Matching Pursuit 방법을 이용한 MR영상법에 관한 연구

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Magnetic Resonance Imaging Using Matching Pursuit

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ABSTRACT

The matching pursuit (MP) algorithm developed by S. Mallat and Z. Zhang is applied to magnetic resonance (MR) imaging. Since matching pursuit is a greedy algorithm to find waveforms which are the best match for an object-signal, the signal can be decomposed with a few iterations. In this paper, we propose an application of the MP algorithm to the MR imaging to reduce imaging time. Inner products of residual signals and selected waveforms in the MP algorithm are derived from the MR signals by excitation of RF pulses which are Fourier transforms of selected waveforms. Results from computer simulations demonstrate that the imaging time is reduced by using the MP algorithm and further a progressive *reconstruction can* be *achieved*.

Keywords: matching pursuit in magnetic resonance imaging, adaptive coding, rapid imaging, non-Fourier imaging.

1. Introduction

Since Fourier zeugmatography had been introduced[1,2], most of magnetic resonance (MR) imaging techniques have been based on Fourier transform[3]. In Fourier Transform MR imaging, signals are sampled and acquired in k-space or Fourier domain of the image. The image is reconstructed by inverse Fourier transform of the signals. The spatial resolution of the image is achieved by acquiring sampled signals with a frequency bandwidth which occupies N×N array in the *k*-space[4,5]. Therefore, the conventional Fourier transform-MR imaging has a shortcoming that it takes a long time to obtain an image with a high spatial resolution.

To reduce imaging time, rapid imaging techniques have been proposed in the last decade, *e.g.*, Echo Planar Imaging (EPI), Steady State Free Precession (SSFP), Rapid Acquisition Relaxation Enhanced (RARE) and DANTE fast imaging[6-7]. These techniques have achieved the reduction of imaging time by using fast data acquisition of the *k*-space and therefore they usually require high performance of MR hardware such as strong gradients or RF systems. On the other hand, other methods using non-Fourier methods have been proposed to tackle the problem. They are feature recognizing MRI, local featured MRI, singular value decomposition and Karhunan-Loeve (K-L) expansion[8-9]. In these methods, Fourier bases conventionally employed in MRI are replaced by other bases which are generally founded in the sense that few bases capture most of the information of the image.

Recently, S. Mallat and Z. Zhang have introduced the matching pursuit (MP) as a new signal decomposition method[10]. Matching pursuit is a recursive algorithm to compute a signal *representation with respect to dictionaries* of elementary building blocks[10]. Since waveforms in the dictionaries are chosen in order to match a signal, the signal can be decomposed into fewer numbers of waveforms compared with Fourier decomposition[10]. In this paper, we apply a matching pursuit algorithm to magnetic resonance imaging and propose an MP-MR imaging method to reduce imaging time. Detailed algorithms for MR imaging are described and they are verified by computer simulations.

2. Theory

2. 1. Image Reconstruction in MR Imaging

In MR imaging, an object to be imaged is located inside a magnetic field which consists of a static magnetic field (B_0) and a spatially gradient magnetic field (G_x, G_y, G_Z). The precession frequencies of spins in the object vary spatially due to spatially varying magnetic field as

$$f_{\vec{\gamma}} = \frac{\gamma}{2\pi}(B_0 + G_{\vec{\gamma}} \cdot \vec{\gamma}), \quad (1)$$

where γ is the gyromagnetic ratio and $\vec{\gamma}$ is the spatial vector of (x, y, z). The spatially varying precession frequency leads to a periodic phase modulation in the spins of the object. After 2-dimensional selection by RF pulse, the MR signal by integrating the spins over the object can be acquired as

$$S_F(m.n) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho(x,y)e^{-ik_x x}e^{-ik_y y}dxdy, \quad (2)$$

where is 2-dimensional spin distribution, $k_x = \gamma G_x Tm$, $k_y = \gamma G_y \delta t\, n$, T is time duration of x-gradient magnetic field, and δt is the ADC sampling time. The object function of spin distribution, $\rho(x, y)$ can be reconstructed by inverse Fourier transform of Eq. (2). The sampled data, $S_F(m.n)$ in k-space are usually acquired column by column where each column is separated by the repetition time (TR). To get a 2-dimensional array of N×N in k-space, therefore, it takes N times of TR.

Another way to encode the MR signal can be carried out using a non-Fourier technique which has been introduced recently, e.g., wavelet transform MR imaging. In this case, a waveform is generated by an RF pulse and is superimposed onto the object function. Then the MR signal will be

$$S_{NF}(m.n) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho(x,y)u_m(x)e^{-i\gamma G_y \delta n y}dxdy$$

$$= \int_{-\infty}^{+\infty} <\rho(x,y), u_m(x)> xe^{-i\gamma G_y \delta n y}dy, \quad (3)$$

where, $u_m(x)$ is the waveform generated by RF pulse and $\langle \cdot \rangle$ is the inner product operator. If the waveform set of $\{u_m(x)\}$ is orthonomal set, *e.g.,* wavelet bases, the object function, can be reconstructed completely[10]. In the non-Fourier encoding method, by optimally choosing $\{u_m(x)\}$, one can reduce the number of encodings even while maintaining a visually sufficient reconstruction of the object. In this paper, to find the optimal basis set for this purpose, we employ the matching pursuit technique which will be described in the following section.

2. 2. Matching Pursuit (MP)

In matching pursuit, a signal f is decomposed into the concatenated sum

$$f = \sum_{n=0}^{k-1} < R^n f, b_{\gamma_n} > b_{\gamma_n} + R^k f$$
$$= f_k + R^k f, \quad (4)$$

where $R^k f$ is the residual signal after k iterations and b_{γ_n} is a waveform which matches the signal. Note that b_{γ_n} belongs to a dictionary Γ, *i.e.,* $\Gamma = \{b_{\gamma_n}\}$. In Eq. (4), the signal can be approximated as f_k after k iterations. With the initial values of $R^0 f = f$ and $f_0 = 0$, the inner products of the residual signals and waveforms from the dictionary are computed, *i.e.,* $\langle R^k f, b_{\gamma_k} \rangle$, and a waveform is chosen so that it closely matches the residual signal $R^k f$, *i.e.,*

$$\left|< R^k f, b_{\gamma_k} >\right| \geq \alpha \sup_{\gamma \in \Gamma}\left|< R^k f, b_\gamma >\right|, \quad (5)$$

where $0 < \alpha \leq 1$. Then the signal f_{k+1} can be spproximated as

$$f_{k+1} = f_k + < R^k f, b_{\gamma_k} > b_{\gamma_k}. \quad (6)$$

The residual signal is updated as

$$R^{k+1} f = R^k f - < R^k f, b_{\gamma_k} > b_{\gamma_k}. \quad (7)$$

After incrementing k, Eqs. (5), (6)and (7) are repeated until some convergence criterion has been satisfied. As seen in Eq. (5), the algorithm greedily chooses a waveform at each iteration so that the selected waveforms are best adapted to approximate parts of the signal. Therefore the signal can be decomposed and reconstructed with a few numbers of iterations[10].

2. 3. Matching Pursuit in MR Imaging

MP provides flexible signal decomposition since the choice of dictionaries is not limited. The time-frequency dictionary provides adaptive decomposition where signal structures are represented by waveforms in the dictionary. Therefore signals are explicitly featured by the scale, frequency and time of selected waveforms. In this paper, a wavelet dictionary is chosen to take advantage of time-frequency information. Since waveforms in this wavelet dictionary are localized spatially, one can achieve an adaptive coding which depends on the object. In the following sections, we will discuss the implementation of matching pursuit algorithm to MR imaging and propose a pulse sequence for the MP-MR technique.

The wavelet dictionary consists of orthonomal wavelet bases. For the image with N×N matrix, N orthonomal wavelet basis functions are in the dictionary. Using this wavelet dictionary, the non-Fourier encoding MR method mentioned in Eq. (3) is employed. The MP algorithm is adopted to find optimal wavelet bases among N orthonomal bases in the dictionary, *i.e.,* using the MP algorithm some wavelet bases are selected so that they are best matched with the image and thereby guaranteeing visually sufficient reconstruction of the object. The reduced optimal wavelet bases, therefore, would lead to the reduction of imaging time.

As seen in Eq. (3), since non-Fourier encoding is applied for 1-dimension in the x-direction, we need a 1-dimentional signal which is representative of the N horizontal lines in the image. The 1-dimensional signal in this paper is chosen by projecting the 2-dimensional image, *e.g.,* by integrating the signal along the reading direction (y-direction). Further, in the MP algorithm, the 1-dimentional projection signal is used to find a reduced set of 1-dimentional wavelet bases which leads the reduction of imaging time. In MR imaging, the projection signal is obtained by acquiring the MR signal with no y-directional gradient, *i.e.,*

$$p(x) = F_x^{-1}[S_F(m,0)] = \int_{-\infty}^{+\infty} \rho(x,y)\,dy. \qquad (8)$$

Using this 1-dimensional projection signal of the image, the MP algorithm is applied with the wavelet dictionary. Since the dictionary chosen in this paper consists of othonomal wavelets, subtracting the projection of one basis from the current residual has no effect on the projections of other bases. Therefore, a wavelet basis ischosen in the dictionary so that it best matches the signal of as

$$|\langle p(x), \omega_{\gamma_k}(x)\rangle| = \sup_{\gamma \in \{\Omega - \{\omega_{\gamma_n}(x) | 0 \le n < k\}\}} = |\langle p(x), \omega_\gamma(x)\rangle| \qquad (9)$$

where $\omega_{\gamma_k}(x)$ is the chosen wavelet basis at k-th iteration and it belongs to the wavelet dictionary, Ω. Note that Eq. (9) is taken from Eq. (5) with the dictionary of orthogonal bases in the MP algorithm. Finding wavelet bases in the dictionary is repeated from k=0 to N-1 so that N wavelet bases in the dictionary are sequenced according to the criterion of Eq. (9). Since the dictionary chosen in this paper consists of othonomal wavelets,

The image signal can be reconstructed sufficiently using some of the wavelet bases among ordered wavelet bases in the dictionary using MP. As an example, figure 1 shows a comparison between a 1-dimensional reconstruction by the MP algorithm with the Harr wavelet dictionary and those by Fourier transform and conventional wavelet transform. As shown in figure 1 (a), the signal reconstructed by MP algorithm with 64 iterations is almost same as the original signal (128 data points) shown in figure 1 (d) while the signals reconstructed by Fourier transform and wavelet transform with the same number of iterations have poor spatial resolution as shown in figure 1 (b) and (c). Therefore a signal can be restored with a few iterations using the MP algorithm.

2. 4. Encoding in MR-MP technique

To apply the ordered wavelet bases to the object, 90 degree-RF pulses are excited with a x-directional gradient as shown in Fig. 2. The RF pulses are generated so that their response to the object is the projection of wavelet bases onto the object. Generally RF pulses are designed by inverse Fourier transform of wavelet bases in case that a linear gradient applied. By superimposing wavelet bases onto the object, the MR signal integrated over the object can be written as

$$S(m.n) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \rho(x,y)\,\omega_{\gamma_m}(x)\,e^{-i\gamma G_y \delta n y}\,dxdy. \qquad (10)$$

Note that $\{\omega_{\gamma_m}(x)\}$ is the set of wavelet bases ordered using MP algorithm as mentioned in the previous section and the MR signal is sampled with the y-directional reading gradient as shown in Fig. 2. Since wavelet bases are chosen optimally so that they have best matched the object function, a reduced set of some MR signals encoded by optimal wavelet bases in the dictionary are sufficient to reconstruct $\rho(x, y)$. The following describe the reconstruction procedure in the MR-MP algorithm

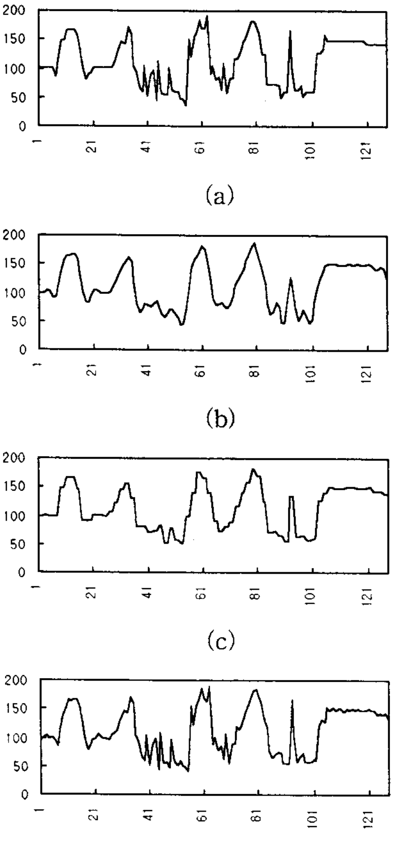

Figure 1. 1-dimensional signals reconstructed by (a) MP algorithm with 64 iterations, (b) Fourier transform with 64 encodings and (c) conventional wavelet transform with 64 encodings. The Harr wavelet dictionary is used. As is seen, (a) is more similar to the original while the signals reconstructed by Fourier transform and wavelet transform have poor spatial resolution. For reference, (d) shows the original signal.

2. 5. Reconstruction: Progressive reconstruction

After acquiring the MR signal at each encoding step, which can be represented as Eq. (10), a 1-dimensional Fourier transform in the reading direction (y-direction) is performed. Then one can obtain an inner product with respect to x, *i.e.*,

$$F_y^{-1}[S(m,n)] = \int_{-\infty}^{+\infty} \rho(x,y)\, \omega_{\gamma^m}(x)\, dx = \langle \rho(x,y), \omega_{\gamma^m}(x) \rangle_x \qquad (11)$$

where $F_y^{-1}[\cdot]$ is the one dimensional inverse Fourier transform in the y-direction. The reconstruction is performed as

$$\rho_{k+1}(x,y) = \rho_k(x,y) + \langle \rho(x,y), \omega_{\gamma_k}(x) \rangle_x \omega_{\gamma_k}(x) \quad (12)$$

where $\rho_{k+1}(x, y)$ is newly updated image with $\rho_{k+1}(x, y) = 0$. As indicated in Eq. (12), computational requirement at each encoding step is as simple as a multiplication and an addition.

Since a basis function at each iteration is selected so that it matches best with the residual signal the resolution of the reconstructed signal or image is progressively improved with each iteration. Therefore one can achieve a progressive reconstruction at each encoding step, *i.e.*, a low resolution image can be obtained with a few iterations and the resolution of the image can be improved with additional encoding steps. Further, one can stop the iterations if the image quality is satisfactory, which leads to a possibly reduced imaging time.

Figure 2 shows a pulse sequence for the matching pursuit algorithm applied to MR imaging. As is seen, the one dimensional projection signal is obtained using a navigator echo which is acquired just after the 90 degree-RF pulse. A reading gradient was applied in the x-direction to obtain the projection signal, but it is not used for 2D imaging. As imentioned previously, 90 degree-RF pulse shapes are determined by bases in the wavelet dictionary, which are arranged in order by the MP criterion of Eq. (9) applied to the 1-dimensional projection signal. Consequently an image is reconstructed from poor resolution to fine resolution with increasing encoding steps. In the following section, we will demonstrate the matching pursuit algorithm in MR imaging and its usefulness for the reduction of imaging time by computer simulations.

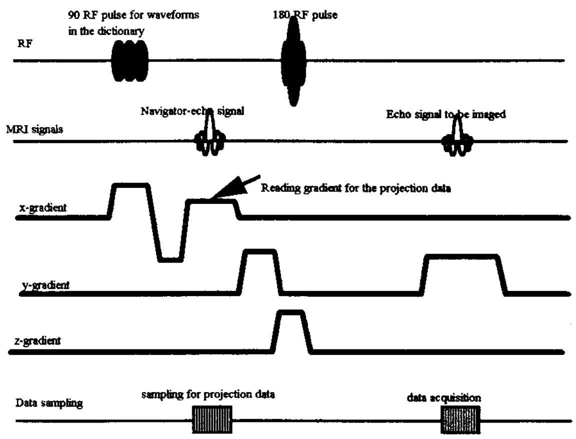

Figure 2. A pulse sequence for matching pursuit algorithm with wavelet dictionary applied to MR imaging.

3. Simulations

To verify the usefulness of the MP algorithm in MR imaging, specifically for reduction of imaging time, computer simulations were performed. A Shepp phantom which had a 128x128 matrix size was used. First, the phantom image was projected along they-direction and the 1D projection signal was obtained. Based on the criterion in Eq. (11), waveforms in Harr wavelet dictionary were selected and sequenced. Then the phantom image was decomposed according to the sequenced waveforms. Finally image reconstruction was performed with selected waveforms in the dictionary. Figure 3 shows a graph which represents the power of the residual image versus the number of iterations. As is seen, to reconstruct the image completely, the proposed algorithm requires fewer iterations or coding steps than Fourier and wavelet transform MR imaging methods.

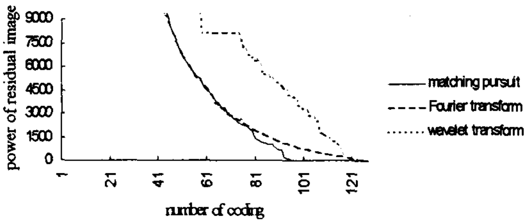

Figure. 3 The power of residual images vs. the number of iterations in case of a phantom image.

As is seen, residual power is negligible after 95 iterations. Therefore the MP algorithm leads to reduction of iterations or encodings thereby reducing imaging time.

4. Conclusions

As a conclusion, the matching pursuit algorithm is applied to MR imaging with a wavelet dictionary. It is found that the proposed technique leads to the reduction of imaging time. Further a progressive reconstruction or hierarchical reconstruction algorithm is achieved using the MP-MR technique. In this paper, the matching pursuit algorithm is modified for MR imaging. Computer simulations were performed with phantom and human images. Results obtained by computer simulations verified that the image could be reconstructed completely with reduced iterations. Therefore, the proposed algorithm is believed to be useful for reducing imaging time. Also progressive reconstruction can be achieved so that it provides the quick recognition of the object during data acquisition. Further, the MP-MR technique proposed in this paper can be applied to motion compensation thanks to its adaptive encoding.

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