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Nuclear Data Compression and Reconstruction via Discrete Wavelet Transform

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Abstract

Discrete Wavelet Transforms (DWTs) are recent mathematics, and begin to be used in various fields. The wavelet transform can be used to compress the signal and image due to its inherent properties. We applied the wavelet transform compression and reconstruction to the neutron cross section data. Numerical tests illustrate that the signal compression using wavelet is very effective to reduce the data saving spaces.

I. Introduction

After the basic concepts of wavelet theory were put forth in a paper by Gabor in 1945, the wavelets have been a very popular topic of conversations in so many different fields of science and engineering such as sound analysis, detection of edges and singularities and solving differential equations. [1,2,3,4] Also some people use wavelets to decompose and reconstruct of physical data.

In this study we concentrate on the compressing and reconstructing of nuclear data by using some Discrete Wavelet Transforms (DWTs).

II. Overview of Wavelets

A wavelet, in the sense of the Discrete Wavelet Transform (DWT), is an orthogonal function that can be applied to a finite group of data. Functionally it is very similar to the Discrete Fourier Transform. Wavelet bases are defined by dilation and translation operation such as: [5]

$$\Phi(x) = \sum_{k=0}^{M-1} c_k \Phi(2x-k), \tag{1}$$

for some $\Phi(x) \in L^2(R)$. Where c_k's are wavelet coefficients and M is referred to as the order of the wavelet.

Generally the area under the wavelet function over the space should be unity, which requires that

$$\sum_k c_k = 2. \tag{2}$$

Because equation (1) is orthogonal to its translations only, we need an equation, which is orthogonal to its dilations for DWT orthogonality. Such a function Ψ exists and is given by the following equation

$$\Psi(x) = \sum_k (-1)^k c_{1-k} \Phi(2x-k). \tag{3}$$

For normalization, equations

$$\sum_k c_k c_{k-2m} = 2\delta_{0m}, \tag{4}$$

$$\sum_k (-1)^k c_{1-k} c_{k-2m} = 0, \tag{5}$$

are required.

Within each family of wavelets are wavelet subclasses distinguished by the number of coefficients and by the level of iteration. Wavelets are classified within a family most often by the number of vanishing moments, which is an extra set of mathematical relationships for the coefficients that must be satisfied. Several kinds of wavelet bases used in this study are shown in Figure 1 and these coefficients (c_k's) are summarized in Table I.

Suppose that a finite sequence or input values s_k^0, $k=1,2,\ldots,K$, where K is a power of two are given. Then the discrete wavelet transform can be written as:

$$\begin{aligned} s_k^j &= \sum_{n=0}^{2N-1} c_n s_{n+2k-1}^{j-1}, \\ d_k^j &= \sum_{n=0}^{2N-1} (-1)^k c_{2N-1-n} s_{n+2k-1}^{j-1}, \end{aligned} \tag{6}$$

and the inverse discrete transform can be written as:

$$\begin{aligned} s_{2n}^{j-1} &= \sum_{k=1}^{N} c_{2k-1} s_{n-k+1}^{j} + \sum_{k=1}^{N} (-1)^{2k-1} c_{2N-2k} d_{n-k+1}^{j}, \\ s_{2n-1}^{j-1} &= \sum_{k=1}^{N} c_{2k-2} s_{n-k+1}^{j} + \sum_{k=1}^{N} (-1)^{2k-1} c_{2N-2k+1} d_{n-k+1}^{j}. \end{aligned} \tag{7}$$

This DWT process is shown in Figure 2.

III. Compression and Reconstruction Using Wavelets

Signal compression method using the wavelet families is similar to the denoising noisy data. In the denoising problem we choose a particular threshold value (e.g., two standard deviation), then set to zero all coefficients that are less than the threshold. [6] The signal compression also uses the thresholding method. If we fix a threshold, set to zero all wavelet-transformed signal less than that, and do inverse transform, we can get the reconstruct signal within some error bound.

IV. Numerical Tests and Conclusions

For doing signal compression and reconstruction, we use four wavelet families, 'Haar', 'Daubechies-4', 'Coiflet-3', 'Symmlet-10' and 256 nuclear cross section data provided by Foster Jr. and Glasgow. [7] For various wavelets, s^j's are plotted in Figure 3. From Figure 3, we note that although the number of s^j's is decreased, the shape of signal is preserved. Figure 4 shows the reconstructed signal (that is, nuclear data in this case) and Table II shows the Signal to Noise Ratio (SNR) for each threshold values, 0.01, 0.1, and 1.0. SNR is defined by

$$y_i(t) = u_i(t) + n_i(t),$$
$$SNR = 10 \times \log_{10}\left(\frac{E[|u_i(t)|^2]}{E[|n_i(t)|^2]}\right) [\text{dB}], \qquad (8)$$

where, y_i's are measured signal, u_i's are original signal, and n_i's are noise. Table III shows the number of data that are set to zero during compression.

In conclusion, the compression of signal using wavelets can reduce the data storage within some error bound. If we choose a more proper wavelet family depending on the signal, we can get smaller error. So wavelet compression and reconstruction could be a good way to record nuclear data or power plant operating data in smaller data storage space. This will also facilitate the transmission of the data. Currently, we are also applying the method to the transient measurement data of a nuclear power plant.

References

1. I. Daubechies, "Orthonormal Bases of Compactly Supported Wavelets", *Comm. Pure. Appl. Math.*, **41**, 909-996, 1988
2. G. Strang, "Wavelets and Dilation Equations : A Brief Introduction", *SIAM Review*, **31**, 614-627, 1989
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6. D. L. Donoho, "De-Noising by Soft Thresholding", *IEEE Trans. Info. Thry.*, **41**, 613-627, 1995
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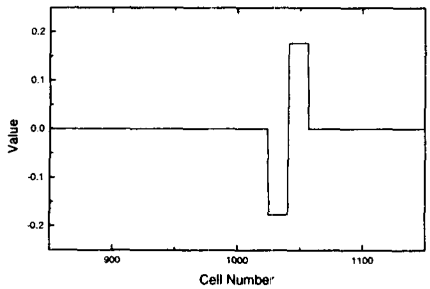
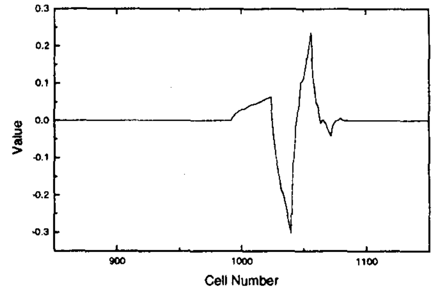
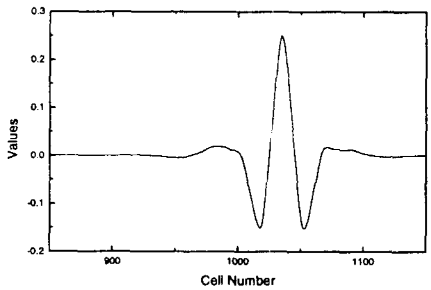


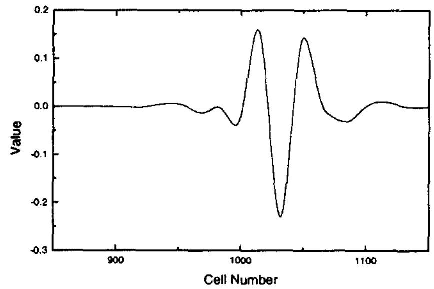
(a) Haar



(b) Daubechies-4



(c) Coiflet-3



(d) Symmlet-10

Figure 1. Several different families of wavelets

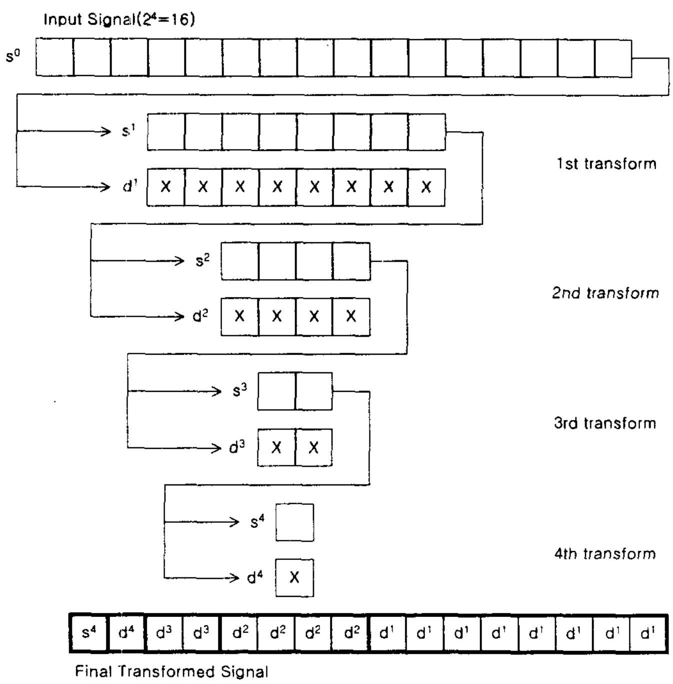
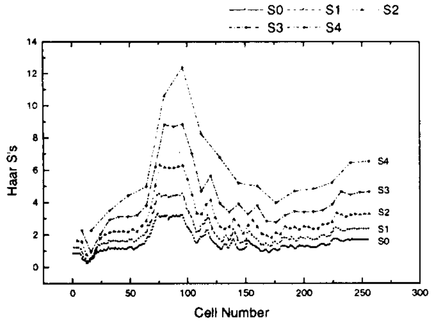
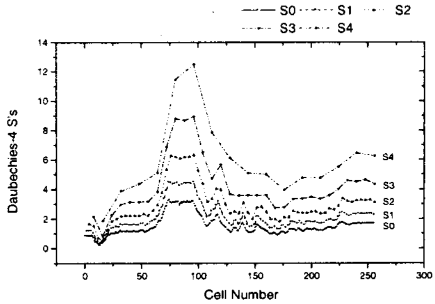


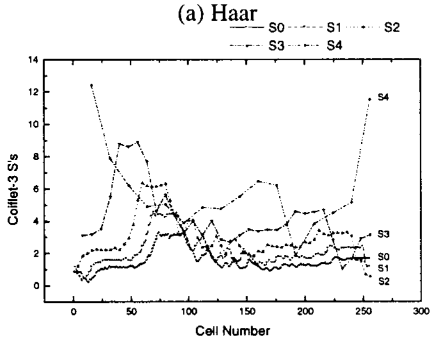
Figure 2. Diagram of discrete wavelet transform for 16 data



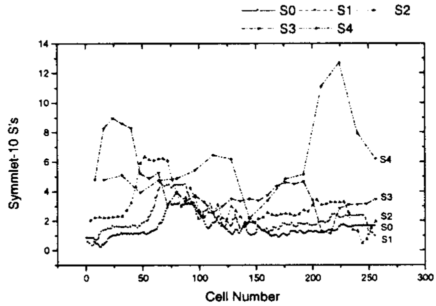
(a) Haar



(b) Daubechies-4

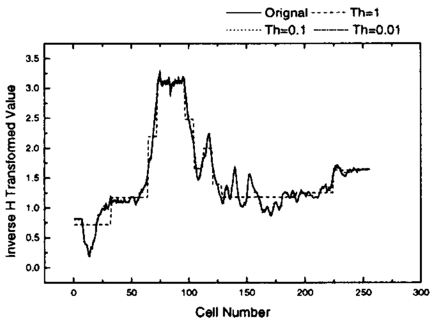


(c) Coiflet-3

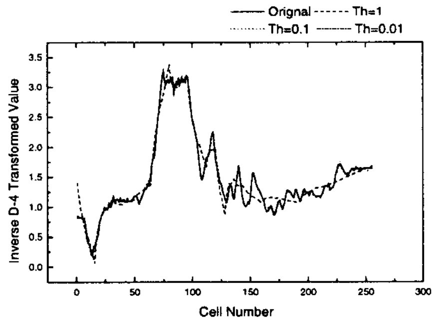


(d) Symmlet-10

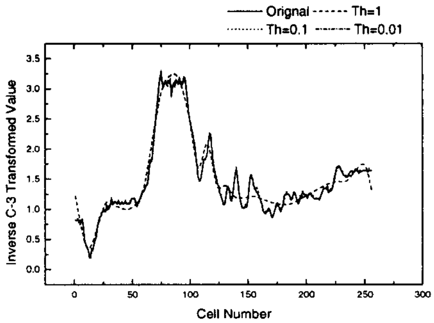
Figure 3. s^j's of each wavelet



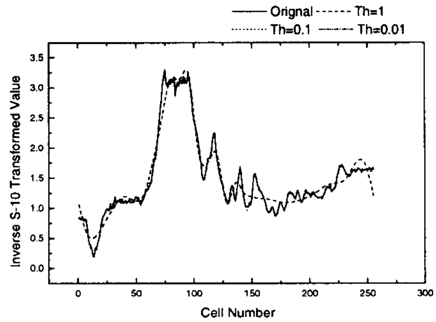
(a) Haar



(b) Daubechies-4



(c) Coiflet-3



(d) Symmlet-10

Figure 4. Reconstructed nuclear data by each wavelet

Table I. Coefficients for several wavelet families

Coefficients	Haar	Daubechies-4	Coiflet-3	Symmlet-10
c_0	1.0	0.6830126974	-0.0053648374	0.0010891704
c_1	1.0	1.1830126966	0.0110062534	0.0001352450
c_2	-	0.3169873009	0.0331671208	-0.0122206427
c_3	-	-0.1830126983	-0.0930155286	-0.0020723639
c_4	-	-	-0.0864415271	0.0649509243
c_5	-	-	0.5730066671	0.0164188696
c_6	-	-	1.1225705100	-0.2255589739
c_7	-	-	0.6059671487	-0.1002402153
c_8	-	-	-0.1015402820	0.6670713428
c_9	-	-	-0.1163925014	1.0882515350
c_{10}	-	-	0.0488681892	0.5428130096
c_{11}	-	-	0.0224584821	-0.0502565397
c_{12}	-	-	-0.0127392020	-0.0452407725
c_{13}	-	-	-0.0036409178	0.0707035675
c_{14}	-	-	0.0015804102	0.0081528167
c_{15}	-	-	0.0006593304	-0.0287862322
c_{16}	-	-	-0.0001003855	-0.0011375353
c_{17}	-	-	-0.0000489315	0.0064957284
c_{18}	-	-	-	0.0000806612
c_{19}	-	-	-	-0.0006495899

Table II. SNR of each wavelet [dB]

Threshold	Haar	Daubechies-4	Coiflet-3	Symmlet-10
0.01	60.2651	57.4903	55.5732	56.1650
0.10	32.6986	33.1643	33.7586	34.6747
1.00	19.2814	20.6087	20.7201	20.3920

Table III. Number (portion) of Data set to zero

Threshold	Haar	Daubechies-4	Coiflet-3	Symmlet-10
0.01	26(10.15%*)	39(15.23%)	52(20.31%)	49(19.14%)
0.10	163(63.67%)	186(72.66%)	186(72.66%)	187(73.05%)
1.00	235(91.80%)	241(94.14%)	244(95.31%)	243(94.92%)

* Portion is given by (Number of data to be zero) / (Number of total data) * 100 (%)