유사정상상태 해법을 이용한 폐쇄 가스저류층의 장기거동 계산

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Pseudosteady-State Approach to Calculate Long-Time Performance of Closed Gas Reservoirs

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Abstract

This paper considers the applicability of a pseudosteady-state approach to the long-time behavior of real gas flow in a closed reservoir. The method involves a combination of a linearized gas diffusivity equation using a normalized pseudotime and a material balance equation. Comparison with a commercial reservoir simulator showed that highly accurate values of pseudopressure drawdown and well pressure are obtained by the pseudosteady-state approach with much less computational effort.

Introduction

The pseudosteady-state approach has been mathematically established and shown to be an effective tool for analyzing long-time performance of a bounded reservoir with wells producing at constant rates (Lee, 1996). To apply pseudosteady-state approach to gas flow in a closed reservoir, one has to derive a linearized diffusivity equation.

As is well known, the flow of gas through a permeable medium is represented by the diffusivity equation using pseudopressure, p_p, suggested by Al-Hussainy and Ramey (1966).

$$\nabla^2 p_p = \frac{\phi \mu c_t}{k} \frac{\partial p_p}{\partial t} \qquad (1)$$

$$p_p = \int_0^p \frac{2p}{\mu z} dp \qquad (2)$$

Because the diffusivity equation for gas flow in terms of the pseudopressure involves the least number of assumptions, it is considered to be the most rigorous of the various treatments. The nonlinearity in the diffusivity equation, however, has long been recognized as one of the problems in dealing with gas flow. In this paper, a linearized diffusivity equation is derived by introducing a pseudotime. The linearized diffusivity equation allows one to apply pseudosteady-state method and estimate late-time behavior.

Mathematical Formulation

The derivation of the linearized gas diffusivity equation starts by using pseudopressure and normalized pseudotime. Pseudotime, which is an empirical function, has been proposed by many authors like Agarwal (1979), Meunier *et al.* (1987), Reynolds *et al.* (1987), Finjord (1989), and Ding *et al.* (1990).

In order to correlate the real gas pseudopressure solution with analogous liquid solution during boundary-dominated flow, we normalized Agarwal's pseudotime through the following equation:

$$t_{pn} = \mu_i c_{ti} \int_0^t \frac{dt}{\mu c_t} \quad \text{.......} (3)$$

Use of pseudopressure and normalized pseudotime produces a theoretical basis for deriving a linearized gas diffusivity equation:

$$\nabla^2 p_p = \frac{\phi \mu_i c_{ti}}{k} \frac{\partial p_p}{\partial t_{pn}} \quad \text{.......} (4)$$

The dimesionless pseudovariables can be defined as

$$p_{pD} = \frac{\pi k h T_{sc}}{q_{sc} p_{sc} T} \left(p_{pw} - p_p \right) \quad \text{.......} (5)$$

$$t_{pnD} = \frac{k t_{pn}}{\left(\phi \mu c_t \right)_i r_w^2} \quad \text{.......} (6)$$

$$t_{pnDA} = \frac{k t_{pn}}{\left(\phi \mu c_t \right)_i A} \quad \text{.......} (7)$$

Use of Eqs. 5 and 6 in the gas diffusivity Eq. 4 results in an equivalent liquid equation:

$$\nabla^2 p_{pD} = \frac{\partial p_{pD}}{\partial t_{pn_D}} \quad \text{(8)}$$

The close analogy between the diffusivity equation of liquid flow and the linearized real gas flow suggests that the liquid and gas solutions can be correlated at late times provided that we graph the gas solution in terms of t_{pnDA}. For constant rate production, the long-time performance of a gas reservoir can be expressed as

$$\bar{p}_{pD} = 2\pi t_{pnDA} \quad \text{(9)}$$

and

$$\bar{p}_{pD} - p_{pwD} = -\frac{1}{2}\ln\frac{4A}{e^{\gamma}C_A r_w^2} \quad \text{(10)}$$

In this case, the long-time condition refers to constant decline rate of pseudopressure with respect to normalized pseudotime. Therefore, pseudopressure is a linear function of normalized pseudotime, as shown in Eq. 9.

Because the pseudotime approach requires information on the average reservoir pressure and fluid properties, it is impossible to obtain an expression for average pseudopressure in a closed form. For practical purpose, the average real gas potential in our approach is evaluated from overall system material balance considerations. For a single-phase gas reservoir, material balance can be expressed in terms of hydrocarbon pore volume (Dake, 1978):

$$\frac{\int_0^t q(\tau)d\tau}{G} = 1 - \frac{\overline{(p/z)}}{(p/z)_i} \quad \text{(11)}$$

To investigate this approach, a table of gas properties and real gas pseudopressure as function of pressure for any given gas and reservoir conditions is generated. Once $\bar{p}$ is known from Eq. 10 as function of time, $\bar{p}_p$ can be computed from the p_p versus p table. p_w can then be computed from the rearrangement of Eq. 10 and the p_p versus p table.

Numerical Simulation Study

This section provides a comparison of the pseudosteady-state solutions with results generated using a conventional finite-difference commercial reservoir simulator. The pseudosteady-state runs were made using FIDAP (Fluid Dynamics International, 1993) on a Cray Y-MP. The commercial simulator was the VIP model (Western Atlas International, Inc., 1993) and was run on an IBM-590.

To provide an example for predicting the long-time performance of a field-scale reservoir problem, a multiwell, multilayer simulation study was conducted. The model is based on reservoir parameters of the Hugoton field in southern Oklahoma, as shown in Fig. 1 (Fetkovich *et al.*, 1994). Table 1 shows reservoir properties and run parameters. The flowrate for each well hypothetically changed twice during a 800-day simulation period as indicated in Table 2.

Figure 2 shows the comparison of wellbore pressures from both approaches. The quality of the match between two solutions is excellent. The results of Fig. 2 indicate that the pseudosteady-state approach is correct within 2% for all time including infinite-acting and transient periods. Relatively large differences at the early time of each period are due to the difference between transient and pseudosteady-state behavior. Negligible differences at late times confirm the accuracy of pseudosteady-state approach.

Conclusion

Pseudosteady-state approach has been applied to model the flow behavior of closed gas reservoirs with wells producing at constant flowrate during boundary-dominated flow. Comparison with a commercial simulator for reservoirs considered in this study showed that highly accurate values of pseudopressure drawdown and well pressure are obtained by the pseudosteady-state approach with much less computational effort.

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Table 1: Input data for simulation of field-scale gas reservoir.

Reservior Configuration	Length (a) (miles)	3
	Width (b) (miles)	4
	Thickness (h) (ft)	148 (58/50/40)
Reservoir Properties	Permeability (k) (md)	25/10/25
	Porosity (ϕ)	0.35/0.25/0.30
	Compressibility (c_r) (psi^{-1})	1×10^{-6}
	Temperature (T_r) (°F)	100
Fluid Properties	Gas gravity (γ_g)	0.7
	Viscosity (μ_{gi}) (cp)	0.026
	Compressibility (c_{gi}) (psi^{-1})	121.1×10^{-6}
Conditions	Initial pressure (p_i) (psia)	2,000
Run Parameters	Max. press. change (psia)	50
	Max. time step size (day)	5-25
	Min. time step size (day)	0.001

Table 2: Flowrate (Mscf/day) of each well of field-scale gas reservoir.

Well #	Period 1 (0-300)	Period 2 (301-500)	Period 3 (501-800)
1	5,000	2,000	4,000
2	4,500	2,000	3,600
3	4,000	2,000	3,200
4	4,000	1,800	4,000
5	5,000	1,600	4,000
6	5,000	1,800	3,200
7	4,500	1,600	3,200
8	4,000	1,800	3,600
9	5,000	2,000	4,000
10	4,500	1,800	3,200
11	4,000	1,600	3,600
12	4,500	1,600	3,600

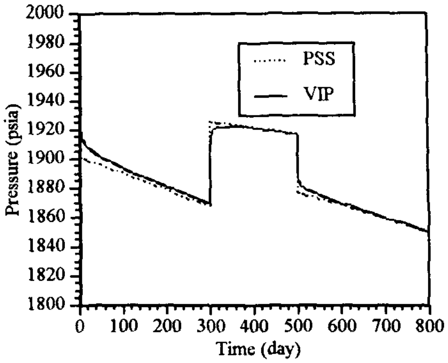

Figure 2: Comparison of well flowing pressure of Well #1.

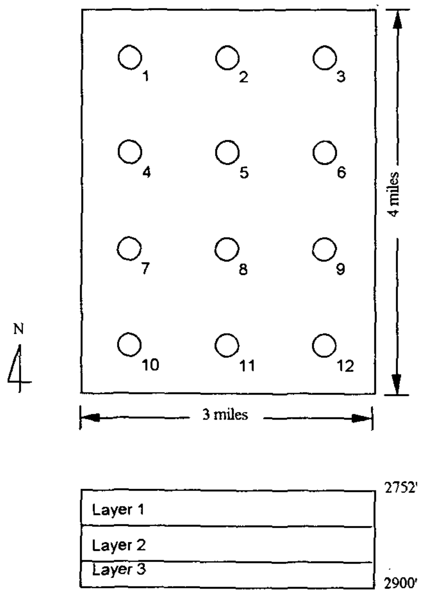

Figure 1: Reservoir model of Oklahoma Hugoton study area (Fetkovich *et al.*, 1994).