

병진변위와 회전변위의 정밀측정을 위한  
새로운 무아레 격자들

New Moiré Gratings for the Fine Visual Measurement of  
Small Linear and Angular Displacements

Jae Heung Jo, Soo Chang, Keun Cheol Yuk\*

Department of Physics, Hannam University

\*Department of Physics Education, Kongju National University

e-mail : jhjo@eve.hannam.ac.kr

The moiré fringes<sup>1</sup> that are produced by two periodic or quasi-periodic structures placed in contact or projected onto another have been widely used for measurement of a rigid-body translational and rotational displacement,<sup>1</sup> strain analysis in deformed bodies,<sup>1</sup> refractive index measurement,<sup>2</sup> visual position indication,<sup>3</sup> surface contour generation,<sup>4</sup> optical alignment,<sup>5</sup> and circular motion detection.<sup>6</sup> In moiré metrology, structures such as parallel, crossed, circular, or radial gratings<sup>7</sup> are used substantially for simplicity in fringe analysis.<sup>1</sup> Sometimes, however, other structures, called elliptic,<sup>1</sup> parabolic,<sup>8</sup> zone plate,<sup>9</sup> skewed radial,<sup>10</sup> spiral,<sup>11</sup> or evolute gratings,<sup>12</sup> are applied in the moiré fringe systems.

The parallel gratings and circular gratings are most widely utilized in many applications, due to their simple patterns and ease of moiré fringe analysis. If two identical parallel gratings or circular gratings are chosen and superimposed, the absolute linear displacements can be measured within an accuracy of one pitch of the grating.<sup>2</sup> However, the moiré patterns from these gratings do not give the direction of displacements in the spatial symmetry of gratings. Such a problem was solved by using many available concentric circular gratings,<sup>5,13</sup> although still it is not easy to precisely determine the linear displacement with the subpitch accuracy using an observer's eyes or optical instruments.<sup>13</sup>

Two parallel line gratings that are angularly displaced to disturb their parallelism create a number of equidistant straight fringes. The relative angular displacement of the gratings can be determined by measuring the spacing of the fringes and their inclination.<sup>1</sup> Moiré fringes of two circular gratings with linear displacement constitute a family of hyperbolas or ellipses. Also, two linearly displaced radial gratings produce a set of circular fringes, of which the circumference passes through the centers of the gratings. The linear displacement can be measured by simply counting the order of the moiré fringes that are formed by two superimposed circular<sup>5</sup> or radial gratings.<sup>10</sup> Moiré fringes of elliptic<sup>1</sup> or parabolic<sup>8</sup> gratings have been studied for strain analysis, and moiré patterns of two superimposed zone gratings have been used to measure inaccuracies in the zone plates.<sup>9</sup> Skewed radial gratings<sup>10</sup> produce a set of circular moiré fringes that move owing to the mutual rotation of the gratings. Spiral<sup>6</sup> or evolute gratings<sup>7,11</sup> also create a set of circular or radial fringes that can be used for angular velocity measurements. The moiré fringe technique with parallel line gratings<sup>1</sup> is not a direct fringe-counting method of determining small angular displacements, because the spacing of the fringes and their inclination must be measured otherwise.

In this paper, an elongated circular grating<sup>14</sup> in the shape of an athletic track is proposed to determine both the moving displacement and its direction with an accuracy of ten times that of the conventional circular gratings. It is composed of line gratings with each ends capped by well matched semicircular gratings. The semicircular gratings have an equal pitch throughout the grating plane except the central bright part where, because of the pitch matching between two different kinds of gratings, only a half width of a full pitch exists. Also, two pieces of semicircular grating in the central region have incomplete pitches. However, for the simplicity of the moiré fringe analysis, we assume that the pitch of semicircular gratings is the same in full radius. The moiré patterns formed by the two elongated circular gratings having different pitches are presented in detail. And these moiré fringes are applied to the measurement of refractive indices of  $C_6H_{12}$  and  $CCl_4$  and the thermal expansion coefficient of an aluminum.

Also, we discuss a modified form of the radial or the parallel grating, the matched radial-parallel (MRP) grating,<sup>15</sup> of which the moiré fringes enable us to determine angular displacements by simply counting the fringe order. This grating is designed so that the central area of a conventional radial grating is replaced by parallel stripes of nonuniform spacings to be matched with the remaining radial stripes. Moiré patterns produced by superimposing the reference MRP grating of angular pitch  $P_\theta$  on the specimen MRP grating of angular pitch  $P_{\theta'} (<P_\theta)$  can provide a direct fringe counting method of determining both the angle and the direction of relative angular displacement of the specimen grating within an accuracy of  $(P_{\theta'}/M_\theta)$ , where  $M_\theta$  is the angular magnification for radial vernier fringes. These MRP gratings can be used for the fine visual indication of angular displacements and rotational alignment.

#### References

- [1] K. Patorski and M. Kujawinska, Handbook of the Moiré Fringe Technique (Elsevier, Amsterdam, 1993).
- [2] Y. Nishijima and G. Oster, J. Opt. Soc. Am. 54, 1 (1964).
- [3] L. O. Vargady, Appl. Opt. 3, 631 (1964).
- [4] D. M. Meadows, W. D. Johnson, and J. B. Allen, Appl. Opt. 9, 942 (1970).
- [5] M. C. King and D. H. Berry, Appl. Opt. 11, 2455 (1972).
- [6] O. Bryngdahl and W. H. Lee, J. Opt. Soc. Am. 64, 1606 (1974).
- [7] G. T. Reid, Opt. Laser Eng. 4, 121 (1983).
- [8] A. Pirard, Anal. Contraintes, Mem. GAMAC 5, 1 (1960).
- [9] H. H. M. Chau, Appl. Opt. 8, 1707 (1969).
- [10] B. Sandler, E. Keren, A. Livnat, and O. Kafri, Appl. Opt. 26, 772 (1987).
- [11] P. Szwaykowski, J. Opt. Soc. Am. A5, 185 (1988).
- [12] P. Szwaykowski and K. Patorski, Appl. Opt. 28, 4679 (1989).
- [13] Y. C. Park and S. W. Kim, Appl. Opt. 33, 5171 (1994).
- [14] J. S. Song, Y. H. Lee, J. H. Jo, S. Chang, and K. C. Yuk, to be published in Opt. Commun. (1998).
- [15] B. J. Kim, J. S. Song, J. T. Kim, J. H. Jo, S. Chang, and K. C. Yuk, Appl. Opt. 36, 2848 (1997).