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A New Hybrid Method for Flow-Dominated Transport

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INTRODUCTION

Recently, since one of the most important societal problems facing us today is the growing incidence of the contamination of coastal sea from variety of sources, several distinct numerical improvements have been made and applying the transport models to flow-dominated transport area. Application of Eulerian numerical models to the solution of sharp-front problems often results in oscillations, phase errors, peak depression, and/or numerical dispersion, unless very fine temporal and spatial steps are adopted. The representative Eulerian scheme is Quadratic Upstream Interpolation for Convective Kinematic with Estimated Streaming Terms (QUICKEST) scheme first presented by Leonard (1979). As the second generation, the mixing Eulerian-Lagrangian method has been proven to provide high accuracy with reduction of oscillations and numerical dispersion. However, its accuracy depends on interpolating algorithm used. This type of method uses a split operator approach in which the advection term is treated by a Lagrangian approach along characteristic paths and the other diffusion term is solvd on Eulerian grids. The Lagrangian approach to advection usually takes either Forward Particle Tracking Method (FPTM) (Garder et al., 1964; Dimou and Adams, 1991) or a single-step Reverse Particle Tracking Method (RPTM) (Holly and Preissmann, 1977; Jun and Lee, 1994; Seo and Kim, 1995): The RPTM method requires the interpolation to evaluate the unknown value between grid points by using the known values of surrounding grid points; and the FPTM requires the four consecutive steps which are somewhat complicated; tracking the concentration front, single-step forward tracking, single-step reverse tracking and finite difference/element approximation.

In this study a new hybrid method is developed for solving flow-dominated transport problems accurately and effectively. The method takes a Lagrangian viewpoint for advection step, introducing moving particles to track their assigned concentration continuously forward. At each time, the concentration of particles is re-assigned through the diffusion step and a new particle is set up at the center of each grid where the concentration is newly diffused. If there is no diffusion effect, the conventional Lagrangian random-walk model requires the numerical implementation of a small number of particles so that it becomes quite economic and effective. However, with diffusion effect, the method requires a large enough number of particles to simulate the diffusion process by random walk (Lee, 1994; Lee and Wang, 1994). That makes the Lagrangian method quite time-consuming and labourous. Therefore, the present method solves the diffusion step on a fixed Eulerian grid but particles move continously forward. It is assumed in the diffusion step as if each particle poses at a cell center. Differently from the FPTM which requires the four consecutive steps, the present method requires only the two steps; forward tracking for advection and finite difference approximation for diffusion.

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FORMULATION

The partial differential equation describing advection and dispersion in two dimensions is written in conservation form as

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=K\frac{\partial^2 C}{\partial x^2}+K\frac{\partial^2 C}{\partial y^2} \tag{1}$$

where, K is the dispersion coefficients and (u, v) are the x, y components of depth-averaged velocity vector. The advection-dispersion equation is solved using split-operator approach which is based on the recognition that the physical phenomena of pollutant transport are represented by superimposing two individual operations, advection and diffusion. Therefore, Eq. (1) can be decoupled into the two elementary operations and solved separately and alternately for each small time increment.

SOLUTION STRATEGY

A new hybrid method proposed here is useful for flow-dominated transport problems. This method takes the switching approach between advection and diffusion prosesses, keeping the concentration particles continuously moving forward rather than a sigle-reverse particle tracking. In this procedure, a big assumption is involved that the variation of concentration within a grid cell is negligible in estimating diffusive effect as similar as the basic concept of finite descrete schemes. However, the results are quite satisfactory since the assumption is adopted in the diffusive process which works the values for smoothing. The following is the computing procedure of this approach :

1) Move particles forward according to pure advection
2) Find the grid mesh where each particle poses
3) Assign the concentration of particle at the grid center
4) Do diffusion step by using Implicit FDM
5) Re-assign the values to corresponding particles
6) Set up a new particle at the node where concentration is newly diffused.

Without diffusion, step 1 is only required and without advection, step 4 is only required.

NUMERICAL EXAMPLES

The numerical model is tested for an instantaneously loaded contaminant under a uniform flow. The one-dimensional analytic solution of Eq. (1) is given by

$$C(x,t)=\frac{M}{\sqrt{2\pi K\Delta t}}\exp\frac{(x-ut)^2}{\sqrt{2\pi K\Delta t}} \tag{2}$$

A point source of M=3,000kg/ m^2 having a deviation of zero is initially given and transported downward for a time of 12,800sec by a uniform current of 0.5m/s. The node spacing and time step chosen are 50m and 100sec, respectively. Figures 1 and 2 show the numerical solutions by the present method is very close to the analytic solution.

The present method is easily applied to two dimensions. The two-dimensional analytic solution is given by

$$C(x,y,t)=\frac{M}{4\pi K\Delta t}\exp\frac{(x-ut)^2+y^2}{4\pi K\Delta t} \tag{3}$$

The initial profile has a point source of M=1,000,000kg/ m^3 and is transported in the x direction for 20,000sec by a uniform current of 0.1m/s. The node spacing and time step chosen are 100m and 100sec, respectively. The computational times required by the various schemes for 2D example are shown on Table 1 in which the values indicate ratios of computational times to the present method (2 m^2/s). The computational effort required by five-point Hermite polynomial method is more than four times than that required by the present method. The ratio for random-walk method is resulted from 100,000 particles released into flow fields. Figures 3, 4 and 5 show the results by the five-point method, random-walk method (100,000 particles) and the present method, respectively. It can be seen from Fig. 5 the present method is very consistent with analytical solutions.

Table 1. Coparison of computational time

Method Types / K (m^2/s)	Five-point method	Present method	Random walk method
2	4.4	1	5.1
10	7.5	1.05	5.1

(a) (b)

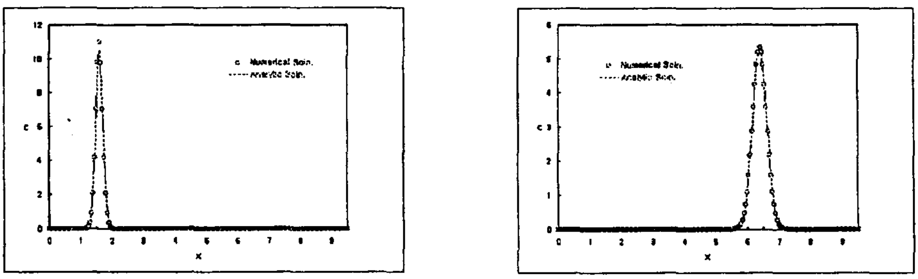

Fig. 1. Comparison between analytic solutions and results by the present study (K= 2 m^2/s, and (a) t=3200sec, (b) t=12800sec)

(a) (b)

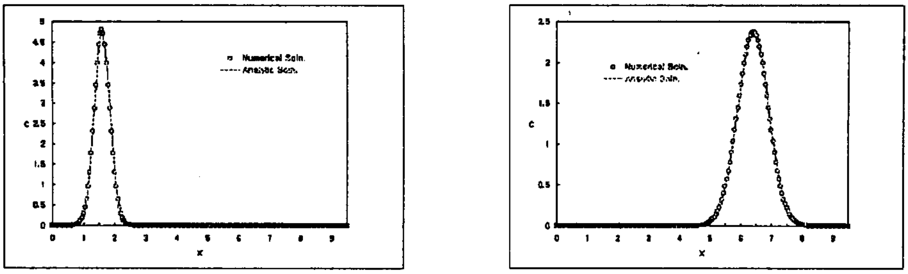

Fig. 2. Comparison between analytic solutions and results by the present study (K= 10 m^2/s, and (a) t=3200sec, (b) t=12800sec)

(a) (b)

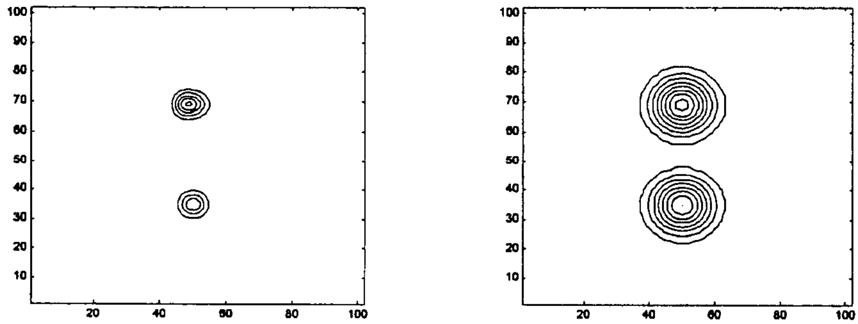

Fig. 3. Comparison between analytic solutions (lower) and results by five-point Hermite polynomial method (upper) (t=20000sec, and (a) K= 2 m^2/s (b) K= 10 m^2/s)

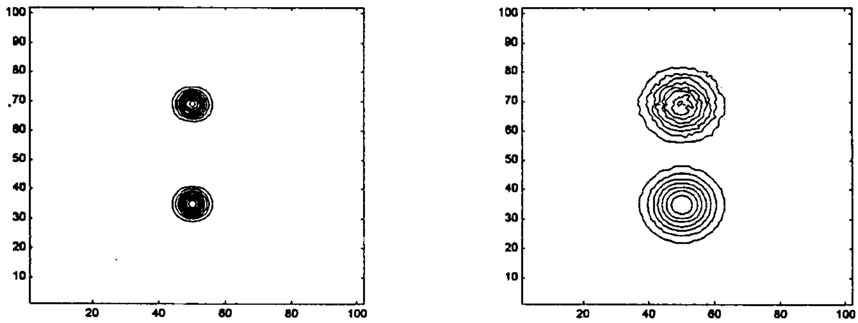

Fig. 4. Comparison between analytic solutions (lower) and results by the random walk method (upper) (t=20000sec, and (a) K= 2 m^2/s (b) K= 10 m^2/s)

(a) (b)

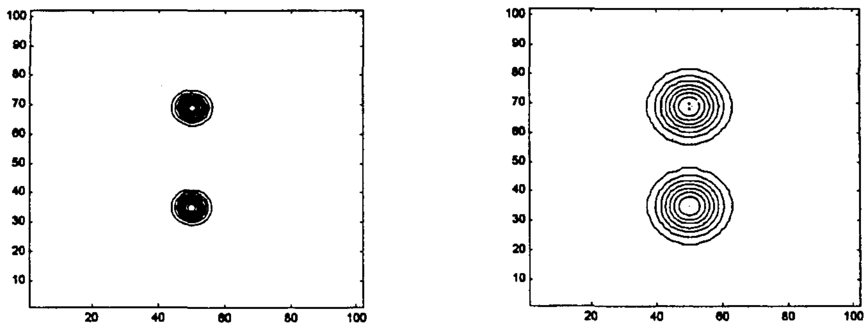

Fig. 5. Comparison between analytic solutions (lower) and results by the present method (upper) (t=20000sec, and (a) K= 2 m^2/s (b) K= 10 m^2/s)

CONCLUSION

The hybrid method proposed here requires only the two steps without need to use the interpolation; forward tracking for advection and finite difference approximation for diffusion. The present method has a high accuracy and fast computation compared with other schemes such as five-point Hermite polynomial method and random-walk Lagrangian method. Based on the results, it is concluded that the present method is easily applied to various flow-dominated fields and very effective in calculating advectiive diffusion not only in one-dimensional cases but also in two-dimensional ones. For the initial point source, the results show the present method works very well.

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