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Maximum Entropy Algorithm and its Implementation for the Neutral Beam Profile Measurement

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Abstract

A tomography algorithm to maximize the entropy of image using Lagrangian multiplier technique and conjugate gradient method has been designed for the measurement of 2D spatial distribution of intense neutral beams of KSTAR NBI(Korea Superconducting Tokamak Advanced Research Neutral Beam Injector), which is now being designed. A possible detection system was assumed and a numerical simulation has been implemented to test the reconstruction quality of given beam profiles. This algorithm has the good applicability for sparse projection data and thus, can be used for the neutral beam tomography.

1. Introduction

In neutral beam injectors, the measurement of 2D spatial distribution of beam power may be useful to study factors which lead to maximize power transmission to the target and minimize interception on duct walls. If they have low power and short pulse length, such a measurement can be done calorimetrically or electrically with probes that are injected into the beam. However, at the KSTAR NBI which is now on the stage of beginning, the requirements for the beam power and pulse length capability are about 8MW and 300sec[1]. So it is necessary to develop indirect methods of beam diagnostics. In the JT-60[2] and the JET NBI[3], it was done by detecting Hα radiation from the beam.

We can get projection data from measuring Hα radiation with various detectors e.g. CCDs, PM tubes etc. but they are highly underdetermined because of engineering constraints such as space reservations and money. So we need an algorithm to reconstruct 2D beam profile distribution with these limited projection data.

Maximum entropy is a widely used technique in such a situation[4]. We developed the maximum entropy based algorithm using Lagrangian multipliers and conjugate direction iterative method. This paper will represent it and its numerical test for the neutral beam profile monitoring.

2. Reconstruction Algorithm

A. Maximum entropy method

The problem of image reconstruction from projections is a sort of inverse problem[5] like

$$\Phi f - h = \theta \qquad (1)$$

where Φ is a system matrix

f is a image vector

h is a projection data vector

θ is a null vector

But for the inverse problem with incomplete data, there can be many different feasible image vectors. Therefore it is a matter of practical necessity to select a single image from the feasible set. The main idea of maximum entropy method is to choose the single feasible image which has the greatest configurational entropy; $S(f) = -\sum_i \frac{f_i}{\sum f_i} \ln(\frac{f_i}{\sum f_i})$. We choose the image with maximum entropy not because it is more likely but because it is maximally noncommittal[4] about unmeasured parameters.

B. Algorithm

To find the image with maximum entropy among feasible images can be regarded as a constrained optimization problem which maximizes

$$S(f) = -\sum_{i=0}^{N_m-1} \frac{f_i}{I_o} \ln(\frac{f_i}{I_o}) \tag{2}$$

where $I_o = \sum_{i=0}^{N_m-1} f_i$; total emission

N_m is the number of meshes

with constraints

$$G(f) = \Phi f - h = \theta \tag{3}$$

Using lagrangian multipliers[5], we have

$$L(f,L) = S(f) + < G(f), L > = -\sum_{i=0}^{N_m-1} \frac{f_i}{I_o} \ln(\frac{f_i}{I_o}) - \sum_{j=0}^{J-1} \Lambda_j (h_j - \sum_{i=0}^{N_m-1} \Phi_{ji} f_i) \tag{4}$$

where J is the total number of projection data

from $\frac{\partial L}{\partial f_i} = 0$, we have

$$f_i = (\frac{1}{I_o} e)^{-1} \prod_{j=0}^{J-1} \exp(I_o \Lambda_j \Phi_{ji}) \tag{5}$$

and from (3) and (5), we have the system of nonlinear equations

$$G_j = \sum_{i=0}^{N_m-1} \Phi_{ji} (\frac{1}{I_o} e)^{-1} \prod_{k=0}^{J-1} \exp(I_o \Lambda_k \Phi_{ki}) - h_j = 0, \qquad j=0, 1, 2, \ldots, J-1 \tag{6}$$

This system of nonlinear equations can be solved by minimizing the function[6]

$$F = \sum_{j=0}^{J-1} G_j^2 \tag{7}$$

The function minimization can be done iteratively by conjugate direction line minimization method[7].

3. Numerical Experiment

Numerical Experiment was done as following procedure.

(1) Possible Profile Measurement System Modeling & System Matrix Calculation

(2) Numerical Phantom and its Projection Data

(3) Use the MEM algorithm to reconstruct the original numerical phantom

A possible profile measurement system is shown in Fig 1.The CCD camera is chosen as Hα radiation detector because it's easy to get and cheap. The reconstruction area is the region of our interest and the region is divided by meshes that decide the resolution. Through this modeling, we can calculate the weighting factor of each mesh to each CCD pixel. We made the program that generate the system matrix Φ if the geometry parameters are given. Here we gave

R=500mm, W=230mm, H=290mm, W_m=23mm, H_m=29mm, C=30°, P=3°, N=32

Usual neutral beam profile is smooth that we designed the numerical phantom like Fig 3(a) and its projection data generated by the system matrix with no noise assumption are shown in Fig 2.

Now we get the information about system matrix Φ and projection data h, but we don't know total emission I_o which is needed as a parameter of MEM algorithm as explained in 2.B. So it must be estimated with h and Φ. Here, it is estimated through global averaging scheme[8], which is

$$I_o = N_m \sum_{j=0}^{J-1} h_j / \sum_{j=0}^{J-1} \sum_{i=0}^{N_m-1} \Phi_{ji} \qquad (8)$$

With these three input parameters, the minimization of Eq.(7) was performed iteratively. Initially, the $\{\Lambda_j\}$ were set equal and the algorithm then iterated $\{\Lambda_j\}$ toward the target solution with conjugate gradient search directions. In Fig 3(b), we can see the result of reconstruction.

4. Conclusion

As you can see in Fig 3(b), the reconstructed image seems almost the same with the original numerical phantom. So it can be possible to apply this MEM algorithm to the neutral beam profile measurement.

Actual projection data may be very noisy and it is necessary to simulate MEM in such a situation. And we must optimize the number of cameras and reconstruction resolution. The program modification to reduce iteration time is also needed.

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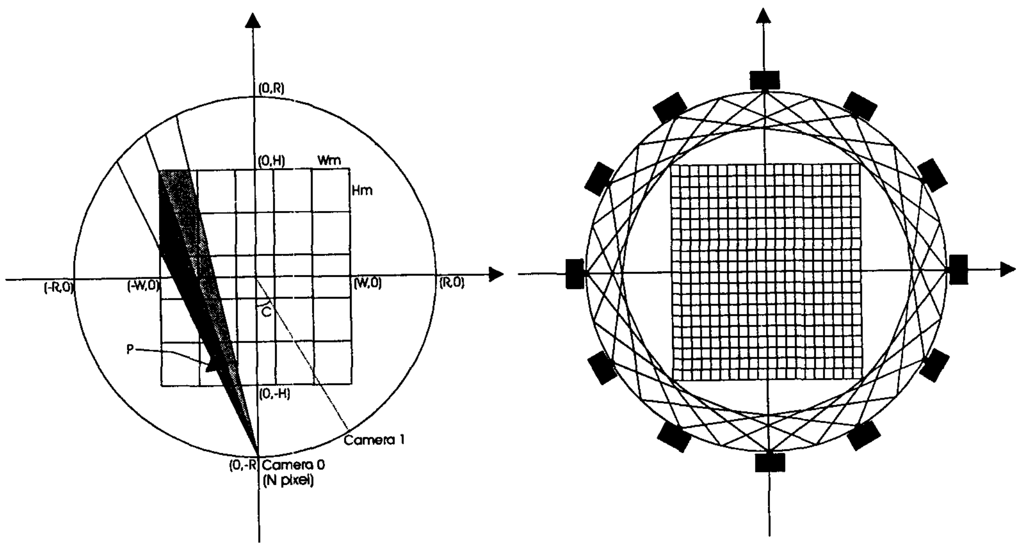


(a) (b)

Fig 1. (a) The modeling of a possible detection system. The detector is assumed as a CCD camera and Hα can be collected through a pinhole with a shape of fan beam. Cameras can be located at the distance R from the center around the beam tube. P is the angle that each CCD pixel can span and each camera has N pixels. C is the angle between each camera and equal for all cameras. The reconstruction area is 2W wide and 2H high. The mesh size is W_m wide and H_m high. (b) R=500mm, W=230mm, H=290mm, W_m=23mm, H_m=29mm, C=30°, P=3°, N=32

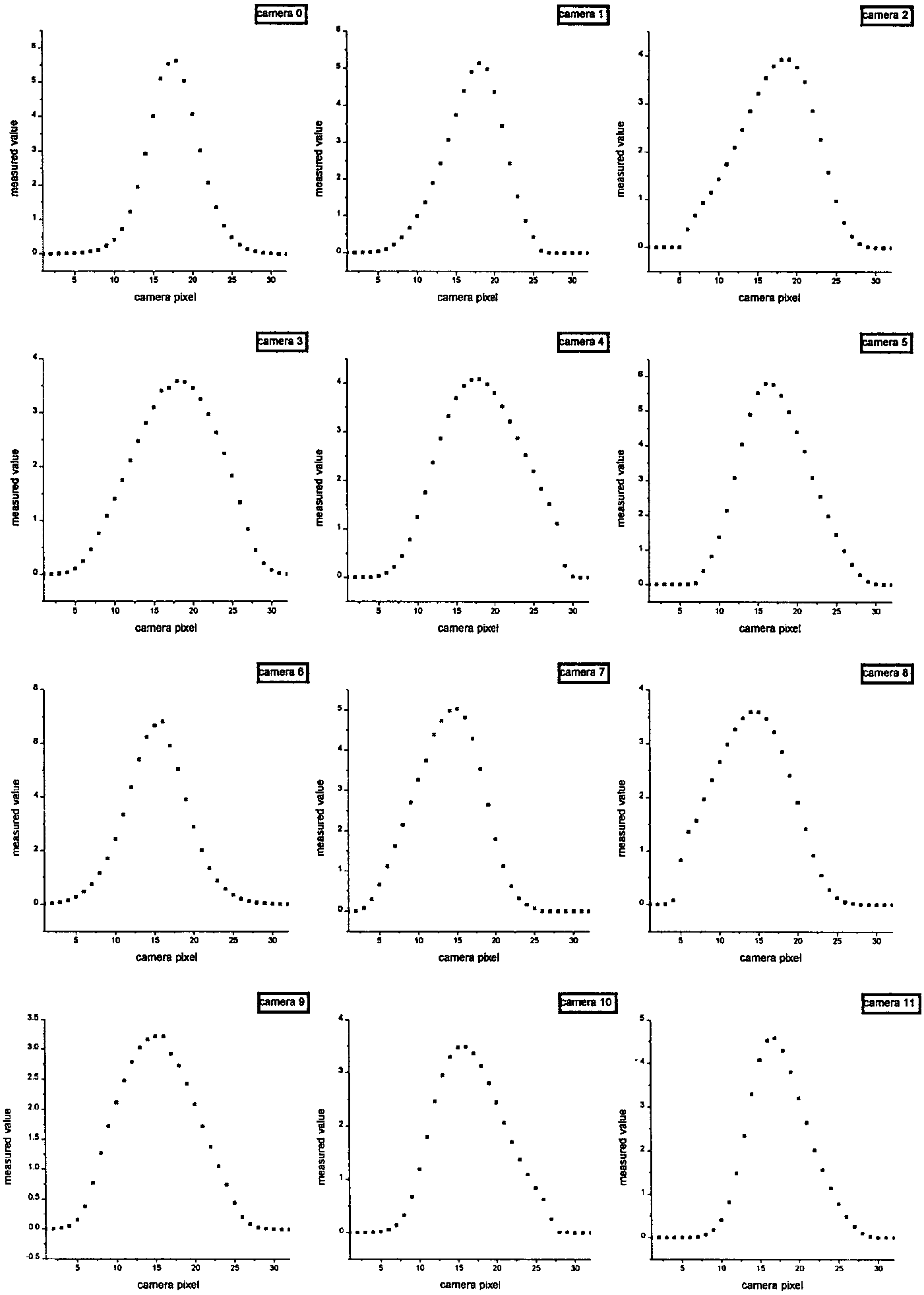
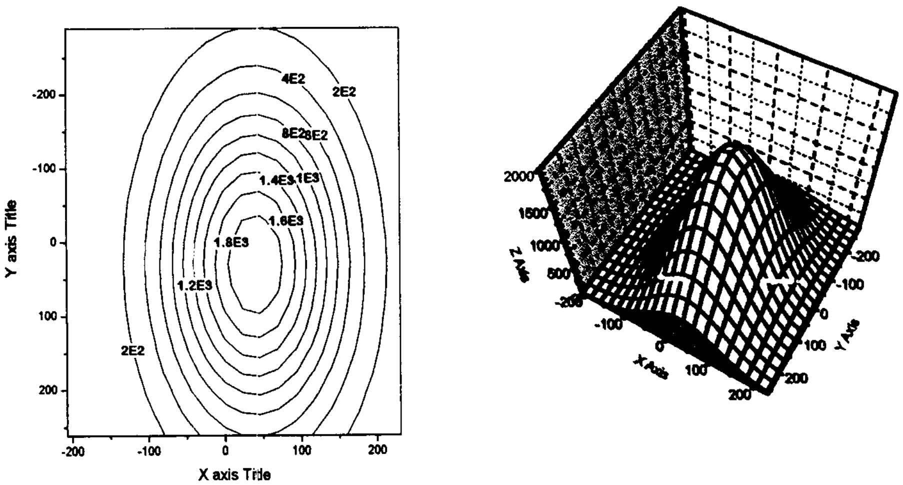
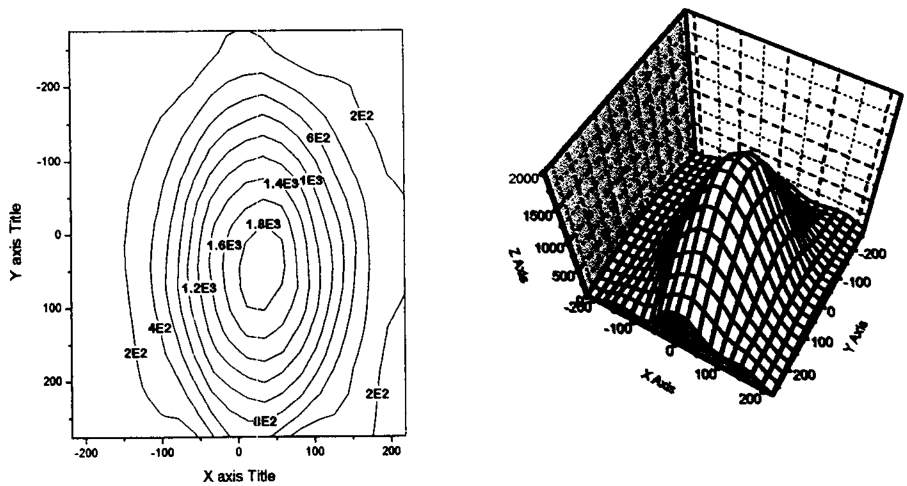


Fig 2. The projected data. The numerical phantom in Fig 3(a) was projected to each camera of Fig 1(b).



(a)



(b)

Fig 3. (a) A numerical phantom and (b) Its reconstructed image using MEM under the geometry of Fig 1(b).