An Analysis of the Control Limit in p-chart Applying Binomial Distribution Using Commercial Software

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ABSTRACT

The p chart approximate to the normal distribution has a difficulty to analyze the process condition precisely when the negative LCL is occurred. Furthermore, the probability of Type Ⅰ error increases compared with using its original binomial distribution. For a long time the p chart has been used as approximated to the normal distribution because of its easy use. However, it becomes rapid and convenient to calculate the binomial distribution through the development of computer and software, so it is strongly suggested to use the binomial distribution determining control limits to reduce the probability of Type Ⅰ error.

In this study, I suggest that the control limits can be designed in use of binomial distribution and they can be utilized without special software by illustrating the certain work for establishing p-chart with the commercial one(EXCEL).

1. Introduction

Statistical process control(SPC) is by now a commonly used quality assurance tool. One of the most SPC techniques is the use of control charts for maintaining process in state of control.

In control, chart control limits are upper control limit(UCL) and lower control limit(LCL). A process is judged out of control when it, with a certain probability, performs worse than expected; in such a case action should be taken to make the necessary corrections to the process. But the process may also be "out-of control" in the sense that it has become better than expected; in such a situation useful information can be sought about how the new-found quality improvement may be sustained. This is an extremely valuable opportunity not be missed, as continuous improvement is now the cornerstone of modern quality management.

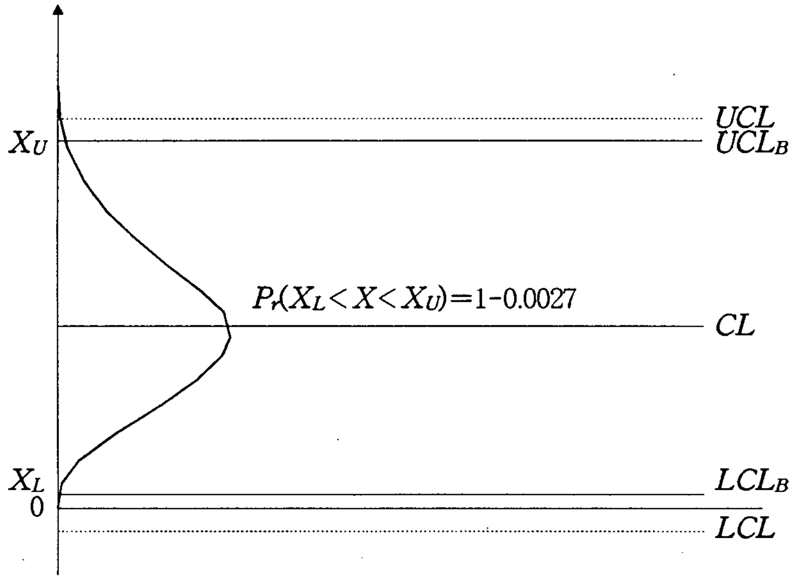
One of such control charts, p-chart for the control of proportion nonconforming, faces some implementation problems when the proportion nonconforming p is low and the size of each sample n is not large enough. In particular, based on 3σ deviation from process mean, LCL is often below zero which has no meaning. It is often advised in textbooks or journal papers that this LCL be ignored or zero be taken instead as the lower limit. However, such practice contradicts the fundamental idea of SPC by control charts[11].

The purpose of this study is to suggest that the control limits can be designed in use of binomial distribution and they can be utilized without special software by illustrating the certain work for establishing p-chart with the commercial one(EXCEL).

2. Control Limits in p-chart and Type I Error

In figure 1, you can find the basic concepts for conventional control limits approximated to the normal distribution and for the control limits applied to the binomial distribution in p-chart. In conventional control limit, the upper control probability limit and the lower control probability limit are determined by $\mu \pm 3\sigma$ - the μ and σ are the average and the standard deviation, those which binomial distribution is approximated to the normal distribution. Therefore, the upper control probability and the lower control probability make a symmetry between CL, which can be minus in some case. Since the control limits applied to binomial distribution are determined by probability, LCL becomes 0 or above it and can't be minus, and it makes an asymmetric line.

As above, in the existing control limits approximated to the normal distribution, LCL is not considered to be minus and almost users have used traditional techniques for without any doubt.



note) UCL : upper control limit applying normal distribution
LCL : lower control limit applying normal distribution
UCL_B : upper probability limit applying binomial distribution
LCL_B : lower probability limit applying binomial distribution

(Figure 1) Conventional Control Limits and Probability Limits for the p-chart

3. Control Limits in p-chart applying Binomial Distribution

If a control chart user set the upper control probability limit up to 1-0.00135 and the lower control probability limit below 0.00135, he can get out the control limits applying binomial distribution through following process.

$$k_U = \min\left\{k: \sum_{i=0}^{k}\binom{n}{i} p'^{i}(1-p')^{n-i} \geq 1-0.00135\right\} \tag{1}$$

$$k_L = \max\left\{k: \sum_{i=0}^{k}\binom{n}{i} p'^{i}(1-p')^{n-i} \leq 0.00135\right\} \tag{2}$$

$$CL = \bar{p} \tag{3}$$

where k_U: upper number nonconforming

k_L : lower number nonconforming

k : 0, 1, 2, ⋯, n

k_U of formula (1) is the minimum value among k of left binomial distribution cumulative probability value larger than 1-0.00135 of right one with increasing i the variable from 0. And, k_L of formula (2) is the maximum value among k of left binomial distribution cumulative probability value less than 0.00135 of right with increasing i the variable from 0.

Formula (4) and formula (5) are achieved when you get control limits of p -chart applying the binomial distribution by using k_U, k_L from formula (1) and from formula (2).

$$UCL_B = \frac{k_U}{n} \tag{4}$$

$$LCL_B = \frac{k_L}{n} \tag{5}$$

$$CL = \bar{p} \tag{6}$$

The UCL_B of formula (4) means the fraction defective, which the binomial distribution cumulative probability is larger than 1-0.00135 and the LCL_B of formula (5) means the fraction defective, which the binomial distribution cumulative probability is less than 1-0.00135. CL uses $\bar{p}$ the process average proportion as it is.

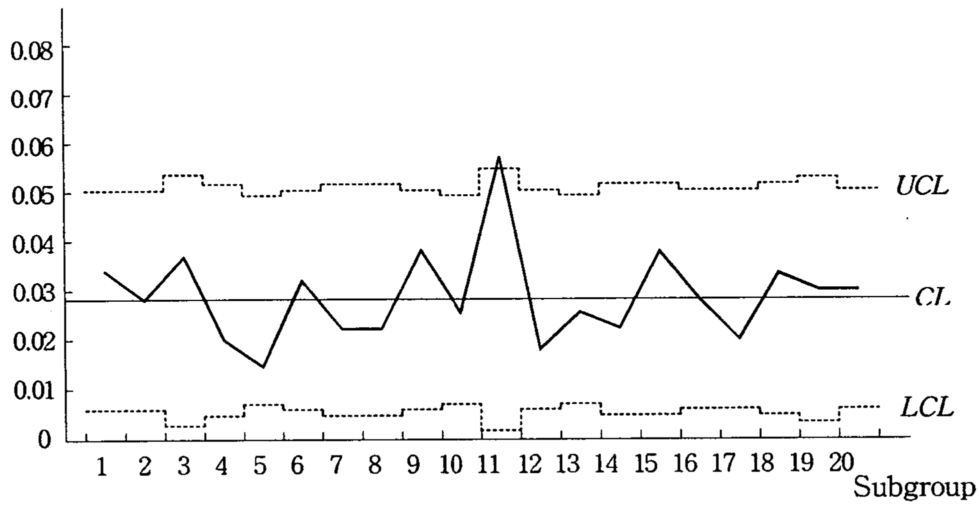
4. Case Study

Table 1 is the data sheet for bearing defect proportion of A company. Ramdom sampled in Lot and sample size is differed as 350~550 pieces.

(Table 1) p-chart data sheet in A company

Date	Lot No.	n	x	p	UCL	LCL
8.24	1	500	17	0.0340	0.0504	0.0060
25	2	500	14	0.0280	0.0504	0.0060
26	3	380	14	0.0368	0.0536	0.0027
27	4	450	9	0.0200	0.0516	0.0048
28	5	550	8	0.0145	0.0493	0.0070
30	6	500	16	0.0320	0.0504	0.0060
31	7	450	10	0.0222	0.0516	0.0048
9. 1	8	450	10	0.0222	0.0516	0.0048
2	9	500	19	0.0380	0.0504	0.0060
3	10	550	14	0.0255	0.0493	0.0070
4	11	350	20	0.0571	0.0547	0.0016
5	12	500	9	0.0180	0.0504	0.0060
6	13	550	14	0.0255	0.0493	0.0070
7	14	450	10	0.0222	0.0516	0.0048
8	15	450	17	0.0378	0.0516	0.0048
9	16	500	14	0.0280	0.0504	0.0060
10	17	500	10	0.0200	0.0504	0.0060
11	18	450	15	0.0333	0.0516	0.0048
12	19	400	12	0.0300	0.0530	0.0033
13	20	500	15	0.0300	0.0504	0.0060
합계		9480	267	0.0282		

Figure 2 is the one which the above data the made in use of conventional method.



(Figure 2) Conventional p-chart in A company

In the above control chart drawn in the way of approximating to the normal distribution, no point is out of UCL and LCL. Process average proportion is

0.0282 and sample average size n is not much as 474. Say the property of binomial distribution, approximated to the normal distribution $p' \leq 0.5$ and under the condition of $np'>5$, since the sample size is 474 and the average proportion nonconforming is 0.028, np is 13.4. Because of the general property of binomial distribution, it has been used with being approximated to the normal distribution.

To analyze, in more detail, the process of A company, the binomial distribution probability for the average proportion nonconforming of 0.282 is calculated on. It is need to be compared with the case, approximated to the normal distribution. First of all, the results of binomial distribution property for the process average proportion nonconforming of 0.0282 appear on Figure 3.

Microsoft Excel - case_bi.xls

	A	B	C	D	E
1	$\bar{x}$	474		k_U	25
2	$\bar{p}$	0.0282		k_L	3
3					
4	x	Pr(x=X)	Pr(x≤X)		
5	0	0.000001	0.000001	LCL	
6	1	0.000018	0.000019	LCL	
7	2	0.000124	0.000143	LCL	
8	3	0.000565	0.000708	LCL	
9	4	0.001927	0.002634	4	
10	5	0.005248	0.007883	5	
11	6	0.011889	0.019772	6	
27	22	0.007706	0.990805	22	
28	23	0.004389	0.995194	23	
29	24	0.002390	0.997584	24	
30	25	0.001247	0.998830	UCL	
31	26	0.000624	0.999454	UCL	
32	27	0.000300	0.999755	UCL	
33	28	0.000139	0.999893	UCL	
34	29	0.000062	0.999955	UCL	
35	30	0.000027	0.999982	UCL	

DataSheet / BiPr / Graph / BiDataSheet

(Figure 3) Binomial Distribution probability of average proportion in A company

To get UCL_B and LCL_B, the control limits applying binomial distribution, with the method applying the binomial distribution suggested in chapter 3, k_U and k_L should firstly be calculated on.

$$k_U = \min\left\{k: \sum_{i=0}^{k}\binom{n}{i}p'^i(1-p')^{n-i} \geq 1-0.00135\right\} = 25$$

$$k_L = \max\left\{k: \sum_{i=0}^{k}\binom{n}{i}p'^i(1-p')^{n-i} \leq 0.00135\right\} = 3$$

The results of UCL_B and LCL_B for each group by applying k_U and k_L to the formula (4) after then appear on Figure 4.

$$\left.\begin{aligned} UCL_B &= \frac{k_U}{n} = \frac{25}{n} \\ UCL_B &= \frac{k_U}{n} = \frac{3}{n} \\ CL &= \bar{p} = \frac{267}{9480} = 0.0282 \end{aligned}\right\} \qquad (7)$$

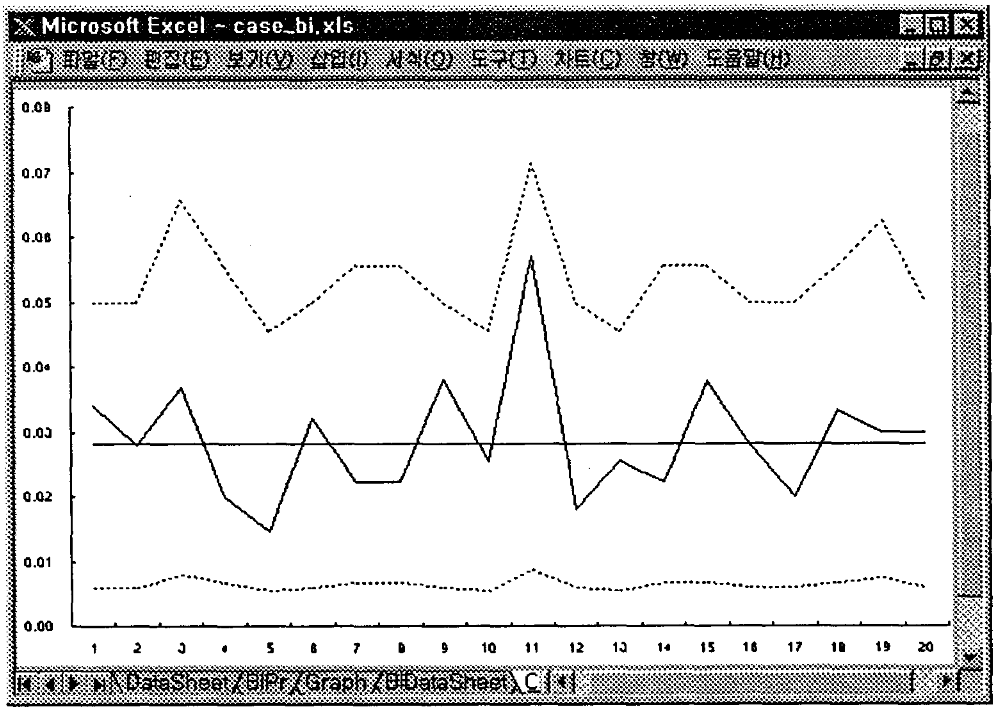
Microsoft Excel - case_bi.xls

	A	B	C	D	E	F	G	H
1	k_U	25						
2	k_L	3						
3	CL	0.0282						
4								
5	lot No	n	x	p	UCL_B	LCL_B	CL	
6	1	500	17	0.0340	0.0500	0.0060	0.0282	
7	2	500	14	0.0280	0.0500	0.0060	0.0282	
8	3	380	14	0.0368	0.0658	0.0079	0.0282	
9	4	450	9	0.0200	0.0556	0.0067	0.0282	
10	5	550	8	0.0145	0.0455	0.0055	0.0282	
11	6	500	16	0.0320	0.0500	0.0060	0.0282	
12	7	450	10	0.0222	0.0556	0.0067	0.0282	
13	8	450	10	0.0222	0.0556	0.0067	0.0282	
14	9	500	19	0.0380	0.0500	0.0060	0.0282	
15	10	550	14	0.0255	0.0455	0.0055	0.0282	
16	11	350	20	0.0571	0.0714	0.0086	0.0282	
17	12	500	9	0.0180	0.0500	0.0060	0.0282	
18	13	550	14	0.0255	0.0455	0.0055	0.0282	
19	14	450	10	0.0222	0.0556	0.0067	0.0282	
20	15	450	17	0.0378	0.0556	0.0067	0.0282	
21	16	500	14	0.0280	0.0500	0.0060	0.0282	
22	17	500	10	0.0200	0.0500	0.0060	0.0282	
23	18	450	15	0.0333	0.0556	0.0067	0.0282	
24	19	400	12	0.0300	0.0625	0.0075	0.0282	
25	20	500	15	0.0300	0.0500	0.0060	0.0282	
26								
27		9480	267	0.0282				

BiPr / Graph / BiDataSheet / Chart

(Figure 4) p-chart data sheet applying Binomial Distribution in A company

The p-chart applying binomial distribution according to Figure 4 data sheet appear on Figure 5. The 11th group that is out of control limits in case with the conventional method approximated to the normal distribution can be seen as being in control limit in case with method applying binomial distribution. Though the control limit line approximated to the normal distribution is symmetrical between the upper and lower sides, the control limit line applying binomial distribution is not symmetrical. It is because the control limits for each group is set by probability limit that is varied according to n of each group since n is not same.



(Figure 5) p-chart applying Binomial Distribution in A company

Therefore, since you can set the control limits more correctly when you utilize binomial distribution rather than the normal distribution, so that you can improve the sensing ability to find out the abnormality.

Consequently, it have to change from existing the method approximated to the normal distribution to the control limits applying the binomial distribution.

5. Conclusion

As a result of consecutive endeavoring to reduce nonconformities of the process, the proportion nonconforming in the product has decreased to the ppm unit, compared with that of the early days of Control Chart.

The p chart approximate to the normal distribution has a difficulty to analyze the process condition precisely when the negative LCL is occurred. Furthermore, the probability of Type I error increases compared with applying its original binomial distribution. For a long time The p chart has been used as approximated to the normal distribution because of its easy use. However, it becomes rapid and convenient to calculate the binomial distribution through the development of computer and software, so it is strongly suggested to apply the binomial distribution determining control limits to reduce the probability of Type I error.

This study designed the control limits of p chart using the binomial distribution and the case study of a company shows the differences between two distributions and emphasize applying the binomial distribution compared to the approximate normal distribution.

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