OPPORTUNISTIC AGE REPLACEMENT POLICY

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Abstract

This paper proposes an opportunistic age replacement policy. The system has two types of failures. Type I failures (minor failures) are removed by minimal repairs, whereas type II failures are removed by replacements. Type I and type II failures are age-dependent. A system is replaced at type II failure (catastrophic failure) or at the opportunity after age T, whichever occurs first. The cost of the minimal repair of the system at age z depends on the random part C(z) and the deterministic part c(z). The opportunity arises according to a Poisson process, independent of failures of the component. The expected cost rate is obtained. The optimal T* which would minimize the cost rate is discussed. Various special cases are considered. Finally, a numerical example is given.

Keywords: Maintenance; Optimization; Opportunity; Reliability; Repair; Replacement

1. Introduction

It is of great importance to avoid the failure of a system during actual operation when such an event is costly and /or dangerous. In such situations, one important area of interest in reliability theory is the study of various maintenance policies in order to reduce the occurrence of system failure.

Barlow and Hunter [1] considered the case of periodic replacement or overhaul at times T, 2T, 3T, ... (for some T>0) and minimal repair if the system failed otherwise. They considered cost c_2 of replacement and c_1 for each minimal repair. This model has been generalized by Beichelt [2], Boland [3], Berg et al. [4], Sheu [5], and Sheu and Jhang [6]. These policies are commonly used with complex systems such as computers, airplanes, and large motor. For preventive replacements of components of these units to be cost effective, execution has to be delayed to some moment in time at which the unit is not required for service. Such idle moments can be created by many mechanisms, e.g., by breakdowns of the other units in a series configuration with the unit in question, and in such cases we speak of maintenance opportunities. Unfortunately, in most cases opportunities cannot be predicted in advance and because of this random occurrence, traditional maintenance planning fails to make effective use of them.

In one type of opportunity-based replacement models (introduced by Jorgenson et al.[7]) there are two classes of components. Failure of components in one class creates opportunities for the preventive replacement of components in the other. There are many variants of this model; e.g., the opportunity-creating component may or may not itself be preventively replaced, it may have an exponentially or a generally distributed lifetime, other components may be replaced at higher costs outside opportunities, etc. Jorgenson et al. [7] considered this type of model and provided formulas for the operating characteristics. Sethi [8] considered the case of two independent components with general discrete IFR distributions and showed that there exists an optimal policy of the control-limit type. Other examples of this type of opportunity model are those given by Vergin and Scriabin [9], Van der Duyn Schouten and Vanneste [10], and Bäckert and Rippin [11], who used discrete-time Markov decision chains. Berg [12] also considered a two-unit opportunity model with continuous time and derived partial differential equations for the joint probability density function of the ages of the components. Recently, Dekker and Dijkstra [13] consider opportunity-based age replacement for the case of exponentially distributed times between opportunities. This paper proposes an opportunity-based age replacement policy. A system has two types of failures. Type I failures (minor failures) are removed by minimal repairs, whereas type II failures are removed by replacements. Type I and type II failure are age-dependent. A system is replaced at type II failure (catastrophic failure) or at the opportunity after age T, whichever occurs first. The cost of the minimal repair of the system at age z depends on the random part C(z) and the deterministic part c(z). The opportunity arises according to a Poisson process, independent of failures of the component.

In the second section the model is described. Then the total expected long-run cost per unit time is found. The optimal T* which would minimize the cost rate is discussed. In the third section various special cases are included. In the last section a numerical example is given.

2. General Model

We consider an opportunistic age replacement model in which minimal repair or replacement takes place according to the following scheme. A system has two types of failures when it fails at age z. Type I failure (minor failure) occurs with probability q(z) and is corrected with minimal repair, whereas type II failure (catastrophic failure) occurs with probability p(z)=1-q(z) and a unit has to be replaced. A system is replaced at type II failure or at the opportunity after age T, whichever occurs first. The opportunity arises according to a Poisson process, independent of failures of the component. Let the random variable W denote the time between successive opportunities. W has an exponential distribution. Let E[W] denote its finite mean and g_W denote its probability density function. Let c_f denote the cost of replacement at type II failure. Let c_p denote the cost of replacement at the opportunity after age T. The cost of the minimal repair at age z is ϕ(C(z),c(z)) where C(z) is the age-dependent random part, c(z) is the age-dependent deterministic part, and ϕ is a positive nondecreasing and continuous function. Suppose that the random part C(z) at age z has distribution L_z(x), density function l_z(x) and finite mean E[C(z)]. After a replacement the procedure is repeated. We assume all failures are instantly detected and repaired. We also assume $c_f > c_p$.

Assume that the system has a failure time distribution F(x) with finite mean μ and has a density f(x). Then, the failure rate (or the hazard rate) is r(x)=f(x) / $\bar{F}$(x) and the cumulative hazard is R(x)= $\int_0^x$ r(y)dy, which has a relation $\bar{F}$(x) =exp {-R(x)}, where $\bar{F}$(x)=1-F(x). It is further assumed that the failure rate r(x) is continuous, strictly increasing, and remain undisturbed by minimal repair.

If T=∞, then the survival distribution of the time between successive type II failure is given by

$$\bar{F}_p(z) = \exp\{-\int_0^z p(x)r(x)dx\}. \tag{1}$$

See Beichelt [3] or Block et al. [14] for derivation of this result.

Let $Y_1, Y_2, \ldots$ be i.i.d. random variables with survival distribution $\bar{F}_p$ and Y_i^* denote the length of the i-th successive replacement cycle for i=1,2, ... Let R_i^* denote the operational cost over the renewal interval Y_i^*. Thus $\{(Y_i^*, R_i^*)\}$ constitutes a renewal reward process. If D(t) denotes the expected cost of operating the unit over time interval [0,t], then it is well-known that

$$\lim_{t\to\infty}\frac{D(t)}{t}=\frac{E[R_1^*]}{E[Y_1^*]}, \tag{2}$$

(see, e.g., Ross [15], p.52). We shall denote the right-hand side of (2) by B(T;p).

We can get the survival distribution of Y_1 is given by $\overline{F}_p(z)=\exp\{-\int_0^z p(x)r(x)dx\}$.

Thus, the expected length of a replacement cycle is given by

$$E[Y_1^*]=\int_0^\infty \int_0^{T+w}\overline{F}_P(z)dzg_W(w)dw. \tag{3}$$

Therefore, the total expected cost in a replacement cycle is

$$E[R_1^*]=\int_0^\infty[c_fF_p(T+w)+c_p\overline{F}_p(T+w)+\int_0^{T+w}h(z)\overline{F}_P(z)q(z)r(z)dz]g_W(w)dw. \tag{4}$$

For the infinite-horizon case we want to obtain optimal T* which minimizes B(T;p), the total expected long-run cost per unit time. Recall that

$$B(T;p)=\{\int_0^\infty[c_p+(c_f-c_p)F_p(T+w)+\int_0^{T+w}h(z)\overline{F}_P(z)q(z)r(z)dz]g_W(w)dw\}$$

$$\Big/\int_0^\infty \int_0^{T+w}\overline{F}_P(z)dzg_W(w)dw. \tag{5}$$

Theorem 1. If (c_f-c_p) p(z)r(z)+h(z)q(z)r(z) is continuous and increases to M_1 (M_1>0), $M_1\cdot E[W]\le c_p$, and $\int_0^\infty\int_0^\infty[M_1-(c_f-c_p)p(z)r(z)-h(z)q(z)r(z)]\overline{F}_p(z)dzg_W(w)dw-c_p>0$ then there exists at least one finite positive T* which minimizes the total expected long-run cost per unit time B(T;p) and the expected cost rate is

$$B(T^*;p)=\{\int_0^\infty[(c_f-c_p)p(T^*+w)r(T^*+w)$$

$$+h(T^*+w)q(T^*+w)R(T^*+w)]\overline{F}_p(T^*+w)g_W(w)dw\}\Big/\int_0^\infty\overline{F}_p(T^*+w)g_W(w)dw. \tag{6}$$

Furthermore, if (c_f-c_p) p(z)r(z)+ h(z)q(z)r(z) is strictly increasing, then T* is unique.

3. Special Cases

Case 1. (p(z)=, $\phi(C(z),c(z))=0$): Dekker and Dijkstra [13] considered this case.

Case 2.(W is degenerated at 0, p(z)=0, and $\phi(C(z),c(z))=c(z)$): This is the case considered by Boland [3].

Case 3. (W is degenerated at 0, and $\phi(C(z),c(z))=C(z)$): This is considered by Berg et al. [4].

Case 4. (W is degenerated at 0, and $\phi(C(z),c(z))=c$) : This is the case considered by Beichelt [2].

4. A Numerical Example

In the numerical analysis here we shall consider the system with Weibull distribution which is one of the most common in reliability studies. The p.d.f. of the Weibull distribution with shape parameter beta and scale parameter θ is given by

$$f(t)=\frac{\beta}{\theta}(\frac{t}{\theta})^{\beta-1}\exp\{-(\frac{t}{\theta})^{\beta}\},\beta,\theta>0, \quad (7)$$

and the time between opportunities W is given by

$$g_W(w)=\lambda e^{-\lambda\omega},\lambda>0. \quad (8)$$

If an operating system fails at age z, it is either replaced with a new system with probability

$$p(z)=1-\int_0^{\delta(z)c_\infty} l(x)dx, \quad (9)$$

or it undergoes minimal repair with probability

$$q(z)=1-p(z) \quad (10)$$

and $h(z)=\frac{1}{q(z)}\int_0^{\delta(z)c_\infty} xl(x)dx+c(z)$, (11) where $\delta(z)=\delta e^{-az}$ with $0\le\delta\le 1$ and $a\ge 0$.

An algorithm for solving this opportunistic age replacement problem is given as follows:

Input. $c_f,c_p,c_\infty,c(z),\delta,a,f(\cdot),l(\cdot),g_W(\cdot)$.

Step 1. Compute p(z), q(z), h(z) as defined by (9), (10), (11) respectively.

Step 2. Compute r(z) and $\overline{F}_p(z)$ as defined by (1).

Step 3. Compute B(T;p) as defined by (5).

Step 4. Find the optimal T* to minimize B(T;p).

Output.

T*= optimal opportunity-based age replacement time.

B(T*;p)= optimal expected cost per unit time over an infinite horizon.

Stop. End.

From the numerical results, we can derive the following remarks:

(i) some improvement can be made in the minimum cost per unit time if one allows for minimal repair at failure;

(ii) from Table 1, the minimum cost per unit time will be reduced when the probability of minimal repairing is age-dependent;

(iii) it can be seen that the present models are a generalization of previously known policies.

Table 1. Opportunistic age replacement policy for the Weibull distribution with $\beta = 2, \theta = 1012.2, c_f = 1200, c_p = 1000, c_\infty = 1000$, $\phi(C(z), c(z)) = C(z) + c(z)$, c(z)=0.3z, $C \sim N(300, 60^2), W \sim Exponential(\frac{1}{450})$.

Dekker and Dijkstra's result				
q(y)	δ	a	T	B(T*;p)
0	0	0	3316.8	1.338
Our results				
q(y)	δ	a	T	B(T*;p)
1	1	0	749.4	1.663
*	1	0.00210	2170.2	0.884
0.9	0.377	0	844.3	1.562
*	0.377	0.00041	3047.7	1.238
0.8	0.3505	0	937.8	1.493
*	0.3505	0.00034	3069.9	1.250
0.7	0.3315	0	1037.7	1.441
*	0.3315	0.00030	3087.3	1.261
0.5	0.3	0	1270.1	1.370
*	0.3	0.00022	3081.9	1.281
0	0	0	3316.8	1.338

* : age-dependent

References

1. Barlow, R. E. and Hunter, L. C., "Optimum Preventive Maintenance Policies," *Operations Research*, Vol. 8, 90-100, 1960.
2. Beichelt, F., "A General Preventive Maintenance Policy," *Math. Operationsforsch. u. Statist.*, Vol. 7, 927-932, 1976.
3. Boland, P. J., "Periodic Replacement when Minimal Repair Costs Vary with Time," *Naval Research Logistics Quarterly*, 1982, Vol. 29, 541-546, 1982.
4. Berg, M., Bienvenu, M. and Cléroux, R., "Age Replacement Policy with Age-Dependent Minimal Repair," *INFOR*, Vol. 24, 26-32, 1986.
5. Sheu, S. H., "A Generalized Block Replacement Policy with Minimal repair and General Random repair Costs for a Multi-unit System," *Journal of Operational Research Society*, Vol. 42, 331-341, 1991.
6. Sheu, S. H. and Jhang, J. P., "A Generalied Group Maintenance Policy," *European Journal of Operational Research*, Vol. 96, 232-247, 1996.
7. Jorgensen, D. W., McCall, J. J. and Radner, R., "Optimal Replacement Policy," North-Holland, Amsterdam, 1967.
8. Sethi, D. P. S., "Opportunistic Replacement Policies in The Theory and Applications of Reliability," *I. N. Shimi and C. P. Tsokos, Eds., Academic Press, New York*, Vol. 1, 443-447, 1977.
9. Vergin, R. C. and Scriabin, M., "Maintenance Scheduling for Multicomponent Equipment," *AIIE Transactions*, Vol. 9, 297-305, 1977.
10 Van der Duyn Schouten, F. A. and Vanneste, S. G., "Analysis and Compotation of (n, N)-strategies for Maintenance of a Two-Component System," *European Journal of Operational Research*, Vol. 48, 260-274, 1990.
11. Bäckert, W and Rippin, D. W. T., "The Determinaion Maintenance Strategies for Plants Subject to Breakdown," *Computer and Chemical Engineering*, Vol. 9, 113-126, 1985.
12. Berg, M. B., "General Trigger-Off Replacement Procedures for Two-unit Systems," *Naval Research Logistics Quarterly*, Vol. 25, 15-29, 1978.
13. Dekker, R. and Dijkstra, M. C., "Opportunity-Based Age Replacement: Exponentially

Distributed Times Between Opportunities," *Naval Research Logistics*, Vol. 39, 175-190, 1992.

14. Block, H. W., Borges, W. S. and Savits, T. H., "Age-Dependent Minimal Repair," *Journal of Applied Probability*, Vol. 22, 370-385, 1985.

15. Ross, S. M., "Applied Probability Models with Optimization Applications," Holden-Day, San-Francisco, 1970.