Derivation of Ideal Function proposed by Taguchi for Dynamic Systems with . Double Signals - the brake system

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Abstract

Dr. Taguchi proposed three models of ideal functions for dynamic systems with double signal factors. He also gave examples for each model yet without derivation. It will be difficult for other engineers to follow because Dr. Taguchi didn't show us how he obtained those models. Actually we can analyze each example from engineering aspect based on basic mechanism. In this paper we use brake systems to illustrate our approach of derivation and obtain a different form of ideal function from what Taguchi proposed. Our purpose is to provide an example that engineers can imitate and solve his problem at hand.

1.Introduction

Taguchi proposed three types of ideal function for dynamic systems with double signal factors as follows[1]:

a. $y = \beta \mathrm{M}\mathrm{M}^*$ (e.g., brake system) (1)

b. $y = \beta \dfrac{\mathrm{M}}{\mathrm{M}^*}$ (e.g., plastic beam and resistor) (2)

c. $y = \left[\bar{\beta} + \beta^* (\mathrm{M}^* - \overline{\mathrm{M}}^*)\right]\mathrm{M}$ (eg., tubing extrusion process and injection molding) (3)

where M, M^* are called technical function signal and adjustment signal, and y is the output response. For example, M is the master cylinder pressure and M^* is the pad surface area and y is the torque generated in brake systems. But he did not give us enough derivation to follow. Naturally one should raise the following questions:

Q1: Are those models correct?

Q2: Are those models the only models for dynamic system with two signals?

Q3: How to choose signal factors? .

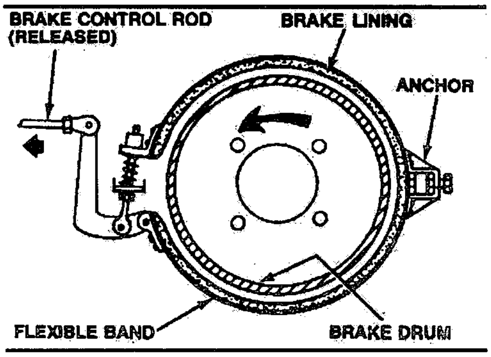

Figure 1. External contracting-band brake.

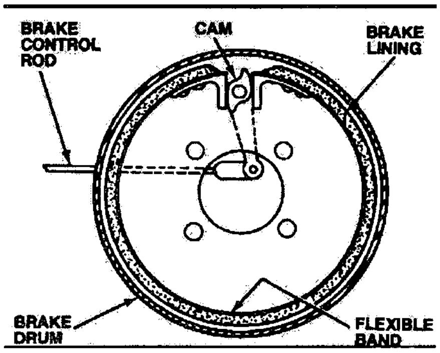

Figure 2. Internal expanding-band brake.

In the rest of this paper, we start with overview of evolution of brake system, then through principles of brake operation we derive out the ideal function.

2.Brake System Overview[2,3]

The brake system is the most important safety device on any vehicle. Since the invention of the wheel, most vehicles have used some sort of brakes to stop the vehicle. However, as vehicle weights and speeds increased, existing brake designs were improved and new systems were developed to maintain adequate braking power.

Vehicle brake systems are divided into two subsystems: the service brakes and the parking brakes. The former operated on all four wheels and stop the car under normal driving condition, the latter operate on only two wheels and are used to hold a parked car in position.

In this section, we will focus on service brake and introduce almost all the automotive brake systems chronologically. Surely, modern brake systems are much more complex than described and there are many variations among the systems now in use.

2.1 External Contracting-Band Brakes

In Figure 1 the brake shoe was replaced with a flexible metal brake band lined with a suitable friction material. The band was wrapped around the outside of the brake drum. When the brakes were applied, a lever mechanism pulled the band tight around the drum to slow the car. However, the external contracting-band brake has some problems. The brake lining material was exposed to the weather which resulted in poor braking in the wet, and dirt and grit from roads caused the brake lining to wear very rapidly.

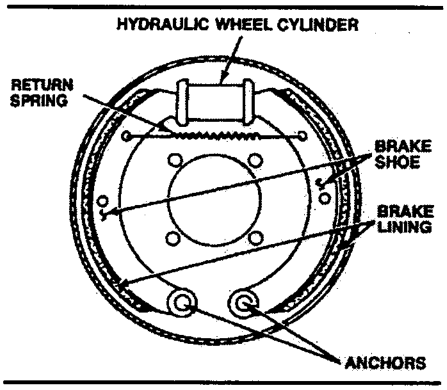

Figure 3. Internal expanding-shoe brake

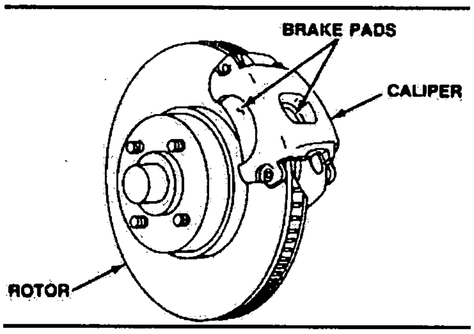

Figure 4. A disc brake friction assembly.

2.2 Internal Expending-Band Brakes

To solve the problems of the external contracting-band brake, car manufacturers introduced the internal expending-band brakes in Figure 2. This design moved the band to the inside of the brake drum where it was actuated by a rotating cam. Though the design improved the problems of external contracting-band brake, new problem is caused. If the brakes were used hard enough for heat to expend the drums, braking power was lost because the band was unable to extend out far enough to maintain solid drum contact. Besides, the enclosed band was not exposed to direct flow of cooling air, so the lining material could more easily become overheated and damaged, which reduced braking efficiency further.

2.3 Internal Expanding-Shoe Brakes

The cure for the problem of internal expending-band brake was internal expending-shoe brake in Figure 3, which replaced the flexible band with two rigid brake shoes. In later systems, including all modern automobile drum brakes, use a hydraulic cylinder to force the shoe out against the brake drum.

2.4 Disc Brakes

As highway speeds, traffic congestion, and the size and weight of cars continued to increase, it was apparent that the drum brake were unable to adequately dissipate the heat generated during reaped hard braking. After a number of stops, the brake lining overheated, braking performance would decrease. In order to safely stop a heavy automobile from highway speeds, the disc brake was adopted. Disc brake in Figure 4 operated by two brake pads against a spinning disc, or rotor. In order to generate the same braking power as a drum brake with the same hydraulic pressure,

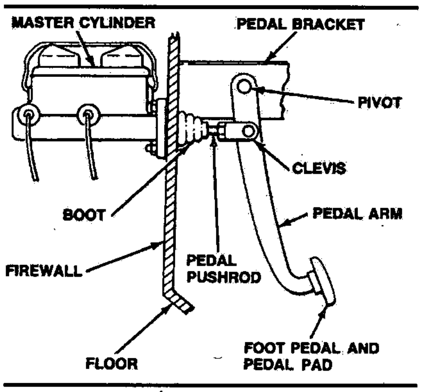

Figure 5. A typical Pedal System.

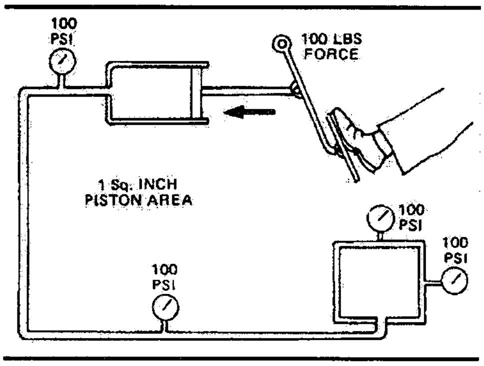

Figure 6. Fluid will transmit pressure equally throughout the hydraulic circuit.

servo action, such as power booster, is used in modern design.

3. Principles of Brake Operation[2,3]

Brake systems work in accordance with unchanging physical "laws" or principles. The principles involved with vehicle brake system include kinetic energy, mechanics, hydraulics, friction, torque, and heat.

3.1 Kinetic Energy

Energy is the ability or capacity to do work. Physics tells us that energy cannot be created or destroyed. It can, however, be transferred from one form to another.

Every moving object possesses kinetic energy, and the amount of that energy is determined by the object's mass and speed. The job of the brake systems is to dispose of that energy in a safe and controlled manner.

3.2 Mechanical Principles

The primary mechanical principle used to increase applied force in brake systems is leverage. In a typical suspended brake pedal system in Figure 5, the pedal arm is the lever, the pivot point is the fulcrum, and the force is applied at the foot pedal pad. Leverage provides a mechanical advantage that, at the brake pedal, is called pedal ratio. For example, a ratio of 5 to 1 is common for manual brakes, which means that a force of 10 pounds at the brake pedal will result in a force of 50 pounds at the pedal pushrod.

Furthermore, leverage is used in both the service and parking brake systems to increase braking force in order to make it easy for driver to control the adequate amount of force applied.

3.3 Hydraulic Principles

In addition to the mechanical advantage provided by leverage, all modern cars also use hydraulic pressure to help increase brake application force. It is designed for three basic purposes as follows: transmit motion from the driver's foot to the brake shoes, transmit force along with motion, and multiply that force by varying amounts to the different wheel assemblies.

Hydraulics is based on the principles that liquids, being fluid, can flow easily through complicated paths, yet cannot be compressed. Another important feature is that when liquids transmit pressure, that pressure will be transmitted equally in all directions, it's also called Constancy of Pressure. This is a simplified version of Pascal's law in Figure 6.

3.4 Friction and Heat

The simplest way to stop a car is to convert the kinetic energy of the moving car to heat through the friction. The rolling friction of the axle shafts and wheel bearings, the flexing friction of the sidewalls and treads of the tires, and the molecular friction of the air drag over the body and under-carriage all create small amounts of heat. Besides, brakes are essentially heat machines. They generate heat from friction by rubbing the shoes against the rotating rotors or drums. The friction generated through braking can create large amounts of heat. As the kinetic energy becomes heat energy, the car will decelerate and come to a stop.

3.5 Torque

As the brakes are applied, a retarding force is created. Since it is a rotating force to resist the turning of the rotor or drum, it is often called brake torque. The amount of torque varies with the radius of the rotor or drum, the coefficient of friction of the lining and the application force at the lining.

Another important engineering term commonly related to brake torque is swept area. This is the area of the rotors or drums that is swept or rubbed by the shoes or pads.

4.Derivation of Ideal Function for Disc Brake System

We can summarize previous discussion on principles of brake system and obtain a flow chart in Figure 7 indicating steps between to stop a car and car stops.

Taguchi pointed out in [4] "*All kinds of functions are energy transformations. Therefore, the product designer must identify what is input, what is output, and what is ideal function...*".

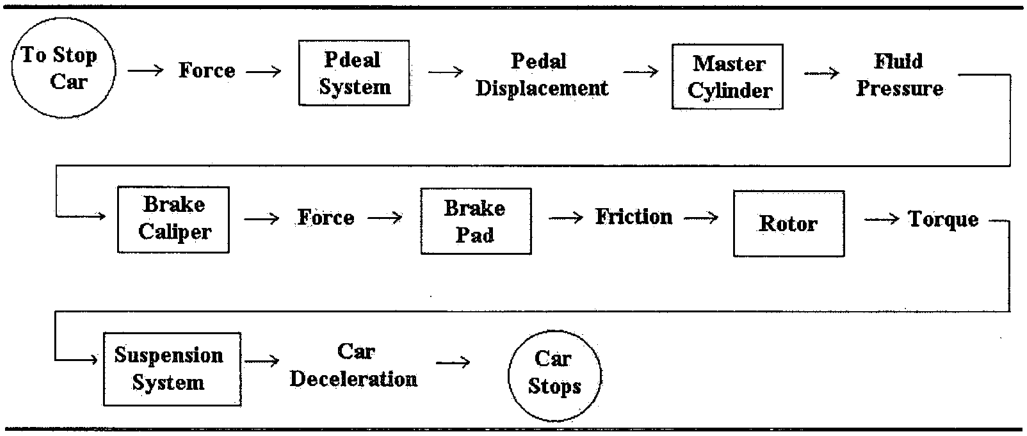

Figure 7. Steps of process when disc brake system is applied.

To derive the ideal function for brake systems in Figure 7, we first decompose it into pedal system, hydraulic system and torque system, then within each subsystem we discuss the issue of energy transformation.

4.1 Pedal System

The pedal pushrod transmits force to the power booster or master cylinder. The sketch of pedal system is displayed as Figure 5. By the application of the leverage, clockwise torque and counter-clockwise torque should be equal. That is

$$F_p \cdot b = F_f \cdot a \tag{4}$$

where

a = distance from pivot to foot pedal(inches)

b = distance from pivot to clevis (inches)

F_p = mechanical force generated by leverage

F_f = the force generated by foot (pounds)

Thus, we obtain the force applied on the pushrod

$$F_p = F_f \cdot \frac{b}{a} \tag{5}$$

It is clear that the magnitude of F_p is determined by two factors: the force generated by foot, the quantity of ratio *b/a*. The quantity of *b/a* is the ratio of the weight lifted to the force applied, and is called mechanical advantage. The greater the ratio is, the larger generated force operated on the pushrod creates.

4.2 Hydraulic System

Pressure can enter a hydraulic system in several ways. We use a piston as the pressure input and one or more pistons for the output. Usually, the input piston is called master cylinder. The amount of pressure in a system is determined by the following two factors: the area of the master cylinder, and the amount of force applied to the brake pedal multiplied by the mechanical advantage of the pedal ratio. Theoretically, the relationship between the force and area is as follows[2]:

$$P = \frac{F}{A} \tag{6}$$

where:

F = mechanical force applied to the piston

A = piston area in square inches

P = pressure in psi

The above mathematical equation describes how mechanical force at the brake pedal pushrod is applied to master cylinder piston area and convert into brake system hydraulic pressure.

As described at previous section, we know that the fluids have the properties of noncompressibility and constancy of pressure. The latter property tells us the pressure will be transmitted equally in all directions.

That is

$$\frac{F_p}{A_1} = \frac{F_c}{A_2} = P_M \tag{7}$$

where

A_1 = the area of master cylinder in square inches

A_2 = the area of output piston

F_c = the force applied to brake caliper

F_p = the force generated by the mechanical lever in pedal system

P_M = the pressure of master cylinder

Thus, we can get the force applied on the caliper:

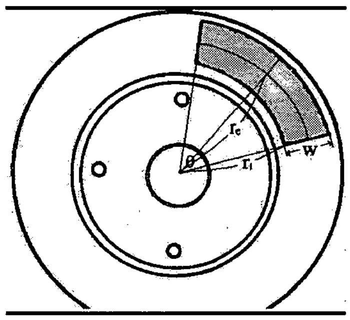

Figure 8. Sketch of rotor and brake pad.

$$F_c = \frac{F_p}{A_1} \times A_2 = P_M \times A_2 \tag{8}$$

4.3 Torque System

Our purpose is to find out the relationship between the brake torque y, master cylinder pressure and pad surface area. In theory, the torque is the product of the force and force arm. That means

$$y = F_a \cdot R_{\perp} \tag{9}$$

where

y = torque generated by braking

F_a = force

$R_{\perp}$ = the lever arm whose distance is perpendicular to the line of action of the force

As shown in Figure 8, we derive the torque generated by brake operated (one tire) as follows[5]:

$$y = 2P_M \cdot A_2 \cdot \mu \cdot r_e \tag{10}$$

where

P_M = pressure of master cylinder

A_2 = the area of caliper piston

r_1 = pad radius to inner edge

r_e = significant radius

w = pad width

μ = the friction coefficient between brake pad and rotor

θ = angle of brake pad expands

Comparing (10) with $y = \beta MM^*$ where M is the master cylinder pressure and M^* is the pad surface area as Taguchi proposed, one can observe that M^* should be caliper piston area instead of brake pad area and moreover r_e is missing.

5.Conclusion

Taguchi asked engineers to design a product/process such that its function is as close to the ideal function as possible. So, to derive the ideal function for any system is the first step in quality engineering. Using brake system as an example, through analysis of energy we obtain the ideal function which differs from what Taguchi proposed.

Dynamic systems are almost everywhere. Models of ideal functions should not be limited to those three given by Taguchi. Engineers can learn from our approach and step out the first step in quality engineering. Once engineers derive the ideal function of their problems, it is up to them to choose appropriate factors as signals.

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