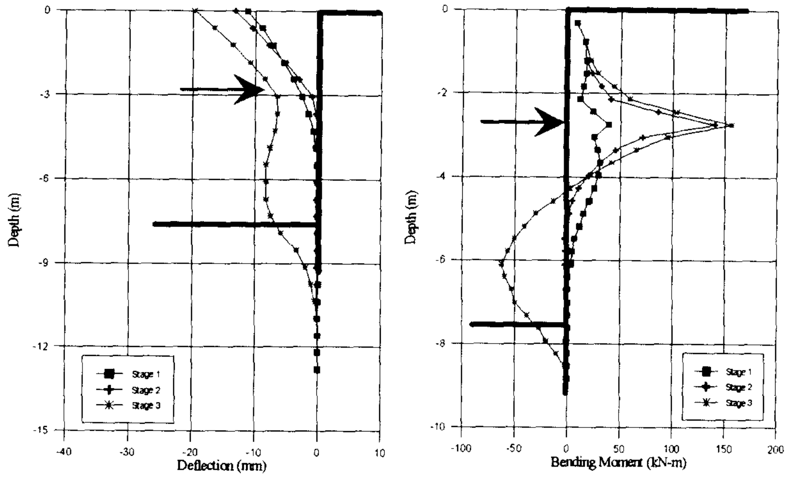


Figure 4. Section of One Row Wall



Note : Stage 1-Excavation to 3.1 m
Stage 2-Stressing Anchor at 2.7 m
Stage 3-Excavation to 7.6 m

Figure 5. Measured Deflection and Bending Moment

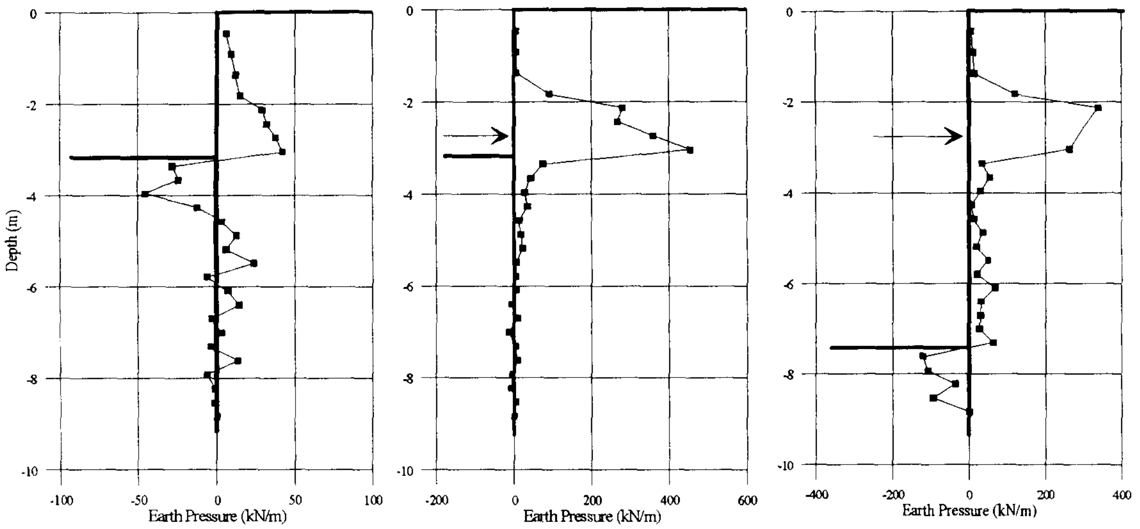
4. Earth Pressure Distribution by Cubic Spline Interpolation

The earth pressure behind the wall can be measured directly by a load cell. An alternative way to measure the earth pressure on the wall is the double differentiation of the bending moment profile. The basic differential equations for a bending are;

$$p = EI\frac{d^4y}{dz^4} = EI\frac{d^2}{dz^2}\left(\frac{d^2y}{dz^2}\right) = EI\frac{d^2}{dz^2}\left(\frac{M}{EI}\right) = \frac{d^2M}{dz^2} \qquad (2)$$

where, M is the bending moment, EI lateral stiffness of the beam, V the shear force, p the pressure on the beam, y lateral deflection of the beam, z the depth.

A cubic spline interpolation can be used for the differentiation of the bending moment data which are commonly scattered. The precision of the differentiation depends on the extent to which the bending moment data is scattered. Since the cubic spline function gives continuous second derivatives, the double differentiation can be performed by applying the cubic function and the interpolation technique. A cubic spline interpolation and double differentiation were performed by using IMSL subroutine package and the earth pressure distribution obtained are shown on Figure 5 at different construction stages.



(a) Excavation to 3.1 m (10 ft) (b) Stressing Anchor at 2.7 m (9 ft) (c) Excavation to 7.6 m (25 ft)

Figure 6. Earth Pressure Distribution at Different Construction Stages

5. Experimental Earth Pressure-Deflection Curves

The earth pressure-deflection (p-y) relationship can be represented by the active earth pressure, the passive pressure and the horizontal subgrade modulus as shown on Figure 7 (a). Once the horizontal subgrade modulus for the tieback wall is obtained, the p-y curves can be constructed. The horizontal subgrade modulus may be available for laterally loaded piles, but not for flexible retaining walls. The horizontal subgrade modulus for tieback walls proposed by Pfister et al. (1982) is based on the experience and was not clearly evaluated.

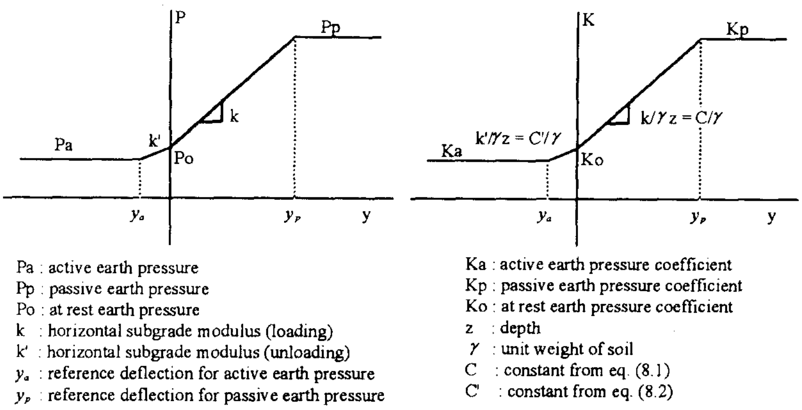
If the soil behind the retaining wall can be assumed to be homogeneous and the horizontal subgrade modulus can be assumed to increase with depth, which appears to be reasonable assumptions for the soil condition at the site, the horizontal subgrade modulus can be written as the following;

$$k_h = Cz \quad \text{for loading} \qquad (3)$$

$$k_h' = C'z \quad \text{for unloading} \qquad (4)$$

where, k_h is the horizontal subgrade modulus, C the constant, z the depth.

The normalized p-y curve can be obtained by dividing p-y curve by the unit weight of the soil and by the depth which gives the active earth pressure coefficient K_a, the passive earth pressure coefficient K_p, the at rest earth pressure coefficient K_o and the reference deflection y_a, y_p as shown on Figure 7 (b). Thus, the normalized earth pressure can be related to the measured deflection of the wall regardless of the depth. The measured deflection which is related to the earth pressure is a certain amount of deflection between consecutive stages in order to consider the plastic movement of the p-y curve. The normalized earth pressure (earth pressure coefficient) related to the deflection is shown on Figure 8.



(a) earth pressure-deflection relationship

(b) normalized earth pressure-deflection relationship

Figure 7. Earth Pressure-Deflection Relationship

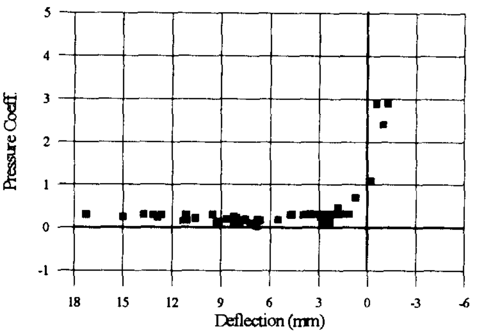


Figure 8. Experimental P-y Curve

The average active earth pressure coefficient was taken to be 0.15, which is smaller than Rankine active earth pressure coefficient. The probable reasons may be the large wall friction, some cohesion, and the arching of the soil. Since there were not enough data for the passive side, the passive earth pressure coefficient was extrapolated by using Kp=1/Ka relationship and the passive coefficient of 6.6 was obtained.

The experimental p-y curve gives an idea of what the horizontal subgrade modulus may be for the flexible retaining wall. For example, at a depth z, the horizontal subgrade modulus k can be estimated as follows;

$$k = \frac{(K_p - K_o)}{y_p}\gamma z \quad \text{for loading} \qquad (5)$$

$$k' = \frac{(K_o - K_a)}{y_a}\gamma z \quad \text{for unloading} \qquad (6)$$

The earth pressure-deflection curve can be established based on this relationship, and gives the constant reference deflections for the active and passive earth pressures as shown Figure 7 (a).

6. Recommended P-y Curves

The earth pressure-deflection (p-y) curves involved in the beam-column analysis of tieback wall consists of the wall p-y curves for presumptively plane strain condition and the pile p-y curves for the soldier pile and woodlagging wall.

For walls in sand with a vertical face and horizontal ground, the following soil reactions and deflection are recommended:

$$P_a = (K_a \sigma_{ov}^{'} \cos\delta + u)b \qquad (7)$$

$$P_o = ((1 - \sin\phi)\sqrt{OCR}\ \sigma_{ov}' + u)b \tag{8}$$

$$P_p = (K_p \sigma_{ov}' \cos\delta + u)b \tag{9}$$

$$y_a = 1.3 \text{ mm} \qquad y_p = 13 \text{ mm} \tag{10}$$

where, σ_{ov}' is the vertical effective stress at depth z, K_a the Coulomb's active earth pressure coefficient, K_p the Coulomb's passive earth pressure coefficient, OCR the overconsolidation ratio, u the pore pressure, δ the wall friction angle, b the width of the wall considered.

For walls in clay with a vertical face and horizontal ground, the following soil reactions and deflection are recommended;

(effective stress analysis or long term analysis)

$$P_a = (K_a \sigma_{ov}' - 2c\sqrt{K_a} + u)b \tag{11}$$

$$P_o = ((1 - \sin\phi)\sqrt{OCR}\ \sigma_{ov}' + u)b \tag{12}$$

$$P_p = (K_p \sigma_{ov}' + 2c\sqrt{K_p} + u)b \tag{13}$$

(total stress analysis or short term analysis)

$$P_a = (\sigma_{ov} - 2S_u)b \tag{14}$$

$$P_o = \sigma_{ov}\, b \tag{15}$$

$$P_p = (\sigma_{ov} + 2S_u)b \tag{16}$$

where, σ_{ov} is the vertical total stress at depth z, K_a the Rankine's active earth pressure coefficient, K_p the Rankine's passive earth pressure coefficient, S_u the undrained shear strength of the clay. Reference deflections are presented in Table 1.

Table 1. Reference Deflections for Clay

Reference Deflection	S_u < 200 kN/m^2	200 < S_u < 400 kN/m^2	S_u > 400 kN/m^2
y_a (mm)	5	4	3
y_p (mm)	25	20	10

For piles below the excavation in sand, the p-y curves proposed by O'Neill and Murchison (1983) are recommended. These p-y curves were modified as follows to take into consideration the fact that the ground surface is not horizontal.

$$P_{us} = (C_1 z + C_2 D)\sigma_{ov}' - (-1)^j K_a \sigma_e' b \tag{17}$$

$$P_{ud} = C_3 D\, \sigma_{ov}' - (-1)^j K_a \sigma_e' b \tag{18}$$

$$P = AP_u \tanh\left(\frac{kz}{AP_u} y\right) \tag{19}$$

$$A = 3 - 0.8\frac{z}{D} \geq 0.9 \tag{20}$$

where, P_{us} is the ultimate soil reaction above the critical depth, P_{ud} the ultimate soil reaction below the critical depth, k, C1, C2, C3 are the coefficient (refer to O'Neill and Murchison, 1983), D the pile diameter or width, σ_{ov}' the vertical effective stress at depth z, K_a the active earth pressure coefficient, σ_e' the vertical effective stress at excavation level, b the width of the wall considered, j=1 if the wall moves away from the excavation and j=2 if the wall moves toward the excavation.

For piles below the excavation in clay, the p-y curves were proposed by using Reese's ultimate soil reaction for clay. As in the case of sand, these p-y curves were calibrated to better match the case histories. The elasto-plastic p-y curves are defined as follows by an ultimate value P_u and a deflection y_c necessary to mobilize P_u:

$$P_{us} = A(\sigma_{ov} D + 2S_u D + 2.83 S_u z) - (-1)^j p_e b \tag{21}$$

$$P_{ud} = 11 A S_u D - (-1)^j p_e b \tag{22}$$

$$p_e = K_a \sigma_e' - 2c\sqrt{K_a} \quad \text{for a long term analysis} \tag{23}$$

$$p_e = \sigma_e - 2S_u \quad \text{for a short term analysis} \tag{24}$$

$$y_c = 18 \text{ mm} \quad \text{if } S_u < 200 \text{ kN/m}^2 \tag{25}$$

$$y_c = 13 \text{ mm} \quad \text{if } 200 < S_u < 400 \text{ kN/m}^2 \tag{26}$$

$$y_c = 2.5 \text{ mm} \quad \text{if } S_u > 400 \text{ kN/m}^2 \tag{27}$$

where, A is equal to 0.2 at z=0, to 0.5 for 0<z<2D, and to 1 for z>2D.

For anchors in tieback walls, the anchor load-deflection curves can be simulated with the ultimate anchor capacity and the elastic slope for the tendon as follows:

$$P_h = Ky_h = \left(\frac{AE}{L_u + \frac{1}{2}L_b}\right)y_h \qquad (28)$$

where, P_h is the horizontal anchor load, A the cross section area of tendon, E elastic modulus of steel tendon, L_u the unbonded length of the anchor, L_b the bonded length, y_h the horizontal deflection of the anchor.

7. Simulation of Construction Sequence

The construction stages of the tieback wall consist of a series of unloading and loading process, which are caused by the excavation and the anchor stressing. The conceptual methodology for the simulation of the stages is to keep track of the p-y path and updating the p-y curves from one stage to the next stage (Briaud and 김낙경, 1998).

At the first construction stage, the unloading path caused by the excavation is from the at rest condition to the active state condition (A-B-C on Figure 9). For the second stage of anchor stressing, the path C-D-E on Figure 9 is followed by the soil behind the wall. In order to consider the plastic movement from the hysteretic properties of soil, the p-y curves need to be updated for the next stage by shifting the p-y curves as much as the plastic movement. The offset amount of deflection which represents the plastic movement for the second stage p-y curves is shown on Figure 9.

The offset amount $y_{off}(i,j)$ for (j)th construction stage at (i)th node, can be obtained by comparing the deflection $y_{off}(i,j)$ with the p-y curves as follows:

$$If \;\; y(i,j) > y_p(i,j), \quad y_{off}(i,j) = y(i,j) - y_p(i,j) \qquad (29)$$

$$If \;\; y_a(i,j) < y(i,j) < y_p(i,j), \quad y_{off}(i,j) = 0 \qquad (30)$$

$$If \;\; y(i,j) < y_a(i,j), \quad y_{off}(i,j) = y(i,j) - y_a(i,j) \qquad (31)$$

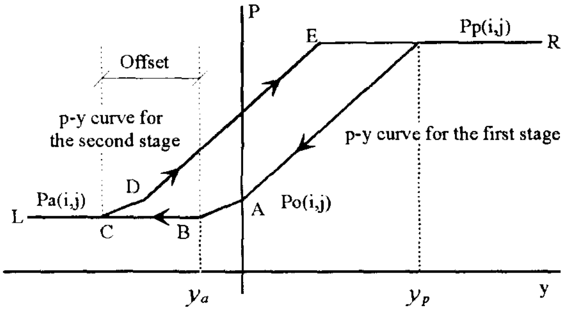


Figure 9. Simulation of Construction Stages

The p-y curves for the first stage are the recommended p-y curves presented earlier. After each (j)th stage, the p-y curves for the (j+1)th stage are updated by moving the reference deflection as much as offset distance obtained from the (j)th stage. The reference deflections for the p-y curves for the (j+1)th stage are as follows:

$$y_a(i,j+1) = y_a(i,j) + y_{off}(i,j) \tag{32}$$

$$y_o(i,j+1) = y_o(i,j) + y_{off}(i,j) \tag{33}$$

$$y_p(i,j+1) = y_p(i,j) + y_{off}(i,j) \tag{34}$$

8. Comparison with Case Histories

The beam-column analysis was performed with the recommended p-y curves for two case histories. The 'sequence' approach and 'no sequence' runs were performed. The case histories include the test wall in sand at Texas A&M University and the Lima tieback wall in clay. The comparisons between the beam-column results and the measurements were made for TAMU test wall and shown on Figure 10.

The Lima wall is an 8.2 m high drilled shaft and woodlagging wall in the city of Lima, Ohio (Lockwood, 1988). The soil at the site consists of a very stiff clay with an undrained shear strength of 158 kN/m^2, drained friction angle of 35°, cohesion of 16.3 kN/m^2, total unit weight of 21.1 kN/m^3, overconsolidation ratio of 2.5. The drilled shaft diameter is 0.76 m and the length is 12.8 m. The reinforcement of the shaft is made of a double channel C-15x33.9. The excavation height is 8.2 m with anchors at depths of 2.4 m and 4.9 m with an inclination of 20°. The comparisons were made and shown on Figure 11.

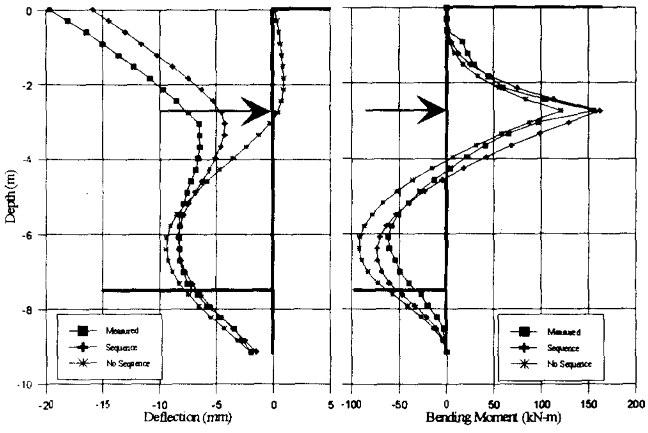


Figure 10. TAMU Test Wall in Sand

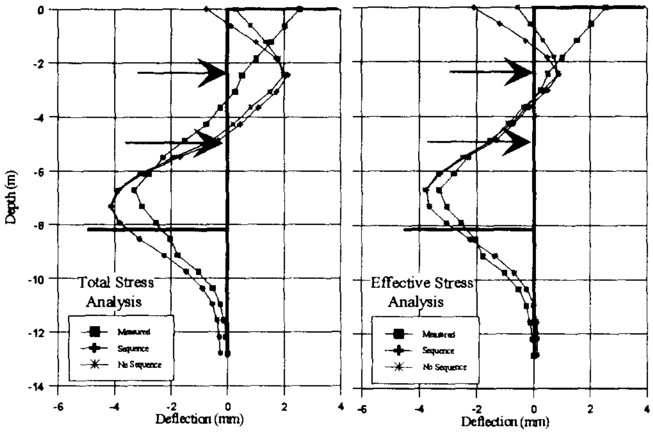


Figure 11. Lima Tieback Wall in Clay

9. Conclusions

The following conclusions and recommendations are drawn.

1. The recommendations are made concerning the earth pressure-deflection curves for the beam-column modeling of tieback walls. The p-y curves for the plane strain condition (wall type) and for the single pile (soldier pile and woodlagging wall) were recommended.
2. The earth pressure acting on the tieback wall was obtained from the bending moment measurements by using the cubic spline function. The normalized earth pressure-deflection relationship was developed and calibrated for the test wall.
3. The simulation of the construction sequence was developed. The p-y path method is used to handle the soil hysteresis during the excavation steps (unloading) and the anchor stressing steps (reloading). The simulation of the construction technique gives better prediction of wall deflection and the bending moment.
4. The beam-column method with the proposed p-y curves should be limited to the case that the anchor bonded length is secured far enough to be fixed in the unmoving soil mass. If the location of the anchor bonded zone is within the active soil wedge, the beam-column method should not be used.

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