Proceeding of the Korean Nuclear Society Autumn Meeting
Taegu, Korea, October 1997

Axial Shape Index Calculation for the 3-Level Excore Detector

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Abstract

A new method based on the alternating conditional expectation (ACE) algorithm is developed to calculate axial shape index (ASI) for the 3-level excore detector. The ACE algorithm, a type of non-parametric regression algorithms, yields an optimal relationship between a dependent variable and multiple independent variables. In this study, the simple correlation between ASI and excore detector signals is developed using the Younggwang nuclear power plant unit 3 (YGN-3) data without any preprocessing on the relationships between independent variables and dependent variable. The numerical results show that simple correlations exist between the three excore signals and ASI of the core. The accuracy of the new method is much better than those of the current CPC and COLSS algorithms.

I. Introduction

Excore detector plays important roles in safe operation of the nuclear power plant. In PWR, safety-grade excore detectors mounted in the reactor cavity region monitor the core conditions including power level, power distribution. The detector signals are used in calculating the safety-related parameters and accurate evaluation of such parameters is very important for both safety and high performance of the nuclear power plant. Axial shape index (ASI), which is defined as the fractional power difference between the upper-half and the lower-half of the core, is an important information that the safety-grade excore detector should generate.

There are two typical types of excore detectors used in nuclear power plants, one is 2-level (top and bottom) and the other one is 3-level (top, middle, and bottom). For the 2-level excore detector, it is relatively easy to calculate the ASI information using the excore signals. This is because the top detector mainly responds to power of upper-half core and the bottom detector signal is highly related to power of the lower-half core. However, in the case of the 3-level detector, which is used in CE-type nuclear power plants, it is not easy to find an accurate relationship between detector signals and ASI of the core. This is due to the fact that both upper-half and lower-half power contribute to the signal of the middle detector.

In CPC (Core Protection Calculator), which is the core protection system of CE-type plants, the safety-related parameters such as DNBR, LPD (Local Power Density), ASI etc. are calculated using the excore detector signals[1]. In current method, axial power distribution is obtained for 20 axial nodes through a power synthesis algorithm. Then ASI is calculated using the resulting axial power distribution. The accuracy of ASI calculated in the current CPC, is fairly poor. Poor quality of ASI is one of the factors that limit flexible operation of the reactor core. The objective of the present work is to develop a methodology based on the ACE algorithm to calculate accurate ASI using three signals of the 3-level excore detector.

The ACE method is a generalized regression algorithm that yields an optimal relationship between a dependent variable y and multiple independent variables $\{ x_n , n = 1,\cdots, p \}$. The objective of the ACE algorithm is to find optimal transformations $\theta(y)$ and $\{ \phi_n(x_n),\ n = 1,\cdots, p \}$ which maximize the statistical correlation between $\theta(y)$ and $\sum \phi_n(x_n)$ without a priori estimate of the functional forms

$\theta(y)$ and $\phi_n(x_n)$. Once the optimal transformations are obtained, simple regression analysis is performed to determine the functional forms for the transformed dependent and independent variables.

II. The ACE Algorithm

The ACE algorithm was formally derived [2] through a functional analysis approach. Instead of repeating the formal derivation, we begin with a physical justification of the ACE algorithm, which serves as a heuristic derivation of the basic algorithm. The approach we take is based on a physical interpretation of the conditional expectation for a set of discrete data points[3, 4]. We use a bivariate formulation to illustrate the concept and make the necessary extension to multivariate regression problems.

For a bivariate regression problem with a set of N experimental data points $\{ (x_i, y_i),\ i = 1,\cdots, N \}$, we wish to find a transformation $\theta(y)$ of the dependent variable y and a functional fit $\phi(x)$ such that the square error in the regression of $\theta(y_i)$ and $\phi(x_i)$

$$e^2 = \frac{1}{N}\sum_{i=1}^{N}[\theta(y_i) - \phi(x_i)]^2 \tag{1}$$

is minimized. We assume that the optimal transformations, $\theta(y)$ and $\phi(x)$ exist. And we also assume, without loss of generality, that the optimal transformations, $\theta(y)$ and $\phi(x)$, minimizing Eq. (1) are properly normalized such that $E[\theta(y)] = E[\phi(x)] = 0$ and $E^2[\theta(y)] = 1$.

With a judicious selection of the transformation $\theta(y)$, the error in Eq. (1) could vanish, in principle, if $\theta(y_i)$ equals $\phi(x_i)$ for every point. In practice, however, this idealized situation will not materialize because the experimental data contain random noises and so do $\theta(y_i)$ and $\phi(x_i)$. Thus, a smooth functional representation $\theta(y)$ cannot be equated exactly to $\phi(x)$ at every data point. Instead, $\theta(y_i)$ is considered, in the ACE algorithm, the expectation of several realizations of $\phi(x)$ for the i^{th} point, rather than a single unique realization $\phi(x_i)$ as in conventional regression analysis. Thus, we interpret $\theta(y_i)$ as a conditional expectation $E[\phi(x)|y = y_i]$ to minimize Eq. (1). In most regression problems, in practice, there is usually only one value y_i, and hence one value $\phi(x_i)$, for the i^{th} data point, and the conditional expectation $\theta(y_i)$ has to be evaluated with the neighboring values $\{ \phi(x_j),\ j = i - M,\cdots, i + M \}$, for some M. In the simplest approach, $\theta(y_i)$ could be determined as an arithmetic average of the neighboring data. In general, some kind of weighted average over the neighboring data may be taken as the conditional expectation.

With this smoothing concept, the transformation $\theta(y)$ at the i^{th} point is obtained as $S[\phi(x)|y = y_i]$ instead of exact conditional expectation. We may equivalently consider $\phi(x_j)$ as the conditional expectation $E[\theta(y)|x = x_j]$ = $S[\theta(y)|x = x_j]$. Therefore the optimal transformations, $\theta(y)$ and $\phi(x)$, may be defined as

$$\theta(y) = \frac{S[\phi(x)|y]}{\|S[\phi(x)|y]\|}, \quad \phi(x) = S[\theta(y)|x]. \tag{2}$$

In practical problem, the smoothing operator, $S[\theta|y]$ can be defined as weighted sum of θ for a given interval of y. The ACE algorithm consists of an iterative use of the two smoothing operations of Eq. (2) in alternating directions.

Based on the bivariate derivation of the ACE algorithm, we can generalize Eq. (2) for a multivariate problem, given a set of experimental data $\{ (y_i, x_{1i},\cdots, x_{pi}),\ i = 1,\cdots, N \}$:

$$\theta(y) = \frac{S\left(\sum \phi_n(x_n)|y\right)}{\left\|S\left(\sum \phi_n(x_n)|y\right)\right\|}, \quad \phi_n(x_n) = S\left(\theta(y) - \sum_{l \neq n}^{p} \phi_l(x_l)|y\right) \tag{3}$$

The optimal transformations, θ and $\phi_1, \cdots, \phi_p$ cannot be obtained directly because they are coupled to each other through Eqs. (3). Thus, the ACE algorithm requires the following iterative procedures:

1. *Initialization.* $\theta^0(y) = y / \|y\|$ and $\phi_1^0(x_1) = \cdots \phi_p^0(x_p) = 0$.
2. *Inner iteration.* Sort $\theta(y)$ and $\{ \phi_l(x_l),\ l = 1, \cdots, p$ and $l \neq n \}$ in an ascending order of $\phi_n(x_n)$ and evaluate $\phi_n(x_n)$ for iteration step t using the second equation of Eq. (3). And then iterate until squared error fails to decrease.
3. *Outer iteration.* Sort $\{ \phi_n(x_n),\ n = 1, \cdots, p \}$ in an ascending order of $\theta(y)$ and calculate $\theta(y)$ using the first equation of Eq. (3). Continue with step 2 until squared error between $\theta(y)$ and $\phi_n(x_n)$ fails to decrease.

When convergence is attained, the data in each transformed variable are usually smooth and slowly varying. Selecting simple functional forms for the transformations, we perform standard regression analysis for each transformation and finally obtain the functional form of y versus $x_1, \cdots, x_p$ if $\theta(y)$ has an inverse function.

III. Application of ACE Algorithm to 3-Level Excore Detector Signals

We now apply the ACE algorithm to 3-level ex-core detector signals to get a correlation between those signals and ASI values. In YGN unit 3, 8 channels of instrumentation are furnished for ex-core neutron flux detection, 2 for startup, 2 for control, and 4 for safety. In current CPC algorithm, the axial power distribution is calculated through a power synthesis method and the ASI value is calculated using the resulted axial power distribution. The power synthesis method is based on a simple correlation between excore detector signals and core power distribution. The fitting coefficients are determined using a set of measured data, which are collected during startup of the core cycle. Table 1 shows the excore detector signals and the corresponding ASI values for the second cycle of YGN unit 3.

Table I. Reference data for ASI and 3-level ex-core detector signals

d_1	d_2	d_3	ASI	d_1	d_2	d_3	ASI
0.3660	0.3903	0.2437	-0.1607	0.3554	0.3915	0.2530	-0.1179
0.3653	0.3903	0.2444	-0.1570	0.3551	0.3914	0.2539	-0.1167
0.3645	0.3905	0.2451	-0.1541	0.3548	0.3914	0.2538	-0.1148
0.3635	0.3907	0.2459	-0.1509	0.3543	0.3917	0.2540	-0.1131
0.3630	0.3907	0.2463	-0.1481	0.3544	0.3916	0.2541	-0.1132
0.3624	0.3908	0.2468	-0.1460	0.3539	0.3914	0.2547	-0.1110
0.3616	0.3911	0.2473	-0.1435	0.3534	0.3914	0.2552	-0.1085
0.3599	0.3912	0.2489	-0.1366	0.3517	0.3913	0.2570	-0.1023
0.3594	0.3915	0.2491	-0.1350	0.3504	0.3915	0.2581	-0.0966
0.3594	0.3912	0.2494	-0.1346	0.3492	0.3918	0.2591	-0.0925
0.3586	0.3914	0.2500	-0.1307	0.3493	0.3916	0.2591	-0.0922
0.3577	0.3916	0.2507	-0.1281	0.3476	0.3918	0.2606	-0.0858
0.3564	0.3917	0.2519	-0.1228	0.3472	0.3918	0.2609	-0.0842
0.3566	0.3916	0.2519	-0.1231	0.3462	0.3919	0.2618	-0.0803
0.3562	0.3915	0.2524	-0.1209				

In Table I, d_1, d_2, and d_3 mean that average ex-core detector signals for top, middle, and bottom detectors, respectively and the values of them are normalized so that the sum of d_1, d_2, and d_3 equals to 1. From the data of Table I, we run the ACE algorithm based on following relations:

$$(x_1, x_2, x_3, y) = (d_1 - d_3, d_2, d_3, ASI) \tag{4}$$

In Eq. (4), we use the difference between the values of upper detector and of bottom detector as the first variable because the basic relationship between this difference and ASI is well known.

Using the CG-ACE code[4], we achieved the converged transformations, with a convergence criterion of 1.0×10^{-6} both for inner and outer iterations and with a windowing factor of 0.5, after 4 outer iterations. We present the transformations, $\phi_n(x_n)$ versus x_n and $\theta(y)$ versus y, obtained through the ACE algorithm in Fig. 1. It is clear from Fig. 1 that the ACE algorithm generates simple functional forms both for the dependent and independent variables. The squared error defined in Eq. (1) is 1.363×10^{-4} for this application. As shown in the figure, we can get clear relationship between $x_1(=d_1-d_3)$ and $y(=ASI)$. Using the plots of Fig. 1 and a standard regression tool, we can obtain simple analytic functions, $\theta(y)$ and $\{\phi_n(x_n),\ n=1,\cdots,3\}$. For example, $\theta(y)$ and $\phi_1(x_1)$ can be represented as first order linear function of x_1 and y, respectively. For the fitting of the other functions in Fig. 1, we used piecewise linear functions. In (b) of Fig. 1, there are some fluctuation in the range of 0.391 and 0.392 in x_2. It is considered as small noise because the scale of ϕ_2 is so small compared to that of ϕ_1.

After individual analytic functions of $\theta(y)$ and $\{\phi_n(x_n),\ n=1,\cdots,3\}$ are obtained, we can get the final form of a new ASI correlation through a simple inversion process and a few manipulations as followings:

$$ASI=9.829\times10^{-2}+A_1+A_2-2.114(d_1-d_3)+A_3d_2+A_4d_3, \quad (5)$$

where the coefficients, A_1 through A_4 in Eq. (5) are given as

$$(A_1,A_3)=\begin{cases}0.583,\ -1.492\ for\ d_2\le0.391\\ 0.0,\ 0.0\ else\end{cases} \qquad (A_2,A_4)=\begin{cases}-0.0584,\ 0.236\ for\ d_3\le0.248\\ 0,\ 0\ for\ d_3\le0.257\\ -0.0277,\ 0.108\ else\end{cases}$$

In order to test the accuracy of the new ASI correlation represented by Eq. (5), we have performed simulation of the ASI data which are sampled during normal power operation of the cycle 2 of YGN unit 3. The data are summarized in Table II.

Table II. Simulation data set for the cycle 2 of YGN unit 3

Burnup (GWD/MTU)	d_1	d_2	d_3	ASI_{ref}	ASI_{CPC}	ASI_{COLSS}
11.240	0.335	0.391	0.274	-0.0205	-0.0336	-0.0242
12.010	0.332	0.390	0.278	-0.0172	-0.0195	-0.0127
13.560	0.331	0.389	0.280	-0.0136	-0.0065	-0.0293
14.330	0.329	0.389	0.282	-0.0033	0.0000	-0.0029
14.590	0.329	0.388	0.283	-0.0212	0.0050	0.0096
15.550	0.330	0.388	0.282	-0.0006	-0.0049	0.0142
16.910	0.329	0.388	0.283	0.0070	-0.0091	-0.0059
17.420	0.329	0.386	0.285	0.0120	-0.0047	0.0115
17.940	0.330	0.385	0.285	0.0007	0.0114	-0.0252
17.940	0.331	0.385	0.284	0.0144	0.0120	0.0010
18.190	0.331	0.385	0.284	-0.0315	0.0105	0.0101
18.460	0.332	0.385	0.283	-0.0035	-0.0109	0.0165
18.460	0.332	0.385	0.284	0.0117	-0.0103	-0.0057
18.710	0.332	0.383	0.285	0.0179	-0.0141	0.0050
18.960	0.332	0.384	0.285	-0.0109	0.0066	-0.0380
19.220	0.333	0.384	0.283	0.0067	0.0049	-0.0018
19.490	0.333	0.383	0.284	-0.0381	0.0050	0.0115
20.220	0.335	0.383	0.282	0.0000	-0.0132	0.0184
20.220	0.333	0.381	0.285	0.0064	-0.0153	-0.0165

In Table II, ASI_{ref}, is the reference ASI obtained using the measured 3-dimensional power and ASI_{CPC}, ASI_{COLSS} mean that ASI values calculated by CPC and COLSS[5], respectively and we used averaged ASI_{CPC} values for 4 channels of CPC values. Comparing Tables I and II, we can see that they are very different data sets. In Table I, the range of ASI is -0.1607 ~ -0.0803. The range of ASI_{ref} in Table II, however, is -0.0381 ~ 0.0144. There are two difficulties to predict ASI_{ref}: one is that ASI_{ref} in Table II has so small values, and second is that the range of ASI in Table I and ASI_{ref} in Table II are different. It means that correlations developed using the data in Table I should extrapolate to predict ASI_{ref} in Table

II.

We calculated ASI using Eq. (5) for the data given in Table II and the prediction results are summarized in Fig. 2(a). For the data in Table II, the average, rms, and maximum errors are 14.41%, 42.38%, and 148.04%, respectively. However, the meaning of relative error in this application is not so much and it might be misunderstanding in some sense because the absolute values of ASI are very small. Rather, the absolute differences between ASI_{ref} and ASI calculated by Eq. (5) are more important to check the accuracy of Eq. (5). The average, rms, and maximum differences of Eq. (5) are -0.000699, 0.0026, and -0.0065, respectively.

Now we compared the accuracy of Eq. (5) to the results of CPC and COLSS. Fig. 2(b) show the comparison results. As shown in the figure, ASI_{cal} calculated by Eq. (5) is much closer to ASI_{ref} than ASI_{CPC} and ASI_{COLSS}. CPC shows the largest error and it become larger as going to end-of-cycle. In case of COLSS, it shows somewhat larger error than Eq. (5). However, we have to remember that COLSS calculates the ASI value using in-core detectors not ex-core detectors.

IV. Summary and Conclusions

In this work, the ACE algorithm is applied to finding the new correlation between signals of the 3-level excore detector and ASI of the reactor core. The ACE algorithm is a generalized regression algorithm that yields an optimal relationship between a dependent variable and multiple independent variables without any preprocessing on the relationships between independent variables and dependent variable. We devleoped ASI correlation, Eq. (5) using ACE algorithm and it has been tested for the operation data of the cycle 2 of YGN unit 3. The average, rms, and maximum differences of Eq. (5) are -0.000699, 0.0026, and -0.0065, respectively. We compared Eq. (5) to CPC and COLSS algorithms and the prediction accuracy of Eq. (5) is much better than those of CPC and COLSS.

Based on previous works and this study, we conclude that ACE algorithm can be used to make correlations for various engineering problems which are available experimental data such as critical heat flux problem, ASI problem and so on.

Acknowledgment

The authors thank J. I. Kim in division of core management for YGN unit 3 for allowing access to the surveillance data of YGN-3 cycle 2.

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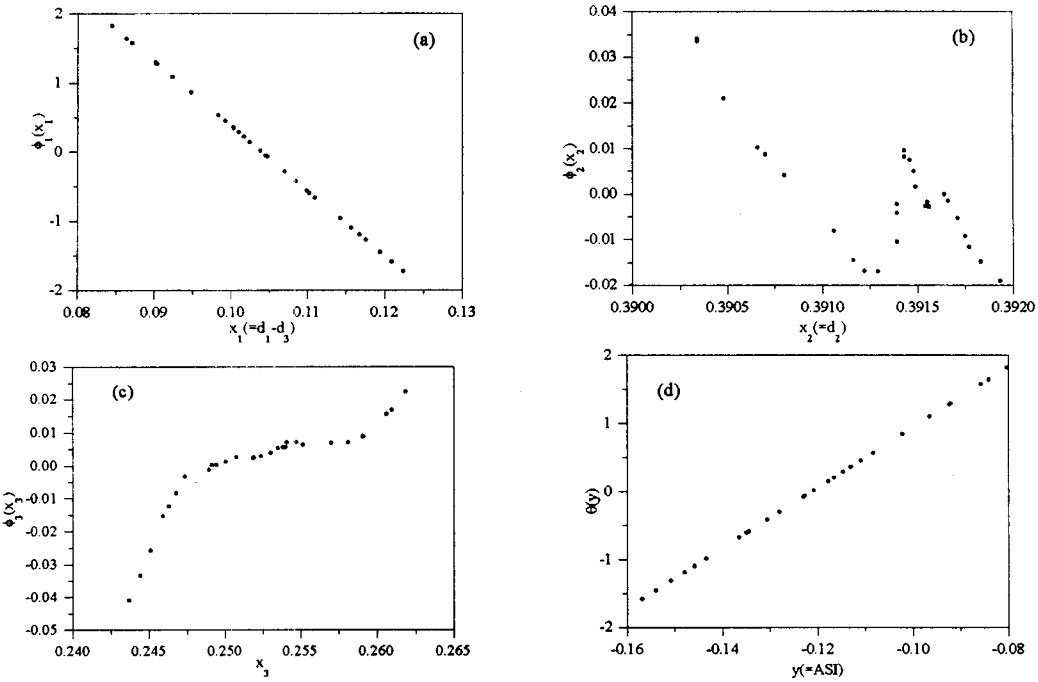
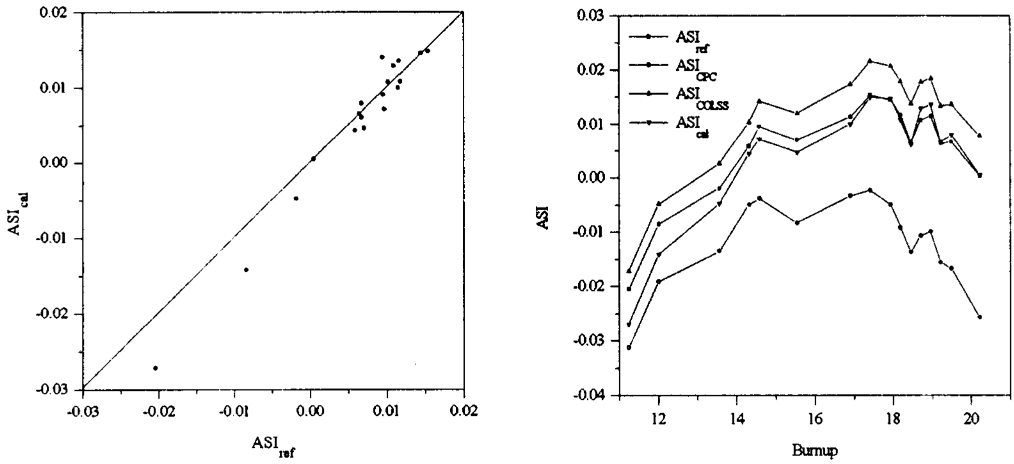


Fig. 1 The results of the ACE algorithm for ASI data



(a) Accuracy of Eq. (5) **(b) Eq. (5) vs CPC and COLSS**

Fig. 2 The comparison results of the prediction accuracy