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Interface Matrix Method in AFEN Framework

Leonid Pogosbekyan, Jin-Young Cho, Young-Jin Kim,
Jae-Man Noh, and Hyung-Kook Joo

Korea Atomic Energy Research Institute

Abstract

In this study, we extend the application of the interface-matrix(IM) method for reflector modeling to Analytic Flux Expansion Nodal (AFEN) method. This include the modifications of the surface-averaged net current continuity and the net leakage balance conditions for IM method in accordance with AFEN fomular. AFEN-interface matrix (AFEN-IM) method has been tested against ZION-1 benchmark problem. The numerical result of AFEN-IM method shows 1.24 % of maximum error and 0.42 % of root-mean square error in assembly power distribution, and 0.006 %$\triangle k$ of neutron multiplication factor. This result proves that the interface-matrix method for reflector modeling can be useful in AFEN method.

1. Introduction

The power distribution in a large pressurized water reactor is significantly affected by the radial reflector which requires adequate modeling of radial reflector to accurately simulate core power distributions. The implicit reflector modeling employs albedo type boundary conditions, while the explicit reflector method treats reflector nodes as calculational nodes in core calculation explicitly. The principal drawback of implicit reflector representation is that the interaction between fuel assemblies and reflector cannot be described easily. The problems in explicit reflector modeling are to generate the equivalent homogenized constants for reflector and to increase the computing time.

Recently, the interface matrix technique for reflector modeling has been developed and has achieved a success in applying to NEM framework[1]. This method employs interface matrix at the core-reflector interface without homogenizing the baffle and water reflector. In this study, we tried to extend the application of the interface matrix method to AFEN method[2], which has been developed to overcome limitation of transverse integration and successfully applied in core calculation. This include the modifications of the surface-averaged net current continuity and the net leakage balance conditions for interface matrix method in accordance with AFEN formula.

2. AFEN-Interface Matrix Method for the Surface Average Flux

In AFEN, the surface-averaged net current at the right surface of node i-1 and left surface of node i can be derived as the following form:

$$\vec{J}^{u}_{i-1,1} = +\,TT2^{u}_{i-1}F_{0}^{ui-1}\vec{\phi}^{ui-1} - TT3^{u}_{i-1}F_{1}^{ui-1}\vec{\phi}^{ui} + PL^{u}_{i}, \tag{1}$$

$$\vec{J}^{u}_{i,0} = -\,TT2^{u}_{i}F_{1}^{ui}\vec{\phi}^{ui+1} + TT3^{u}_{i}F_{0}^{ui}\vec{\phi}^{ui} - PR^{u}_{i}, \tag{2}$$

where F_{1}^{ui-1} and F_{0}^{ui} are the surface flux discontinuity factors at left and right side of the interface, respectively. The surface-averaged net current continuity condition at the interface between the nodes i-1, i is,

$$\vec{J}^{u}_{i-1,1} = \vec{J}^{u}_{i,0}. \tag{3}$$

Therefore the final formula for the net current continuity condition for the surface average flux is written as,

$$TL^{u}_{i}\vec{\phi}^{ui-1} + TC^{u}_{i}\vec{\phi}^{ui} + TR^{u}_{i}\vec{\phi}^{ui+1} = PL^{u}_{i} + PR^{u}_{i}. \tag{4}$$

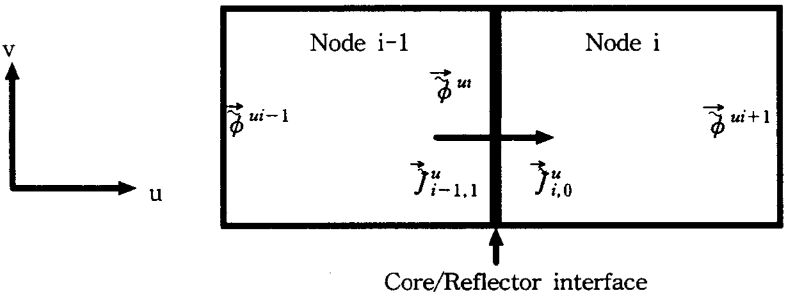

Figure 1. Stencil for surface condition.

In the interface matrix technique, the following relations are supposed at the cell interface,

$$\vec{\phi}^{-ui} = R^{ui}_{11}\vec{\phi}^{+ui} + R^{ui}_{12}\vec{J}^{u}_{i,0}, \tag{5}$$

$$\vec{J}^{u}_{i-1,1} = R^{ui}_{21}\vec{\phi}^{+ui} + R^{ui}_{22}\vec{J}^{u}_{i,0}, \tag{6}$$

$$\vec{\phi}^{-ui} \equiv F_{1}^{ui-1}\vec{\phi}^{ui}, \tag{7a}$$

$$\vec{\phi}^{+ui} \equiv F_{0}^{ui}\vec{\phi}^{ui}. \tag{7b}$$

where, $\vec{\phi}^{-ui}$ and $\vec{\phi}^{+ui}$ denote the fluxes at the left (superscript "-") and right (superscript "+") side of interface. R^{ui}_{11}, R^{ui}_{12}, R^{ui}_{21}, R^{ui}_{22} are the elements of interface matrix, which relates currents and fluxes at core-reflector interface. Eqs. (5) and (6) can replace the standard net current continuity Eq. (3) in AFEN formulation.

In this point, one should note that the number of equation is two, Eqs. (5) and (6), at the interface, but the number of unknown to be defined in the standard AFEN method is only one which is surface-averaged flux. So, we should define another unknown parameter in addition to the surface averaged flux. In this study, we set the discontinuity factor F_{0}^{ui} as an another unknown quantity. In the following procedure, we will derive the equation for the surface flux at the interface. In this equation, the discontinuity factor defined as an additional unknown is excluded and the nodal unknowns are related. In this connection, two cases are considered from the difference of interface position where the interface matrix technique is assigned.

In the case that the interface matrix technique is assigned to the interface between the nodes i-1 and i, one can derive the following equation by manipulating the Eqs. (1), (2), (5), (6), and (7):

$$A_i^u \vec{\phi}^{ui-1} + B_i^u \vec{\phi}^{ui} + C_i^u \vec{\phi}^{ui+1} = G_i^u PL_i^u + PR_i^u \tag{8}$$

where

$$G_i^u = TT3_i^u R_{12}^{ui} + R_{22}^{ui},$$

$$A_i^u = -G_i^u TT2_{i-1}^u F_0^{ui-1},$$

$$B_i^u = (G_i^u TT3_{i-1}^u + TT3_i^u R_{11}^{ui} + R_{21}^{ui}) F_1^{ui-1},$$

$$C_i^u = -TT2_i^u F_1^{ui}.$$

And in the case that the interface matrix technique is assigned to the interface between the nodes i-2 and i-1, one can derive also the following equation:

$$A_i^u \vec{\phi}^{ui-1} + B_i^u \vec{\phi}^{ui} + C_i^u \vec{\phi}^{ui+1} = -G1_{i-1}^u PR_{i-1}^u + PL_i^u + PR_i^u \tag{9}$$

where

$$A_i^u = AA_{i-1}^u F_1^{ui-2},$$

$$AA_{i-1}^u = -TT2_{i-1}^u GG_{i-1}^u,$$

$$G1_{i-1}^u = AA_{i-1}^u R_{12}^{ui-1},$$

$$B_i^u = (G1_{i-1}^u TT2_{i-1}^u + TT3_{i-1}^u) F_1^{ui-1} + TT3_i^u F_0^{ui}, \quad C_i^u = -TT2_i^u F_1^{ui}.$$

Eqs. (8) and (9) are used for the calculation of the surface average fluxes at the core-reflector interface and reflector-reflector interface.

3. AFEN-Interface Matrix Method for the Edge Flux

In the standard AFEN method, the net leakage balance condition for the edge flux is written as

$$(\vec{J}_{u00}^u - \vec{J}_{w10}^u) + (\vec{J}_{u01}^u - \vec{J}_{w11}^u) + (\vec{J}_{u00}^u - \vec{J}_{u01}^u) + (\vec{J}_{w10}^u - \vec{J}_{w11}^u) = 0. \tag{10}$$

For the sake of definiteness, consider the case of L-shaped interface. From the interface matrix technique, the following conditions are supposed at the edge:

$$\vec{J}_{u01}^v = R_{21}[F^c]^{ij} \vec{\phi}_{ij} + R_{22} \vec{J}_{u00}^v, \tag{11a}$$

$$\vec{J}_{w10}^u = R_{21}[F^c]^{ij} \vec{\phi}_{ij} + R_{22} \vec{J}_{u00}^u, \tag{11b}$$

$$[F^c]^{ij-1} \vec{\phi}_{ij} = R_{11}[F^c]^{ij} \vec{\phi}_{ij} + R_{12} \vec{J}_{u00}^v, \tag{12a}$$

$$[F^c]^{i-1,j} \vec{\phi}_{ij} = R_{11}[F^c]^{ij} \vec{\phi}_{ij} + R_{12} \vec{J}_{u00}^u. \tag{12b}$$

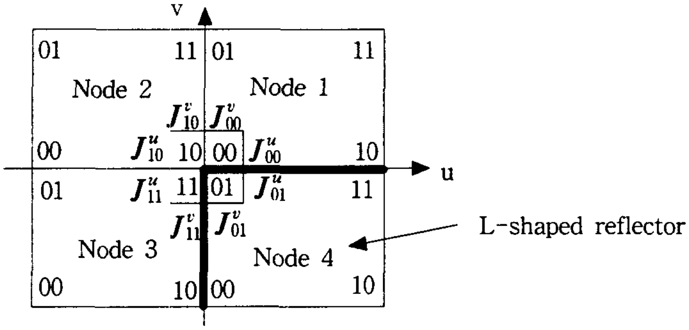

Figure 2. Stencil for edge condition.

Eqs. (11) can be used in the net leakage balance condition of Eq. (10), but Eqs. (12) give another condition which the nodal unknowns must satisfy. In this study, instead of two conditions of Eqs. (12), we will introduce a new condition which does not violate the original condition of Eqs. (12). By adding the two conditions of Eqs. (12), we can get the following equation:

$$([\boldsymbol{F}^c]^{i-1,j}+[\boldsymbol{F}^c]^{i,j-1})\vec{\phi}_{ij}=2\boldsymbol{R}_{11}[\boldsymbol{F}^c]^{i,j}\vec{\phi}_{ij}+\boldsymbol{R}_{12}\boldsymbol{L}_{00}. \tag{13}$$

And we employ $[\boldsymbol{F}^c]^{i,j}$ as an additional degree of freedom, while the discontinuity factors $[\boldsymbol{F}^c]^{i-1,j-1}$, $[\boldsymbol{F}^c]^{i-1,j}$, $[\boldsymbol{F}^c]^{i,j-1}$ have the commonly used sense, or edge flux discontinuity factors.

Introducing Eqs. (11) into Eq. (10) gives

$$\begin{aligned}&(\boldsymbol{F}_{21}[\boldsymbol{F}^c]^{ij}\vec{\phi}_{ij}+\boldsymbol{R}_{22}\vec{J}^v_{u00}-\vec{J}^v_{u01})+(\vec{J}^u_{u01}-\vec{J}^u_{w11})\\&\quad+(\boldsymbol{R}_{21}[\boldsymbol{F}^c]^{ij}\vec{\phi}_{ij}+\boldsymbol{R}_{22}\boldsymbol{J}^u_{u00}-\boldsymbol{J}^u_{w10})+(\boldsymbol{J}^v_{w10}-\boldsymbol{J}^v_{w11})=0.\end{aligned} \tag{14}$$

By using the net leakage expressions, Eq. (14) can be rewritten as,

$$\boldsymbol{R}_{22}\boldsymbol{L}_{00}+2\boldsymbol{R}_{21}[\boldsymbol{F}^c]^{ij}\vec{\phi}_{ij}+\boldsymbol{L}_{01}+\boldsymbol{L}_{10}+\boldsymbol{L}_{11}=0, \tag{15}$$

where

$$\begin{aligned}\boldsymbol{L}_{00}&=\vec{J}^u_{u00}+\vec{J}^v_{u00},\\ \boldsymbol{L}_{10}&=-\vec{J}^u_{w10}+\vec{J}^v_{w10},\\ \boldsymbol{L}_{01}&=\vec{J}^u_{u01}-\vec{J}^v_{u01},\\ \boldsymbol{L}_{11}&=-\vec{J}^u_{w11}-\vec{J}^v_{w11}.\end{aligned}$$

and substituting equations of the standard AFEN method for $\boldsymbol{L}_{00}$, $\boldsymbol{L}_{01}$, $\boldsymbol{L}_{10}$, $\boldsymbol{L}_{11}$ into Eq. (15), one can derive the net leakage balance equation at the L-shaped reflector edge.

$$\boldsymbol{TI}^L_{ij}\vec{\phi}_{i-1j}+\boldsymbol{TI}^R_{ij}\vec{\phi}_{i+1j}+\boldsymbol{TI}^C_{ij}\vec{\phi}_{ij}+\boldsymbol{TI}^T_{ij}\vec{\phi}_{ij-1}+\boldsymbol{TI}^B_{ij}\vec{\phi}_{ij+1}=\vec{qi}_{ij}. \tag{16}$$

One can find that the edge flux discontinuity factor, $[\boldsymbol{F}^c]^{i,j}$, which is defined as an additional degree of freedom, is included in the edge flux coefficient of Eq. (16), $\boldsymbol{TI}^C_{ij}$. But using Eq. (13), we can transform Eq. (16) to the linear form.

4. Numerical Results and Conclusion

The verification of AFEN-IM has been tested against ZION-1 benchmark problem. One node per assembly scheme is used in AFEN-IM method. Additionally, the AFEN calculation with one-dimensionally homogenized reflector parameters was performed. The reference solution in this results was from the 4x4-node-per-assembly nodal calculation with explicit baffle representation.[3]

Figure 3 shows the assemblywise power distribution and multiplication factor for the both AFEN calculations (AFEN-IM and AFEN with 1-dimensional reflector homogenization). The result of AFEN-IM method shows 1.24 % of maximum error and 0.42 % of root mean square error in assembly power distribution and 0.006 %$\triangle$k in multiplication factor, while AFEN with the 1-dimensional reflector homogenization shows 1.90 % and 0.56 % in maximum and root mean square errors in assembly power distribution and 0.006 %$\triangle$k in multiplication factor.

Figure 4 presents the errors in edge and surface-averaged power distribution. The maximum power errors are 6.67 % and 5.76% in AFEN-IM method and AFEN with the 1-dimensional reflector homogenization, respectively.

From the results presented at the Figures 3 and 4, we see that the interface-matrix method for reflector modeling can be useful in AFEN method.

Reference

1. L. Pogosbekyan, et. al. " A New Model for Homogenized Reflectors Based on Interface Matrix Technique," Joint International Conference on Mathematical Methods and Supercomputing for Nuclear Applications, Saratoga, NY (1997).
2. J. M. Noh and N. Z. Cho, "A New Approach of Analytic Function Expansion to Neutron Diffusion Nodal Equation," Nucl. Sci. Eng., v. 116, p. 165 (1994).
3. K. S. Smith, "An Analytic Nodal Method for Solving the Two-Group, Multidimensional, Static and Transient Neutron Diffusion Equations," Nuclear Engineering Thesis, Massachusetts Institute of Technology (1979).

	8	9	10	11	12	13	14	15
H	1.631 0.40 -0.20	1.777 0.35 -0.23	1.535 0.31 -0.22	1.565 0.29 -0.17	1.253 0.21 -0.13	1.164 0.10 -0.13	0.797 -0.03 -0.06	0.505 -0.40 -0.08
I		1.583 0.37 -0.19	1.672 0.33 -0.18	1.395 0.29 -0.12	1.365 0.18 -0.12	1.033 0.13 -0.03	0.918 -0.07 -0.09	0.490 -0.47 -0.35
K			1.446 0.30 -0.14	1.478 0.20 -0.12	1.181 0.13 -0.02	1.081 0.07 0.10	0.719 -0.11 0.08	0.439 -0.70 -0.35
L	k-eff = 1.27483[1] k-eff = 1.27495[2]			1.243 0.16 0.01	1.217 0.04 0.14	0.894 -0.10 0.37	0.719 -0.43 0.19	0.316 -1.14 -1.24
M	Ref. k-eff = 1.27489				1.078 -0.19 0.31	0.849 -0.42 0.71	0.527 -1.23 1.09	
N	max=1.90% RMS=0.56% max=1.24% RMS=0.42%					0.664 -0.94 0.75	0.321 -1.90 0.93	

[1] AFEN, conventional 1D homogenization technique.

[2] AFEN - Interface Matrix method.

Reference: quoted from K.Smith M.S. thesis (NEM 4x4 nodes/assembly, explicit baffle).

Figure 3. Assemblywise power distribution errors on ZION-1 benchmark problem

0.52	0.47	0.49	0.44	0.45	0.38	0.38	0.30	0.28	0.17	0.13	-0.05	-0.21	-0.60	-4.78
-0.08	-0.11	-0.07	-0.10	-0.05	-0.07	-0.04	-0.06	-0.02	-0.08	-0.04	-0.11	-0.11	-0.47	-3.36
		0.45		0.42		0.33		0.26		0.01		-0.17		-4.80
		-0.10		-0.08		-0.06		-0.02		-0.12		-0.17		-2.17
		0.47	0.42	0.42	0.35	0.35	0.26	0.23	0.13	0.09	-0.09	-0.29	-0.76	-4.96
		-0.07	-0.08	-0.05	-0.07	0.00	-0.02	0.02	-0.03	0.01	0.15	-0.31	-0.72	-1.71
				0.35		0.30		0.12		0.04		-0.41		-5.44
				-0.07		0.02		-0.02		0.08		-0.38		-2.18
						0.25	0.16	0.11	0.02	0.03	-0.16	-0.32	-1.18	-6.18
						0.05	0.04	0.09	0.11	0.22	0.07	-0.17	-0.72	0.32
						0.14		0.03		-0.20		-0.59		-6.55
						0.04		0.18		0.29		-0.03		-1.58
								-0.10	-0.24	-0.38	-0.78	-0.73	2.86	-3.03
								0.23	0.35	0.59	0.64	-2.27	-3.95	-5.76
								-0.25		-0.75		-0.92		
								0.31		0.54		1.04		
										-1.04	-1.95	-6.66		
										0.59	0.51	0.16		
										-1.35		-6.67		
										0.94		0.13		
										-2.28	0.66	-2.96		
										0.56	-0.08	1.07		

Conventional 1D homogenization technique
AFEN-IM method

Figure 4. Surface and edge power distribution errors on ZION-1 benchmark problem