STATISTICAL PROCESS CONTROL FOR MULTIPLE DEPENDENT SUBPROCESSES

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ABSTRACT

A cost model, controlling multiple dependent subprocesses with minimum cost, is derived by renewal theory approach. The optimal multiple cause-selecting control chart and individual Y control chart are thus constructed to monitor the specific product quality and overall product quality contributed by the multiple dependent subprocesses. They may be used to maintain the process with minimum cost and effectively distinguish which component of the subprocesses is out of control. The optimal design parameters of the proposed control charts can be determined by minimizing the cost model using simple grid search method. An example is given to illustrate the application of the optimal multiple cause-selecting control chart and individual Y control chart.

1. Introduction

Control charts are important tools of statistical quality control. These charts are used to decide whether a process has achieved a state of statistical control and to maintain current control of a process. Today, most products are produced by several different process steps. In multiple step processes a Shewhart control chart is often used at each individual step. If the steps of the process are independent then using Shewhart control chart at each individual step is a meaningful procedure. However, in many processes the steps are not independent and thus the control charts are difficult to interpret. One approach to solve this problem is to use a multivariate control chart such as Hotelling T^2. The disadvantages of using T^2 control chart are that one must assume that the process quality characteristics are multivariate normal random variables and, once an out-of-control is given, it is often difficult to determine which component of the process is out-of-control. Alternative to this problem was proposed by Zhang (1984). He calls his charts "cause-selecting control charts". The cause-selecting control chart is constructed for a variable only after the observations have been adjusted for the effect of some other random variables. Zhang's cause-selecting control charts are the concepts of overall quality and specific quality. Zhang defines overall quality as that quality due to the current subprocess and any previous subprocesses. Specific quality is that quality which is due only to the current subprocess. The cause-selecting charts are designed to further distinguish between controllable assignable causes and uncontrollable assignable causes. Controllable assignable causes are those assignable causes that affect the current subprocess but no previous subprocesses. The uncontrollable assignable causes are those assignable causes affecting previous processes that cannot be controlled at the current process level. The advantage of this approach is that once an out-of-control signal is given, it is often easy to determine which component of the subprocesses is out of control. Wade and Woodall (1993) review the basic principles of the cause-selecting chart the simple case of a two step process and give an example to illustrate the use of cause-selecting chart. They also examine the relationship between the cause-selecting chart and the multivariate T^2 chart. In their opinion the cause-selecting control chart has some advantages over the T^2 chart.

To use any control chart, three design parameters must be specified; the sample size, the sampling interval, and the number of standard deviation above or below the center line of a control chart. The choice of these design parameters influences the costs of sampling and testing, costs of searching and repairing and costs due to the production of nonconforming items. Therefore, it is logical to consider the design of control charts from an economic viewpoint.

Duncan (1956) first proposed an economic model for the optimal economic design of $\overline{X}$ control chart. He recommended the use of a concept which he called an economic design to obtain the optimal design. The pioneering work of Duncan was later extended by others, including the $\overline{X}$ and R charts employed jointly (Saniga (1977,1979, 1989), Rahim (1989), and Yang(1993)). Rahim et al. (1988) discussed the use of joint $\overline{X}$ and S^2 charts according to economic considerations when sample sizes are moderately large. Collani and Sheil (1989) proposed the economic design of S chart when the assignable cause could only influence the process variance. Yang(1998) first presented the economic design of a simple cause-selecting control chart for a system with a single assignable cause which is assumed to occur in one of the current subprocess or the previous subprocess. However, the multiple assignable-cause cost model for multiple dependent subprocesses has not been addressed. In this paper, we consider a system with multiple dependent subprocesses, the current subprocess and the multiple previous subprocesses, and each subprocess may be influenced by a single assignable cause. The optimal individual Y control chart proposed to monitor the overall product quality and the multiple cause-selecting control chart proposed to monitor the specific product quality are derived by minimizing a multiple assignable-cause cost model which is obtained by extending renewal theory approach. Finally, an example is given to illustrate the design procedure and the application of the optimal multiple cause-selecting control chart and the individual Y control chart.

2. The Derivation of Process Model

Let X1, X2,..., Xk represent the in-coming quality measurements of interest for the preceding k steps of the process and let Y (overall quality or out-going quality) represent the quality measurement of interest for the k+1 (final) step. Suppose that a sample with size one is taken at the end of the final process every h hours and observations (X1i, X2i,..., Xki,Y_i) are measured on the same item of production. To control the overall product quality contributed by the current subprocess and all the preceding processes, we have to use the individual Y control chart on the variable. If the out-going quality to be controlled, Y, depends on k incoming qualities X1, X2,..., Xk then this is the cause-selecting case of multiple causes where we need to use the multiple cause-selecting control chart to control the specific quality resulting from the current subprocess itself.

The difference between simple cause-selecting control chart and multiple cause-selecting control chart is that the function between the out-going quality to be controlled and the in-coming qualities is multiple, not simple. To find out the relationship between the object to be controlled and the multiple in-coming qualities, we often use the multiple linear regression. The multiple cause-selecting chart is then based on values of the out-going quality Y that have been adjusted for the values of in-coming qualities (X_1, X_2,..., X_k). The multiple cause-selecting control chart would be used in conjunction with individual Y control chart to control the overall product quality and the specific product quality simultaneously and may effectively distinguish which component of the multiple dependent processes is out of control. An example is given to illustrate the design procedure of the two control charts, and their application on a manufacturing process steps is also presented.

The procedure in constructing the individual Y control chart and the multiple cause-selecting control chart is illustrated as follows.

Suppose that there are k in-coming qualities, (X_1, X_2,..., X_k), by experiment, we have the m sets of observed data;(y_i; x_{1i}, x_{2i}, ..., x_{ki}), i=1, 2,..., m. The individual Y chart on the Y variable is constructed to monitor the overall product quality. The Y_i values are assumed independent and $Y_i \sim N(\mu, \sigma^2)$ when all the subprocesses are in control. The center line (CL), upper control limit (UCL), and lower control limit (LCL) of the individual Y chart are set at μ, $\mu + k_1\sigma$, and $\mu - k_1\sigma$ respectively, where k_1 is the number of standard deviation above or below the center line of the individual Y chart, μ is the mean of the random variable Y_i, and σ^2 is the variance of the random variable Y_i. Suppose that the overall quality Y is the function of k in-coming qualities X_1, X_2, ..., X_k, and the Y_i values are independent, and specific quality $Y_i|(X_1, X_2,...X_k) \sim N(\mu_i, \sigma^{*2})$ when the process is in control, where $\mu_i = f(x_{1i}, x_{2i}, ...x_{ki})$, and σ^{*2} is a constant. μ_i is the mean of random variable $Y_i|(X_1, X_2,...X_k)$, and σ^{*2} is the variance of random variable $Y_i|(X_1, X_2,...X_k)$. Next, we have to establish a relationship between X_1, X_2,..., X_k and Y, either empirically or theoretically. If the function $f(x_{1i}, x_{2i}, ..., x_{ki})$ was known, the transformation $Z_i = (Y_i - \mu_i)/\sigma^*$ would be used to standardize the Y_i values. The multiple cause-selecting chart is a Shewhart type of control chart for the cause-selecting values Z_i, the values of Y_i adjusted for the effects of X_{1i}, X_{2i}, ..., X_{ki}. Thus, the Z_i's are independent N(0,1) random variables. The center line, upper control limit, and lower control limit for the multiple cause-selecting control chart are set at 0, k_2, and $-k_2$ respectively, where k_2 is the number of standard deviation above or below the center line of the multiple cause-selecting chart. Alternatively, cause-selecting values could also be defined as $(Y_i - \mu_i) \sim N(0, \sigma^{*2})$. Thus, the center line, upper control limit, and lower control limit for the multiple cause-selecting control chart are set at 0, $k_2\sigma^*$, and $-k_2\sigma^*$ respectively. In practice, the true relationship between X_1, X_2, ..., X_k and Y is never known. Hence, the mean of $Y|(X_1, X_2, ..., X_k)$, $E(Y|X_1, X_2, ..., X_k)$, and the variance of $Y|(X_1, X_2, ..., X_k)$, $V(Y|X_1, X_2, ..., X_k)$, have to be estimated from an initial sample of m observations. The estimate for μ_i will be $\hat{Y}_i$, where $\hat{Y}_i$ is the fitted value of $E(Y_i|(X_1, X_2,..., X_k))$. The estimate for σ^* will be $\sqrt{MSE}$, where $\sqrt{MSE}$ is the square root of the mean square error. The model fitting methods and diagnosis see Montgomery and Park (1982), Weisberg (1985), and others. Thus, the upper and lower control limits of the optimal multiple cause-selecting chart are set at $k_2\sqrt{MSE}$ and $-k_2\sqrt{MSE}$ respectively for residuals e_i, where $e_i = Y_i - \hat{Y}_i$. Zhang (1984) estimated σ^* using the average range of the residuals, $\overline{MR}$, where $\overline{MR} = \sum_{i=1}^{m-1} MR_i/(m-1)$, and $MR_i=|e_{i+1}-e_i|$. In his case, $UCL=k_2\overline{MR}$, $CL=0$ and $LCL=-k_2\overline{MR}$.

3. The Derivation of Cost Model

A current process is out of control when it is influenced by a controllable assignable cause, say A_{k+1}. We also assume that there are k uncontrollable assignable causes, say A_1, A_2, A_3,..., and A_k, which can only affect the preceding operations 1, 2, ..., and k respectively. A preceding operation j is out of control when it is influenced by an uncontrollable assignable cause, say A_j, j=1,2,...,k. Assignable causes A_1, A_2, A_3,..., A_k, and A_{k+1} would be allowed to occur in the first step, the second step,..., and the current step of the process simultaneously.

The distributions of the overall quality (Y) and the specific quality $Y|(X_1, X_2, ..., X_k)$ would be changed once any assignable causes occur in the process steps. The general distribution of the overall quality, Y, can be expressed as $N(\mu_s, \sigma^2)$, where s is a subset of all assignable causes. The most general distribution of the specific quality $Y|(X_1, X_2, ..., X_k)$ can be expressed as $N(\mu^*s, \sigma^{*2})$. Other assumptions and the nature of the operation condition are summarized as follows.

(1) The time (T_{Ai}) until the occurrence of assignable cause (A_i) is assumed exponential distribution with parameter λ_i, i=1, 2,...k+1. T_{A1}, T_{A2}, ..., and $T_{A(k+1)}$ are mutually independent.

(2) The time of taking a sample, inspection, and charting are negligible.

(3) The search and repair time is a constant T_{srs} when the process is influenced by the assignable causes in the set S. The search and repair time is a constant T_f when there are at least one false alarm for the two charts.

(4) The search and repair cost is a constant C_{srs} when the process is influenced by the assignable causes in the set S. The search and repair cost is a constant C_f when there are at least one false alarm for the two charts.

(5) A quality cycle is defined as the time between the start of successive in-control period. Then the process is expressed as a series of independent and identical cycles. That is, the process is a renewal process. The accumulated cost per cycle is called the cycle cost. The cycle costs are independent and identically distributed. Such a process is known as a renewal reward process (see Ross 1993).

(6) The cost of sampling and testing is a constant b, b>0.

(7)The process ceases during the search state.

In order to obtain the expression for the expected cycle time (E(T)) using renewal theory approach we have to study the possible states at the end of the first sampling and testing. There are 2^{k+2} possible states. Depending on the state of the system, one can compute the expected residual cycle length (Ri) and the expected residual cost (Ri'). These values, together with the associated probabilities (Pi), lead us to formulate the renewal equation. Consequently, the expected cycle time is

$$E(T)=[(h+P_2T_f)/(1-P1-P2)]+\sum_{i=3}^{2^{k+2}} P_iR_i/(1-P1-P2), \qquad (1)$$

and the expected cycle time is

$$E(C)=[(P1+P2)(b+C_0h)+P2C_f]/(1-P1-P2)+\sum_{i=3}^{2^{k+2}} P_iR_i'/(1-P1-P2). \quad (2)$$

Applying the property of renewal reward process (Ross (1993)), the objective function, the expected cost per unit time, $(E(V_\infty))$ is derived by taking the ratio of the expected cycle cost (E(C)) and the expected cycle time (E(T)); $E(V_\infty)=E(C)/E(T)$. The objective function is the function of design parameters h, k_1 and k_2. Hence, the optimal esign parameters of the proposed control charts can be determined by minimizing the objective function.

4. An Example

We illustrate the application of optimal multiple cause-selecting control chart and Individual Y control chart in this section. Suppose that the approximate optimal values h^*, $k_1{}^*$ and $k_2{}^*$ have been obtained by using an optimization technique. That is, the upper and lower control limits of the optimal individual Y chart are set at $\mu+k_1{}^*\sigma$ and $\mu-k_1{}^*\sigma$ (if μ and σ are unknown then we use $\overline{Y}$ (sample mean) and S (sample standard deviation) to estimate them) respectively for the plotted statistic Y_i. The upper and lower control limits of the optimal multiple cause-selecting chart are set at $k_2{}^*$ and $-k_2{}^*$ respectively for plotted statistic Z_i, or set at $k_2{}^*\sqrt{MSE}$ and $-k_2{}^*\sqrt{MSE}$ respectively for residuals e_i. To monitor the process states, every h^* hours a sample with size one (X_{1i}, X_{2i}, X_{3i}, ..., Y_i) is taken and tested. There are three possible testing results for the multiple process steps. Combination 1 means that all the previous and current process steps are all in control, so the process continues and the next sample is taken after h^* hours. Combination 2 means that some previous process steps are out of control but the current process is in control, hence the process has to be stopped and the preceding process steps need to be check and repaired. Combination 3 may mean that previous process steps are in control but the current process is out of control, hence the process has to be stopped and the current process needs to be check and repaired; or some previous process steps and the current process are out of control, hence the process has to be stopped and the previous process steps and the current process need to be check and repaired.

The multiple cause-selecting chart is used in conjunction with individual Y chart for the multiple dependent process steps. We find that they may distinguish the uncontrollable assignable causes and controllable assignable cause effectively.

5. Conclusions

The multiple cause-selecting chart could be used in conjunction with individual Y chart for the multiple dependent process steps. They may effectively distinguish the uncontrollable assignable causes and the controllable assignable cause. The method of designing the optimal multiple cause-selecting chart and the individual Y chart simultaneously has proposed. The constraints on powers and Type I error probability would be allowed in economic design of control charts. It can be viewed as an improvement to economic design while achieving desirable statistical properties. In practice, if the engineers would like to maintain the processes with minimum cost, desired statistical properties, and determine which component of the subprocesses is out of control effectively then using the optimal cause-selecting control chart and individual Y control chart is preferable. The method proposed can be extended to the case of multiple assignable causes occurred in each of the current process and previous process steps.

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