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Inconsistency in the Average Hydraulic Models Used in Nuclear Reactor Design and Safety Analysis

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Abstract

One of important inconsistencies in the six-equation model predictions has been found to be the force experienced by a single bubble placed in a convergent stream of liquid. Various sets of governing equations yield different amount of forces to hold the bubble stationary in a convergent nozzle. By using the first order potential flow theory, it is found that the six-equation model can not be used to estimate the force experienced by a deformed bubble. The theoretical value of the particle stress of a bubble in a convergent nozzle flow has been found to be a function of the Weber number when bubble distortion is allowed. This force has been calculated by using different sets of governing equations and compared with the theoretical value. It is suggested in this study that the bubble size distribution function can be used to remove the presented inconsistency by relating the interfacial variables with different moments of the bubble size distribution function. This study also shows that the inconsistencies in the thermal-hydraulic governing equation can be removed by mechanistic modeling of the phasic interface.

1. Introduction

Average thermal-hydraulic conservation equations have been extensively applied in nuclear steam supply system safety and design analysis. Among many important modeling efforts, systematic constitution of the average governing equation is the most challenging work, since the closure laws are strongly influenced by the complicated phasic interface geometry (i.e., the flow regimes). If one recalls that constituting average conservation equations is the process of re-introducing thermomechanical microscale information lost during the average process, the understanding of fundamental two-phase flow phenomena is essential. Obviously, the eventual goal of thermal-hydraulic modeling efforts is the development of a mechanistic two-fluid model which can be consistently applied to various flow phenomena. Moreover, the model should work within different flow boundaries in three-dimensional coordinate systems without violating the axiom of thermodynamics. Unfortunately, however, many modeling equations violate the thermodynamic laws due to incorrect constitution (Wallis, 1990, Arnold et al., 1990).

In this study, a review of the thermal-hydraulic models used in contemporary nuclear reactor system analysis computer codes is given. Some characteristics and inconsistencies of different sets of governing equations are identified. The external force holding a single bubble stationary in a convergent stream of liquid is evaluated using both the potential flow theory and the six-equation models used in different nuclear reactor system analysis codes.

2. Homogeneous Mixture Model and Drift-flux Correction

The drift-flux formulation has been conveniently used to describe vapor drifting motion (i.e., the local slip) in a pool of liquid. Through an extensive developmental effort (Ishii, 1977) succeeding the original work (Zuber & Findlay, 1965), the lateral void distribution (i.e., the profile slip) and separated flow geometry effects have been consistently incorporated into the drift-flux framework. The simple and clear idea of the drift-flux model has been a strong attraction for many users. In fact, the accuracy of this model for the bubbly two-phase flow is so high that the current state-of-the-art system analysis code (e.g., RELAP5/MOD3) employed the drift-flux correction to the six-equation model (Putney, 1989).

One of major problems in using the drift-flux formulation is that the drift-flux parameters for a horizontal/inclined flow are not completely modeled yet. For dispersed flows in a horizontal pipe, the local slip is known to be small, through it can be large for separated flows (e.g., stratified flows). Currently, the local slip parameter for slightly inclined pipe flows is frequently modified by multiplying the angle of inclination to the vertical flow slip value without enough validation. Moreover, the dynamic motion of the two-phase flow can not completely be simulated by the drift-flux model since the drift-flux model is based upon the so-called kinematic motion of the flow. There are, however, the good features in the homogeneous mixture model with the drift-flux correction, which are: 1) the homogeneous mixture model is mathematically well-posed (Stuhmiller, 1977) and 2) the equation set is simple and can be easily constituted.

3. Six-Equation Model

Mass, momentum and (total) energy conservation for each phase is modeled and solved with the equation of state in the six-equation model. Since each phase is treated separately, the averaged mass, momentum and energy exchanges between the phases need to be constituted. This requirement generally includes an in-depth understanding of the interfacial phenomena and the interface geometry which depends on the two-phase flow regimes. Among the interfacial momentum exchange terms, the so-called drag force has been developed in an algebraic form and successfully used in the reactor system design analysis codes. On the other hand, the non-drag forces (i.e., the momentum exchanges other than the interfacial drag force) have not been thoroughly understood, except for the two-phase virtual mass force.

Inconsistency in the six-equation model has been discussed by Geurst(1986) and Wallis(1990). When a bubble is placed in a converging nozzle, as shown in Fig.1, the net force needed to make the bubble stationary (F_{ex}) can be found by setting $\alpha_d = 0$ and $v_d = 0$ in the average two-fluid momentum equations, for example, those used in RELAP5/MOD3 (Carson et al., 1990);

$$\rho_c v_c \frac{\partial v_c}{\partial z} = -\frac{\partial p_c}{\partial z} \tag{1a}$$

$$0 = -\frac{\partial p_c}{\partial z} + 2C_{vm}\rho_c v_c \frac{\partial v_c}{\partial z} - F_{ex} \tag{1b}$$

where the wall friction is neglected and the liquid phase interfacial pressure difference is used. Solving Eq.(1) for F_{ex}, we obtain:

$$F_{ex}=\xi\rho_c v_c\frac{dv_c}{dz}=-\xi\frac{dp}{dz} \qquad (2)$$

where $\xi=2C_p+1$.

Wallis (1990) pointed out that ξ =3/2 is the correct value for a solid sphere which is the classical result obtained by Taylor (1928). The two-fluid modeling equations used in RELAP5/MOD3, however, yields ξ =1. It is interesting to note that we can obtain ξ =3/2 from the equations used in CATHARE2 (Barre, 1992) when the virtual mass effect is included.

In general, a bubble is deformable and the shape of a bubble in a converging nozzle may not be spherical due to the pressure distribution around it. Therefore, it is worthwhile to evaluate F_{ex} for a deformable sphere in a converging stream of liquid. Since the potential flow approximation is good for water flows around a bubble, the dimensionless velocity potential for a converging stream around a bubble of radius a can be used to solve the flow field around the bubble. By allowing bubble shape deformation, such as:

$$R^*(\theta)=1+a_1P_1(\cos(\theta))+a_2P_2(\cos(\theta)), \qquad (3)$$

we may solve the first order approximated equation with the following force balance on the bubble surface:

$$p^*_{di}-p^*_{ci}=\left(\frac{1}{R^*_1}+\frac{1}{R^*_2}\right)We^{-2} \qquad (4)$$

where R^*_1 and R^*_2 are dimensionless principal radii of the bubble surface curvature.

The bubble shape solution has been plotted in Fig.2 for a converging nozzle of about 10° (i.e., J=0.134) for four different values of Weber numbers. As can be seen, the bubble shape is a strong function of the Weber number. It should be noted that when the Weber number is increased enough, the solution becomes multi-valued. As shown in Fig.3, the bubble deforms more in a 45° converging nozzle (i.e., J=0.5) than in a 10° nozzle for the same Weber number.

The force experienced by the unit volume of the bubble can be evaluated by:

$$F_{ex}=\frac{F_d}{\frac{4}{3}\pi a^3}=\int\int_{\Omega(R)}p_{di}\,d\Omega(R)=\xi_b\rho_c\frac{dv_c}{dz}=-\xi_b\frac{dp}{dz} \qquad (5)$$

where:

$$\xi_b=\frac{22}{23}+\frac{50}{23}We^{-2} \qquad (6)$$

Eq.(6) has been plotted against the Weber number in Figure 4. As can be seen, ξ_b depends on the Weber number for a deformable bubble. When the inertia of the liquid is much greater than the surface tension force (i.e., We >> 1), ξ_b is very close to 1.0, which is the value obtained from the average-model used in RELAP5/MOD3.

The point here is the interpretation of the phasic pressures and inertial forces of the continuous phase around the bubble. Since the basis of the average six-equation model is that the local interfacial pressures are averaged over a large amount of the phasic interfaces (or large number of samples in the phasic interface ensemble), the interface-averaged variables take forms depending on the geometrical shape of the phasic interface, unless the phasic interface is dynamically modeled. It should be noted that this is significant when the model is applied to separated flow regimes of large phasic pressure differences, for example, in the horizontally stratified two-phase flow. As a matter of fact, this problem is coupled with the dynamic flow regime model which will be discussed in next section.

4. Dynamic Flow Regime Model

In the six-equation model, geometric information of the phasic interface (i.e., the flow regimes and those boundaries) is supplied in the form of correlations. This type of approach often yields unrealistic results since, in general, the correlations between various regimes do not behave in a consistent and smooth manner. Currently, significantly simplified assumptions must be made in order to cover all anticipated conditions. Moreover, in order to provide smooth transitions between correlations, arbitrary criteria for correlation selection and pseudo correlations (e.g., artificial fitting) have often been utilized in current codes to link the correlations together. To improve this situation, one may need to develop a continuously evolving phasic interface, say, as the global void fraction increases.

It can be proposed that the bubble size distribution function could be an excellent tool for relating the interface parameters with the global variables. Let $f(v, \mathbf{r}, t)$ be the bubble size distribution function such that $f(v, \mathbf{r}, t)dv$ is the number of bubbles per unit volume, at vectorial position $\mathbf{r}$ and time t, having volumes between v and $v+dv$. Significantly, the bubble number density (N'''), mean radius ($\overline{R_b}'''$), interfacial area density (A_i'''), and local void fraction (α_d), are related to the zeroth, 1/3, 2/3 and the first moments of the distribution function, and are given by:

$$N'''(\mathbf{r}, t) = \int_0^\infty f(v, \mathbf{r}, t)dv \quad (7)$$

$$\overline{R_b}'''(\mathbf{r}, t) = \frac{1}{N''(\mathbf{r}, t)} \left(\frac{3}{4\pi}\right)^{1/3} \int_0^\infty v^{1/3} f(v, \mathbf{r}, t)dv \quad (8)$$

$$A_i''' = (36\pi) \int_0^\infty v^{2/3} f(v, \mathbf{r}, t)dv \quad (9)$$

and

$$\alpha_d(\mathbf{r}, t) = \int_0^\infty v\, f(v, \mathbf{r}, t)dv \quad (10)$$

The equation describing the bubble size distribution function can be obtained by performing the balance of the loss and gain due to the breakup and formation of bubbles or bubble clusters, and is in general, given by:

$$\frac{\partial f(v, \mathbf{r}, t)}{\partial t} + \nabla \cdot [f(v, \mathbf{r}, t)\ \mathbf{u}_g] = B(v, \mathbf{r}, t) + C(v, \mathbf{r}, t) + S(v, \mathbf{r}, t) \quad (11)$$

where $B(v, \mathbf{r}, t)$, $C(v, \mathbf{r}, t)$ and $S(v, \mathbf{r}, t)$ are the total rate of breakup, formation and a source term per unit volume, respectively, and $\mathbf{u}_g$ is the average velocity of the bubbles. Currently, the solution of Eq.(11) can be obtained for very limited flow conditions since the bubble breakup and formation mechanisms have not been completely understood.

From Eqs.(7) through (11), the distribution of bubble size, we find that $f(v, \mathbf{r}, t)$ is the key parameter. However, there hasn't been found a reliable means of measuring the bubble size distribution directly. The bubble chord length distribution has been measured by a conventional impedance probe but no technique has been proved to be valid to convert the chord length distribution data to the bubble size distribution yet. Therefore, a more sophisticated measurement technique must be developed to obtain the bubble size distribution function. Once the bubble size distribution data is obtained, the modeled breakup, formation and source terms (Luo et al., 1996) in Eq.(11) could be validated.

5. Summary and Conclusion

A critical review of the thermal-hydraulic codes used in the contemporary nuclear reactor thermal-hydraulic system analysis is given, which includes the homogeneous mixture model with the drift-flux and the six-equation model (i.e., the separate flow model). The bubble shape solution in a convergent nozzle shows that the controlling parameters are the Weber number and the slope of the nozzle. Based upon the first order approximation result, a bubble placed in a convergent stream of liquid is deformed and the average force experienced by the bubble is found to depend on the Weber number, while a solid particle develops a fixed amount of particle stress. To resolve this type of dilemma in the six-equation model, the bubble size distribution function has been proposed as a fundamental tool for the dynamic interfacial model.

This study shows that it is important to identify inconsistencies in the constituted governing equations for mechanistic descriptions of two-phase flows. It will be necessary to develop a dynamic model for the phasic interface and perform experimental validations in order to achieve a realistic simulation of multi-dimensional thermal-hydraulic phenomena.

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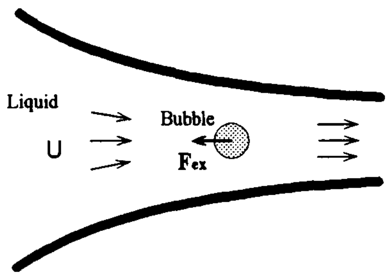

Figure 1. A Bubble Placed in a Converging Nozzle

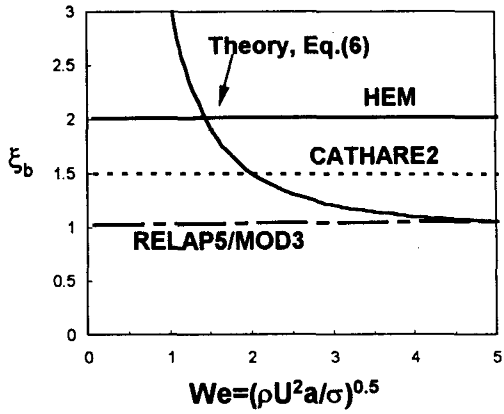

Figure 3. Values of ξ_b from Different Models

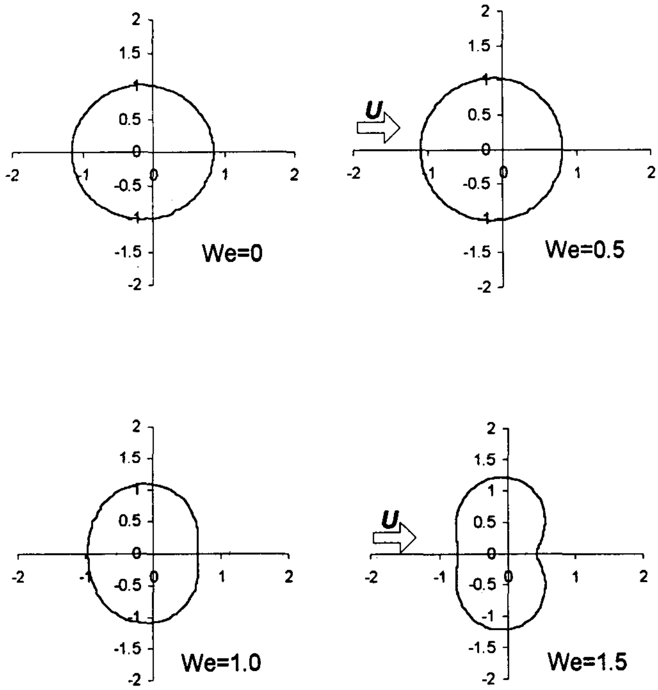

Figure 2. Bubble Shape Solution for 10 Degree Converging Nozzle