Model of Simultaneous Travel time and Activity Duration for worker with Transportation Panel Data

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Abstract

Recent world-wide interest in activity-based travel behavior modeling has generated an entirely new perspective on how the profession views the travel demand process. This paper seeks to further promote the case of activity-based travel behavior models by providing some empirical evidence of relationship between travel time and activity duration decision for worker with transportation panel data.

The travel time from home to work and from work to home, without activity involvement, is estimated by the Ordinary Least Squares (OLS) method. And, the travel time to and from the selected activity and the activity duration are modeled simultaneously by the Three Stage Least Squares (3SLS) method due to the endogenous relationship between travel time and activity duration. Two kinds of models, OLS and 3SLS, include selectivity bias corrections in a discrete/continuous framework, because of the inter-relationship between the choice of activity type/travel mode (discrete) and the travel time/activity duration (continuous). Estimation is undertaken using a sample of over 1300 household two-day trip diaries collected from the same travelers in the Seattle area in 1989. The behavioral consequences of these models provide interesting and provocative findings that should be of value to transportation policy formulation and analysis.

INTRODUCTION

There has been a recent proliferation of activity-based studies that have sought to include time dimension and individual diversity. However, the absence of one universal theory that would be acceptable to everyone is a major reason for the lack of planning applications of activity-based analysis. Another reason for a low acceptance of activity-based approaches in the transportation field is its fragmental development. However, without doubt, existing research on activity-based travel modeling has provided important directions for achieving the goal of applied forecasting.

Another major issue in activity-based dynamic analyses is the fact that travel behavior can not be adequately explained by contributing factors that are observed concurrently with behavior, that is, travel behavior depends upon the history of contributing factors and perhaps on the past trajectory of behavior itself. Therefore, travel behavior models based on cross-sectional data may be seriously biased and, if this bias is not negligible, are of questionable quality (Kitamura, 1988). As a consequence, this motivates the use of panel models that are based on longitudinal observations of individual behavioral units.

From these view points, the objective of this paper is to provide some empirical evidence of relatinship between travel time and activity duration decision for worker with two day transportation panel data. The travel time from home to work and from work to home, without activity involvement, is estimated by the Ordinary Least Squares (OLS) method. And, the travel time to and from the selected activity and the activity duration are modeled simultaneously by the Three Stage Least Squares (3SLS) method due to the endogenous relationship between travel time and activity duration. In the case of a trip-chain, individual travel time and activity duration are summed into total travel time and total activity duration.

MODEL FORMULATION

All-day activity of workers can be splitted into pre-work, while-work, and post-work activity due to significant differences in activity patterns. However, while-work activities are out of the scope of this study and left for further research because most of them are lunch-related and consequently difficult to define in terms of general activity types.

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The travel time from home to work and from work to home, without activity involvement, is estimated by the Ordinary Least Squares (OLS) method. The travel time to and from the selected activity and the activity duration are modeled simultaneously by the Three Stage Least Squares (3SLS) method due to the endogenous relationship between travel time and activity duration. In the case of a trip-chain, individual travel times and activity durations are summed into total travel time and total activity duration. Beside the above issues, two kinds of models, OLS and 3SLS, include selectivity bias corrections in a discrete/continuous framework, because of the inter-relationship between the choice of activity type/travel mode (discrete) and the travel time/activity duration (continuous). Since mode choice is either contemporaneous with, or followed by, the activity type choice, expected probabilities of activity type choices are employed in the mode choice model for correcting the possible endogenous problems.

Model of Travel Time from Home to Work and from Work to Home without Activity

If a traveler selects to go to work in the morning or to go home after work without engaging in activities, it is necessary to estimate the travel time on the roadway. Route travel time has traditionally been assumed to be beyond a traveler's control. However, this assumption is only valid when extremely congested conditions prevail. Travelers have, in general, some control over their travel time because of their ability to alter driving speeds, risk taking behavior, and reaction times (Mannering, Abu-Eisheh, and Arnadottir, 1990).

Therefore, based on the assumption of a traveler's ability controlling over travel time, a continuous linear travel time model (OLS) is defined as,

$$TT_{kt} = \beta_o + \mathbf{X}_{kt}\beta_1 + \mathbf{NW}_{kt}\beta_2 + \mathbf{HB}_{kt}\beta_3 + E[DT_{kt}]\beta_4 + \xi_{kt} \qquad (1)$$

where TT_{kt} is the travel time from home to work or from work to home without an activity for traveler k at time t, $\mathbf{X}_k$ is a vector of traveler k's socioeconomic variables, $\mathbf{NW}_{kt}$ is a vector of traffic congestion/network specific characteristics at time t, $\mathbf{HB}_{kt}$ is a vector of habitual behavior that includes previous activities pursued, DT_{kt} is the travel distance to/from an activity calculated from minimum-path network and this is also replaced by values instrumented by all exogenous values due to its endogenity, ξ_{kt} is an error term assumed to be normally distributed, and the β's are estimable parameters.

However, if the travel times for observed travelers were to be used, a potential for selectivity bias in model estimation would exist. Because travelers are observed selecting the no-activity choice with a particular travel mode and it is not known what the other travelers' behavior, with respect to the ability to control travel time, would have been had they selected other choices. If this selectivity bias is left untreated, the parameter estimates of equation (1) will be biased and inconsistent.

Several methods have been developed to correct for selectivity bias in discrete/continuous models. Among those, the selectivity bias correction term method (developed by Heckman (1976,1978,1979) and extended by Hay (1980) and Dubin and MacFadden (1984) to problems with multiple choices) and the expected value method identified by Mannering and Hensher (1987) will be considered. Hamed and Mannering (1993) applied the selectivity bias correction term method to the estimation of travel time from work to home conditioned on a binary activity/no-activity choice. When multiple discrete choices are involved, Dubin and McFadden (1984) and Mannering and Winston (1991) have shown the expected value method to be particularly convenient. This study extends Hamed and Mannering's work in two ways: one is including travel mode choice as another conditioning point on the travel time, and the other is incorporating response lag effects in explanatory variables ($\mathbf{HB}_{kt}$).

When the selectivity bias is corrected (i.e. using the conditional value of travel time), Equation 1 becomes,

$$\begin{aligned} E[TT_{kt} \mid \text{No-activity}] = {} & \beta_o + \mathbf{X}_{kt}\,\beta_1 + \mathbf{NW}_{kt}\beta_2 + \mathbf{HB}_{kt}\beta_3 + E[DT_{kt}]\beta_4 \\ & + E[\mathbf{MD}_{kt}]\;\beta_5 + E[\xi_{kt} \mid \text{No-activity}] + \eta_{kt} \qquad (2) \end{aligned}$$

where $E[TT_{kt} \mid \text{No-activity}]$ is the conditional expectation of travel time given the no-activity alternative, $\mathbf{MD}_{kt}$ is a vector of indicator variables denoting travel modes for traveler k at time t and this is replaced by its expected values which is the vector of probabilities calculated from the mode choice, $E[\xi_{kt} \mid \text{No-activity}]$ is the conditional expectation of the error term given the no-activity choice, and η_{kt} is a normally distributed error term.

As shown by Hay (1980) and Dubin and McFadden (1984), the closed form of the selectivity bias correction term for the no-activity choice is defined as,

$$E[\xi_{kt} \mid \text{No-activity}] = -(\sqrt{6\sigma^2}\ /\ \pi)\ \rho_{\text{no-activity}}\ [(1-P_{kt})\ln(1-P_{kt})\ /\ P_{kt} + \ln P_{kt}] \qquad (3)$$

where σ^2 is the variance of ξ_{kt} in the entire population (not conditioned on the no-activity choice), $\rho_{\text{no-activity}}$ is the correlation of ξ_{kt} with the unobserved utility associated with the no-activity choice, and P_{kt} is the probability of traveler k selecting the no-activity choice at time t. Entering this into Equation 2 gives,

$$\begin{aligned} E[TT_{kt} \mid \text{No-activity}] = {} & \beta_0 + \mathbf{X}_{kt}\,\beta_1 + \mathbf{NW}_{kt}\beta_2 + \mathbf{HB}_{kt}\beta_3 + E[DT_{kt}]\beta_4 \\ & + E[\mathbf{MD}_{kt}]\ \beta_5 + SB_{kt}\,\gamma_{\text{no-activity}} + \eta_{kt} \qquad (4) \end{aligned}$$

where SB_{kt} is the selectivity correction term calculated as $[(1-P_{kt})\ln(1-P_{kt})\ /\ P_{kt} + \ln P_{kt}]$ and $\gamma_{\text{no-activity}}$ is the coefficient of the selectivity correction term, which equals $-(\sqrt{6\sigma^2}\ /\ \pi)\ \rho_{\text{no-activity}}$.

Model of Simultaneous Travel Time To/From Activity and Activity Duration

If travelers decide to pursue some activities and to take a particular travel mode, the prediction of both the activity duration and the travel time to and from the activity is an indispensable step in tracing all-day activity patterns. Moreover, this represents the presence (during travel time) and non-presence (during activity duration) of a traveler on the transportation network. Intuitively, the travel time to and from the selected activity and the duration of that activity are interrelated; that is, travelers may naturally be willing to accept longer travel times to activities requiring a longer duration and vice versa.

Hamed and Mannering (1993) assumed this relationship and adopted the Three Stage Least Squares (3SLS) method to estimate a simultaneous equations system. However, their analysis was based on the assumption of symmetric travel times (i.e. the travel time to an activity is equal the travel time from the activity). In reality, this is not true, because one-way trips such as home-activity-work and work-activity-home are not symmetric in travel distance and, if the travel time to an activity takes longer than expected, a traveler will try to make a shorter trip, even in the case of a round-trip with a symmetric distance. Thus, in addition to the assumption of the interrelationship between travel time and activity duration, asymmetrical travel times to and from an activity are assumed. As a result, the simultaneous equation structure can be formulated as,

$$TT1_{kt} = \alpha_0 + \alpha_1 TT2_{kt} + \alpha_2 ADUR_{kt} + \mathbf{Z}_{kt}\,\alpha_3 + \delta_{1kt} \qquad (5)$$

$$TT2_{kt} = \beta_0 + \beta_1 TT1_{kt} + \beta_2 ADUR_{kt} + \mathbf{Z}_{kt}\,\beta_3 + \delta_{2kt} \qquad (6)$$

$$ADUR_{kt} = \gamma_0 + \gamma_1 TT1_{kt} + \gamma_2 TT2_{kt} + \mathbf{Z}_{kt}\,\gamma_3 + \delta_{3kt} \qquad (7)$$

$$\mathbf{Z}_{kt} = (\ \mathbf{X}_{kt}\ \mathbf{NW}_{kt}\ \ \mathbf{HB}_{kt}\ \mathbf{AT}_{kt}\ \ \mathbf{MD}_{kt})$$

where $TT1_{kt}$ and $TT2_{kt}$ are travel times to and from an activity by traveler k in minutes respectively, $ADUR_{kt}$ is activity duration in minutes and $\mathbf{Z}_{kt}$ is a vector of subvectors defined as before, and δ_{1kt}, δ_{2kt} and δ_{3kt} are error components allowing contemporaneous correlations .

In the case of a trip-chain, the activity duration is defined as the sum of component activity durations and component travel times. This complexity of travel times and activity durations in a trip-chain can be split into individual components by estimating the number of stops in the trip-chain. Here, random-splitting or some other technique can be applied.

As was the case with the travel time model from home to work or work to home, selectivity bias is present. It results from the facts that the travel time to and from the activity and the duration of the activity are only observed for the activity type and the travel mode that have been selected by the traveler. Thus, $\mathbf{AT}_{kt}$ and $\mathbf{MD}_{kt}$ are replaced by their expected probabilities (i.e. calculated from the activity type choice and the mode choice models respectively). This section also extends the work of Hamed and Mannering (1993) as before mentioned.

4.2 Data

The Puget Sound Transportation Panel (PSTP) was the source of the activity data used in this study. The PSTP is based on two-day travel diaries administered to all members of the sampled households, and includes the four counties of the Seattle-Tacoma metropolitan area. The panel has administered in the fall of 1989 and consisted of 1,713 households.

The range of travel related information collected on the two-day diaries is impressive. The diary includes a wide-range of household socioeconomic data including information on individual household members (each of whom completed trip diaries). The trip diaries themselves contained all relevant information on the type of trip, mode(s) used, arrival and departure times, activity duration, activity locations, distance to and from activities, and whether or not others were jointly undertaking the activity. Although the availability travel information from only two days limits empirical exploration of day-to-day variation in travel patterns and travel/habit formation, it does permit limited estimation of state dependence by modeling second-day travel using information from travel choices on the first day (e.g. previous-day total number of activity stops).

Table 1 presents summary statistics for some selected variables contained in the data set and used in the models. The table includes information for workers. It is noted that the panel data includes an attrition bias and the diary data includes some under-reporting bias. The former issue was dealt by Pendyala, Goulias, Kitamura and Murakami (1993). But, the latter is left for a future research.

Table 1. Sample summary statistics (average unless otherwise noticed)

Variable	Non-Worker	Worker
Sex (% male/female)	33.1/66.9	52.9/47.1
Age of travelers (year)	51.53	38.33
Annual household income ($1K)	38.37	44.80
Household size	2.79	2.95
Number of household vehicles	2.22	2.45
Number of activity-stops per trip chain	3.14	2.64
Percent distribution of home-activity-home trips in pre-work (1-trip/ 2-trip/ 3-trip/ 4-trips)	-	84.3/12.4/2.3/1.1
Percent of activity type (chain/shopping/free/personal/visit-appointment)	50.88/11.65/12.46 /15.78/9.22	32.76/13.66/15.67 /27.79/10.11
Percent of travel mode for activity involvement (SOV/pool drive/pool ride/mix of SOV & pool/bus/park&ride)	43.26/25.53/15.92 /13.95/1.34/0.0	43.39/17.08/11.92 /24.88/1.12/1.62
Percent of commute travel mode without activity involvement (SOV/pool drive/pool ride/bus/park&ride)	-	74.23/3.91/7.81/ 9.20/4.85
Number of observations (travelers)	799	1788

Estimation Results

As discussed before, the commute travel time without an activity involvement was modeled by OLS, and the travel time to/from activity was modeled with activity duration in a simultaneous form. First, the parameter estimates of OLS models are presented and then 3SLS estimates are given.

Model of Travel Time from Home to Work and from Work to Home without Activity

As the case of Hamed and Mannering (1993), the logarithmic functional form was chosen for the commute travel time without an activity involvement model. Due to endogeneity with travel time, mode choice indicators are replaced by expected probabilities as calculated from the estimated mode choice models. Another endogenous variable, travel distance, which is not real distance but minimum-path distance in the network, is instrumented by all exogenous variables, as in the activity-stops choice model.

By visual inspection, both pre-work and post-work models are common in sign and specification for most of non-socio-economic variables. This reflects the fact that pre- and post-work commute travel times without activity involvement are almost symmetric in nature. The instrumented distance variables showed plausibly positive coefficients in all models, indicating that longer distances produce longer travel times.

Finally, the results clearly show that the correction bias term is significantly different from zero. This significance suggests that selectivity bias is clearly present in the equations. This result is consistent with that of Hamed and Mannering (1993). Note that the positive sign of selectivity correction term means negative correlation of the error term ($\rho_{\text{no-activity}}$ in equation 3) in travel time equation with utility associated with no-activity choice (e.g. unobserved factors that cause longer commute time decrease the utility of no-activity choice).

Model of Simultaneous Travel Time to/from Activity and Activity Duration

In the case of activity involvement, travel time to/from activity and activity duration are inter-dependent as mentioned before. Thus, three stage least squares was used to estimate these simultaneously, along with the selection bias correction discussed earlier. Based on the previous descriptive analysis of sample data, workers' travel time to a single-stop activity and travel time from a single-stop activity were found to be asymmetric. Therefore, a three-equations system for workers' single-stop activity are estimated. In the case of trip-chaining, travel time and activity duration are summed respectively and then a two-equations system is estimated for each of pre-work and post-work trip-chains.

First, travel time to an activity is positively correlated with activity duration. This result supports the notion that the decision by a traveler to spend more time in activities is generally taken with the expectation that these activities are less accessible (see Hamed and Mannering, 1993). The most interesting finding is the negative coefficient of the travel time from the activity (TT2) in the activity duration equation. Its implication is such that the pre-work activity plan is restricted by the scheduled work-shift, hence longer expected travel times from the activity decrease the activity duration. However, the travel time from the activity (TT2) in the third equation was found to be positively correlated with the activity duration. That is, longer activity duration comprising longer distance results in longer travel time. This also implies that, even with the tight pre-work schedule, the travel time from the activity can not easily be adjusted because the travel distance is given.

Summing up, these findings come to the following conclusion. When the activity type and destination are chosen, the easiest way to adjust the planned activity involvement in pre-work is to lengthen or shorten the activity duration rather than change travel times.

Turning to 3SLS estimation results of post-work single-stop activity involvement, right-hand side dependent variables showed plausibly consistent results. But it is noted that the right-hand side endogenous variable, travel time from activity (TT2), in the activity duration equation was found to be positive, in coefficient, which is the opposite sign of the result in the single-stop pre-wok model. This is because post-work activity involvement is open-ended in terms of time-schedule. That is, there is no restriction in the time-budget, hence the post-workers do not need to shorten their activity duration due to longer expected travel times. In the travel time equations, as expected, the activity duration (ADUR) is positively correlated with both travel times.

The 3SLS estimation results of pre-work trip-chain showed good system R^2. Like the non-work trip-chain model, the right-hand side dependent variables turned out to be positively correlated to each other. Different from the single-stop pre-work model, the fixed work-shift (tight pre-work schedule) did not impact on the negative feed-back between activity duration and travel time. This reflects the fact that although the latter components of trip-chaining may have the negative feed-back relationship (i.e. expected prolongation of travel time reduce the activity duration), summed-up travel time and activity duration may absorb a relatively small portion of the negative partial correlation.

Finally, the 3SLS estimation results of post-work trip-chain showed positive correlation between travel time (STT) and activity duration (SADUR).

In conclusion, four important findings evolve from the travel time and activity duration models. First, selectivity-bias corrected mode and activity-type indicators also showed considerable variation in specification. This may be caused by the large number of alternatives that preclude a common specification. Second, with a few exceptions, current-day state effects turned out to be negative but previous-day effects were positive indicating activity/travel habits. Third, in 3SLS estimation, consistent relationships between dependent and right-hand side dependent variables were found. These relationships are shown in Figure 1.

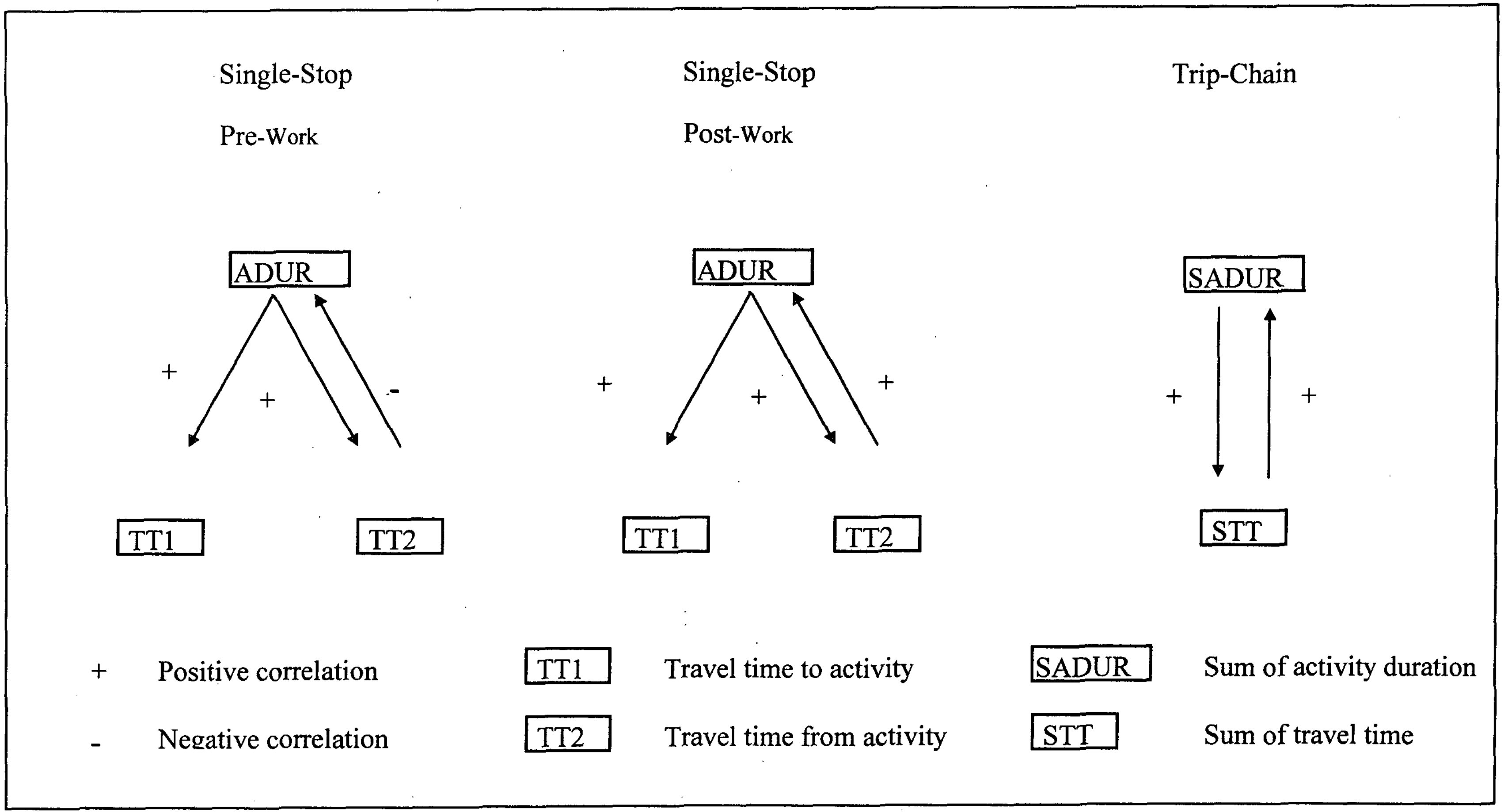

Fig. 1 Relationships between dependent and right-hand side dependent variables

Concluding Remarks

This study formulated a activity-based model that accounted for travel time and activity duration, which are divided into two main categories of pre-work and post-work. The model includes a set of discrete/continuous models. Using data from two-day travel diaries, collected in, econometric models were estimated for 1989 .

The empirical results show the importance of proper econometric specification in estimating activity-based travel behavior models. Habitual behavior was confirmed with highly significant coefficient estimates in most of the models. This underscores the need for multi-day travel data. In addition, current-day state effects (i.e. current-day experience negatively impacts on the same decision) showed consistent results. This notion was supported by the fact that lagged variables were identified and found to be consistently significant.

The model of travel times without activities, showed that the selectivity-bias correction term was significantly different from zero suggesting that the selectivity-bias of the activity/no-activity choice is clearly present in these models. The positive sign of selectivity-bias correction term means a negative correlation of the error term in travel time equation with the utility associated with no-activity choice. Thus, unobserved factors that cause longer commute times decrease the utility of the no-activity choice but increase the utility of the activity while commuting choice.

In the simultaneous travel time and activity duration models, consistent relationships between dependent and right-hand side dependent variables were found (see Figure 1). The pre-work single-stop model had activity duration positively impacting travel time to/from activity. Conversely, the travel time from the activity decreased the activity duration. This finding reflects a tight pre-work schedule, in which a traveler expecting a late arrival at the work-place is more likely to reduce activity duration. The relationship in the post-work single-stop model is the same as the pre-work single-stop model, except for the positive impact of expected travel time from the activity on activity duration. This is because post-work activities are less constrained by scheduling. In the case of trip-chaining, the sum of travel times and the sum of activity durations are positively inter-correlated. This reflects the fact that although the latter components of the trip-chain may have a negative feed-back relationship (i.e. expected prolongation of travel time reduces the activity duration), summed travel time and activity duration may absorb the relatively small portion of the negative partial correlation.

The empirical findings of the model presented in this study should provide a valuable methodological starting point for future activity-based travel-modeling research. In this regard, there are a number of important directions to follow.

First, although the state effects looked significant, it should be tested with appropriate testing methods which can account for heterogeneity.

Second, the findings underscore the need for more extensive data both in terms of the length of the travel observation period (i.e. seven to fourteen day diaries). With such data, day-to-day variations and taste changes, and habit formation could be carefully accounted for in the model structure. In addition, as Ben-Akiva and Lerman (1985) suggested, the coefficients of interest, such as travel distance, time budget, and state dependent coefficients, can be constrained to have the same value across data sets, and all other coefficients can be allowed to vary.

Third, another drawback of the model is in the area of "while-work" activities which is assumed to be constant. To complete an all-day behavior modeling system, while-work activities should be accounted for.

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